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Modified Model of Pressure Gradient Prediction of Vertical Multiphase Flow Based on the Residual Model

Dong Yong[1], Liao Ruiquan[2,3,4], Li Mengxia[1,2,3,4] and Luo Wei[2,3,4]

[1]School of Information and Mathematics, Yangtze University, Jingzhou Hubei 434023, China;
[2]Petroleum Engineering College, Yangtze University, Wuhan Hubei 430100, China;
[3]The Branch of Key Laboratory of CNPC for Oil and Gas Production, Yangtze University, Wuhan Hubei 430100, China;
[4]Key Laboratory of Exploration Technologies for Oil and Gas Resources, Yangtze University, Wuhan Hubei 430100, China;
dongyong80@126.com

Abstract

Aiming at the problem of the large error between the pressure gradient of multiphase flow predicted by the model presented in literature [12] (denoted by Liao model) and the experimental pressure gradient, this paper firstly compares the error between the Liao model and the experimental data, and builds a correlation model (denoted by CM model) between error of the Liao model and gas liquid ratio according to the close relations between the two. And then combining the CM model and the Liao model, a new predicting method of pressure gradient which is denoted by LCM model is built, and the algorithm is given. Finally, this paper gives the numerical experiment results for ninety groups of experimental data. The numerical results show that the average relative error of LCM model is about 3.3% and the one of Liao model is about 38%. Based on the known data with 30% water content and with 90% water content, this paper uses LCM model to calculate the pressure gradient under the case of 60% water content and compares the LCM gradient with the experimental pressure gradient under the same condition. The average relative error of LCM model is 10.44%, but the one of Liao model is 41.54%. The results shows that the LCM model proposed in this paper improves the predicting precision of pressure gradient for multiphase flowing in a certain range.

***Keywords**: multiphase flow; flow pattern; pressure gradient; relative error; model*

1. Introduction

The estimation of pressure gradient of multiphase flow is the important theoretical basis in designing and analyzing the production well. It is also important for cost effective design of production optimization and surface facilities. Multiphase flow is the process of simultaneous flow of two phases or more. In oil or gas production wells the multiphase flow usually consists of oil, gas and water. Some empirical correlations [1-3] are available for predicting pressure gradient during the process of multiphase flow in the wellbore. However, most of the correlations are not reliability because of some reasons that the correlations are empirical and the calculation procedures are also very complicated. Recently several mechanistic modelling models [4-12] which are attempt to model the flow system and then verify the model based on the actual data have been applied to understand and analyze the process of multiphase flow.

When multiphase flow occurs, the phase will have various configurations which are called flow patterns. There are many factors which influence the flow patterns and the

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influence factors are also very complex. So the description of flow pattern is somewhat subjective. In this paper, we will consider the four flow patterns in vertical wellbores which are listed in standard textbooks as bubbly, slug, churn and annular. In the wellbore, different flow patterns will exist at different depth. The Liao model[12] is proposed based on the previous research results. Comparing with the eight common models which are Hagedorn-Brown model [2], Orkiszewski model [3], Aziz model [4], Beggs-Brill model [5], Chierici model [6], Beggs-Brill modified model [7], Mukherjee-Brill model [8] and Hasan model [9], the Liao model [12] has higher precision. But the calculating results based on the experimental data presented in this paper shows that the average relative error of pressure gradient between the Liao model and the experimental data exceeds 40%. So it is necessary to further research the Liao model and build the modified computational model.

Based on the above discussion, we firstly calculate the pressure gradient which is called Liao gradient according to the conditions from the experiment by using the Liao model. We denote the difference value between the Liao gradient and experimental pressure gradient as Liao error, we analyze the relations between the Liao error and experimental conditions. And find that the Liao error and gas liquid ratio have close relations. Hence, we firstly build the modified model of Liao error with respect to GLR (that is gas liquid ratio). And then combining the error predictive model (CM model) and Liao model, we obtain a new predictive model of pressure gradient which is denoted by LCM model. The pressure gradient calculated by LCM model is more in line with the experimental pressure gradient. We also use the new model to deal with the in-situ test data and the results show that the new model has superiority.

2. Results and Analysis of Liao Model Based on the Experimental Data

The experimental data which are used in this paper are described as follows. The pipe is vertical with diameter 75 mm. The roughness of the pipe wall is 0.0002 mm. The experimental mediums are atmosphere, mineral white oil and water. We experimented under three different water contents which are 30%, 60% and 90%. The liquid flow rate is between 10 m3/d and 50 m3/d. The test temperature is between 14 ℃ and 17 ℃. The experimental pressure gradient is between 5.16 kPa and 28.77 kPa. The total number of test points is 90. We use Liao model to calculate the pressure gradient and compare the Liao gradient with the experimental pressure gradient. The comparison results are shown in Figure 1.

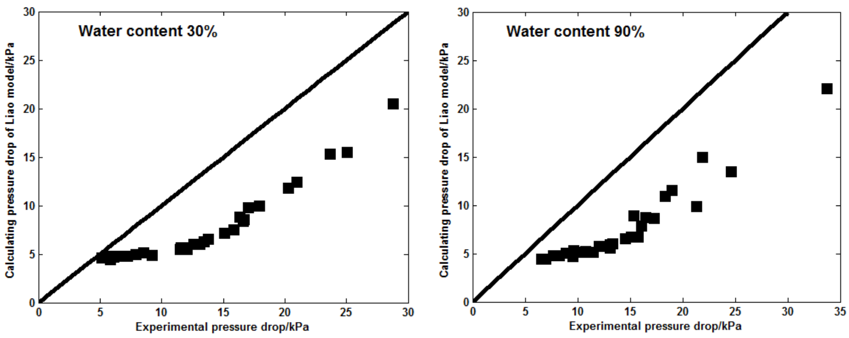

Figure 1: The Comparison Results between the Liao Gradient and the Experimental Pressure Gradient for Different Water Contents

From Figure 1, it can be seen that the calculating pressure gradient is less than the experimental pressure gradient as a whole. And as the water content increases, the Liao error increases. The average absolute error are 5.73 kPa, 5.85 kPa and 6.99 kPa respectively. The average relative error are 40.64%, 41.54% and 48.11%. For the experimental data presented in this paper, the Liao error is a little large. So we consider modifying the original Liao model.

In order to modify Liao model, it is necessary to further analyze the regularity of the Liao error. The Liao predictive error is compared with the gas flow rate, liquid flow rate and experimental pressure gradient respectively. The comparison results are shown in Figure 2.

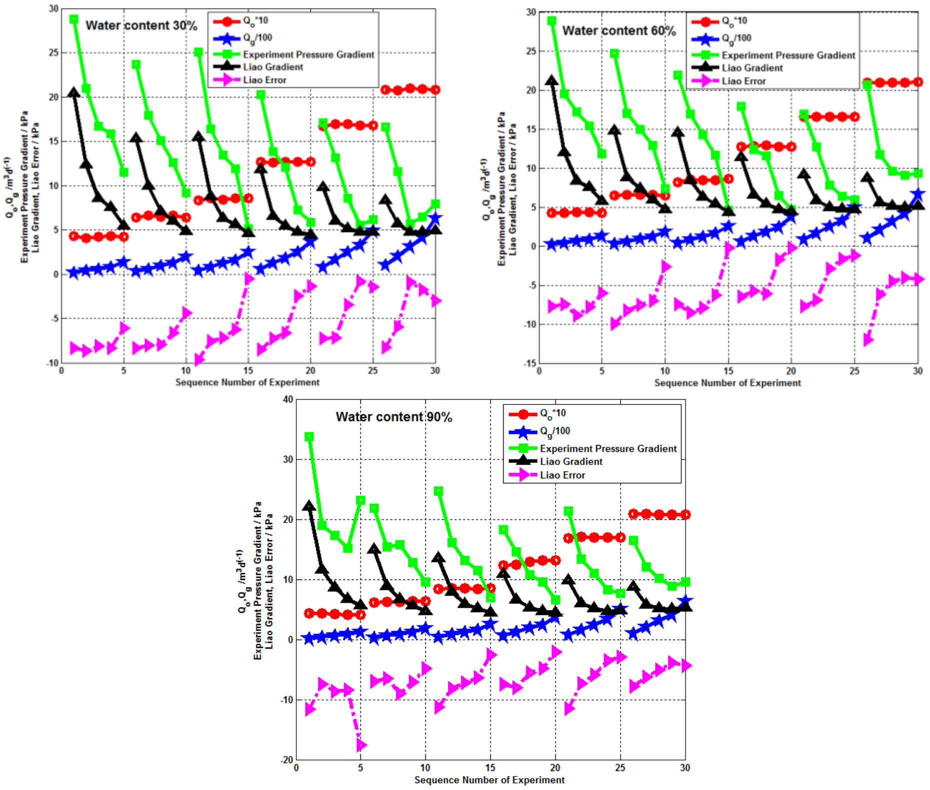

Figure 2. Comparison between Liao Error and other Parameters for Different Water Contents

In Figure 2, the horizontal ordinate denotes the sequence number. The sequence numbers from 1 to 5 corresponds the design flow 10 m^3/d. The sequence numbers from 6 to 10 corresponds the design flow 15 m^3/d. The sequence numbers from 11 to 15 corresponds the design flow 20 m^3/d. The sequence numbers from 16 to 20 corresponds the design flow 30 m^3/d. The sequence numbers from 21 to 25 corresponds the design flow 40 m^3/d. The sequence numbers from 26 to 30 corresponds the design flow 50 m^3/d. The corresponding GLR of sequence numbers 1, 6, 11, 16, 21 and 26 is 50. The corresponding GLR of sequence numbers 2, 7, 12, 17, 22 and 27 is 100. The corresponding GLR of sequence numbers 4, 9, 14, 19, 24 and 29 is 200. The corresponding GLR of sequence numbers 5, 10, 15, 20, 25 and 30 is 300. In order to compare the change regularity of variables conveniently, the liquid flow rate is multiplied by 10, and gas flow rate is multiplied by 0.01.

From Figure 2, it can be seen that Liao error have some similarity with respect to the change regularity of GLR for different water contents when the design flow is the same. The curves of Liao error corresponding to different liquid flow rates have a greater difference in geometric shapes for the same water content. But all curves have the characteristics of quadratic curve.

Based on the above analysis, we can use the quadratic function to model the Liao error.

3. Modified Model of Liao Error (CM model)

The experimental data which are used in this paper are described as follows. The pipe is vertical with diameter 75 mm. The roughness of the pipe wall is 0.0002 mm. The experimental mediums are atmosphere, mineral white oil and water. We experimented under three different water contents which are 30%, 60% and 90%. The liquid flow rate is between 10 m^3/d and 50 m^3/d. The test temperature is between 14 ℃ and 17 ℃. The experimental pressure gradient is between 5.16 kPa and 28.77 kPa. The total number of test points is 90. We use Liao model to calculate the pressure gradient and compare the Liao gradient with the experimental pressure gradient. The comparison results are shown in Figure 1.

According to the above analysis, we use the quadratic function to fit the Liao error. We call this model CM model. CM model is expressed in Eq. 1:

$$y = ax^2 + bx + c \tag{1}$$

In Eq. 1, the horizontal ordinate x denotes the GLR, and the values are 50, 100, 150, 200 and 300. The vertical ordinate y denotes the modified Liao error, that is the value of CM model. The coefficients a, b and c are parameters which are needed to be estimated.

The modified Liao model called LCM model is expressed in Eq. 2:

$$\left(\frac{dp}{dl}\right)_{LCM} = \left(\frac{dp}{dl}\right)_{Liao} - y, \tag{2}$$

where $\left(\frac{dp}{dl}\right)_{LCM}$ denotes the LCM gradient, and $\left(\frac{dp}{dl}\right)_{Liao}$ denotes the Liao gradient. And the relative error of LCM predictive gradient is

$$E_r = \frac{\left(\frac{dp}{dl}\right)_{LCM} - \left(\frac{dp}{dl}\right)_{\exp eriment}}{\left(\frac{dp}{dl}\right)_{\exp eriment}}, \tag{3}$$

where $\left(\frac{dp}{dl}\right)_{\exp eriment}$ denotes the experimental pressure gradient.

Firstly, we need to determine the coefficients a, b and c. For the given water content and liquid flow rate, the ordered pairs (x_i, y_i), $i = 1, 2, \cdots, 5$ are substituted into Eq. 1 and the least square method is used to determine the coefficients a, b and c. If the water content is 60% and the liquid flow rate is 20 m^3/d, the CM model determined by the above method is

$$y = 0.0002x^2 - 0.0469x - 5.7833 \quad . \tag{4}$$

The calculating value of the CM model and the Liao error value under the given conditions are shown in Figure 3.

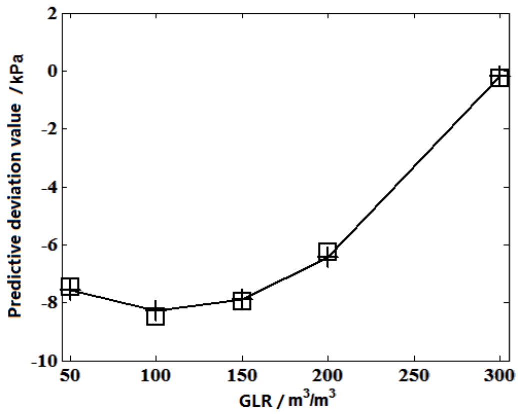

Figure 3. The Results of CM Model and Liao Error For 60% Water Content and Liquid Flow Rate 20 M^3/D

In Figure 3, block symbols show the Liao error at the given GLR. The graph of formula (4) has been drawn above. Cross symbols show the calculating values of CM model at the given GLR.

From Figure 3, it can be seen that the geometrical morphology of the curve of each modified Liao error is different for different water content and liquid flow rate. Hence the undetermined coefficients in Eq. 1 is different for different water content and liquid flow rate. We must fit the experimental pressure gradient values to determine the coefficients a , b and c .

Table 1 gives the values of undetermined coefficients a , b and c for different water content and liquid flow rate.

Table 1. Coefficients of CM Model for Different Water Content and Liquid Flow Rate in Vertical Pipe with Diameter 75 Mm

Experimental condition	Liquid flow rate	undetermined coefficients
water content: 30%	10 m^3/d	$(a,b,c)=(0.0001,-0.0160,-7.6502)$
	15 m^3/d	$(a,b,c)=(0.0001,-0.0066,-8.1251)$
	20 m^3/d	$(a,b,c)=(0.0001,-0.0007,-9.3376)$
	30 m^3/d	$(a,b,c)=(-0.0000,0.0391,-10.7253)$
	40m^3/d	$(a,b,c)=(-0.0001,0.0737,-11.5492)$
	50 m^3/d	$(a,b,c)=(-0.0003,0.1177,-13.7995)$
water content: 60%	10m^3/d	$(a,b,c)=(0.0001,-0.0258,-6.4518)$
	15 m^3/d	$(a,b,c)=(0.0001,0.0072,-10.1079)$
	20 m^3/d	$(a,b,c)=(0.0002,-0.0469,-5.7833)$
	30 m^3/d	$(a,b,c)=(0.0000,0.0140,-7.5769)$
	40m^3/d	$(a,b,c)=(-0.0001,0.0759,-11.8189)$
	50 m^3/d	$(a,b,c)=(-0.0003,0.1212,-16.7714)$
water content: 90%	10m^3/d	$(a,b,c)=(-0.0004,0.1211,-16.3511)$
	15 m^3/d	$(a,b,c)=(0.0001,-0.0393,-4.9377)$
	20 m^3/d	$(a,b,c)=(-0.0000,0.0373,-12.5685)$
	30 m^3/d	$(a,b,c)=(0.0000,0.0096,-8.3906)$
	40m^3/d	$(a,b,c)=(-0.0002,0.0939,-15.5911)$
	50 m^3/d	$(a,b,c)=(-0.0001,0.0524,-10.2728)$

4. Algorithm of LCM Model

In the LCM model, Liao model is quoted completely. For different incline angles, different water contents, different liquid flow rates and different GLR, the model can be used. In the above section, we give the CM model under the experimental conditions. But we need to further clear that how to determine the modified value of error for any given water content, liquid flow rate and GLR based on the CM model under the known experimental conditions. This paper uses the linear interpolation method to solve this problem. The pressure gradient calculated by the linear interpolation method is called LCM gradient under non-experimental conditions.

The notations are described as follows. The upper limit and lower limit of the water content in the experiment are denoted by F_{WU} and F_{WD}, respectively. The upper limit and lower limit of the liquid flow rate are denoted by Q_{LU} and Q_{LD}, respectively. The upper limit and lower limit of gas liquid ratio are denoted by GLR_U and GLR_D, respectively. Given that the water content is F_W, the liquid flow rate is Q_L and the gas liquid rate is GLR, we predict the pressure gradient by using LCM model.

Step 1 If $F_{WD} \le F_W \le F_{WU}$, $Q_{LD} \le Q_L \le Q_{LU}$ and $GLR_D \le GLR \le GLR_U$, then do Step 2. Otherwise, terminate procedure.

Step 2 Choose two water contents which are adjacent to the given water content F_W from the experimental data showed in Table. 1. Denote the two water contents by F_{W1} and F_{W2}, respectively. They satisfy $F_{W1} \le F_W \le F_{W2}$. Choose two liquid flow rates which are adjacent to the given water content Q_L from the experimental data. Denote the two liquid flow rates by Q_{L1} and Q_{L2}, respectively. They satisfy $Q_{L1} \le Q_L \le Q_{L2}$. From Table. 1, four modified CM models can been determined. The CM model corresponding to F_{W1} and Q_{L1} is denoted by f_{11}. The CM model corresponding to F_{W1} and Q_{L2} is denoted by f_{12}. The CM model corresponding to F_{W2} and Q_{L1} is denoted by f_{21}. The CM model corresponding to F_{W2} and Q_{L2} is denoted by f_{22}.

Step 3 According to f_{11} and f_{12}, use the linear interpolation method to determine the CM model f_1 given that water content is F_{W1} and liquid flow rate is Q_L. It shows in Eq. 5.

$$f_1 = \frac{Q_L - Q_{L2}}{Q_{L1} - Q_{L2}} f_{11} + \frac{Q_L - Q_{L1}}{Q_{L2} - Q_{L1}} f_{12} \tag{5}$$

Analogously, CM model f_2 is determined in Eq. 6.

$$f_2 = \frac{Q_L - Q_{L2}}{Q_{L1} - Q_{L2}} f_{21} + \frac{Q_L - Q_{L1}}{Q_{L2} - Q_{L1}} f_{22} \tag{6}$$

Step 4 According to f_1 and f_2, use the linear interpolation method to determine CM model f given that water content is F_W and liquid flow rate is Q_L. It shows in Eq. 7.

$$f = \frac{F_W - F_{W2}}{F_{W1} - F_{W2}} f_1 + \frac{F_W - F_{W1}}{F_{W2} - F_{W1}} f_2 \tag{7}$$

Step 5 Obtain the CM model value f_0 for the water content F_W, liquid flow rate Q_L and gas liquid ratio GLR by substituting the gas liquid ratio GLR into Eq. 7. The LCM gradient is expressed in Eq. 8.

$$\left(\frac{dp}{dl}\right)_{LCM} = \left(\frac{dp}{dl}\right)_{Liao} - f_0$$

(8)

5. Numerical Results and Analysis of LCM Model

In this section, we verify the LCM model by numerical experiment. Firstly, we analyze the prediction accuracy of LCM model based on the experimental conditions. Secondly, we calculate the pressure gradient with 60% water content by using LCM model, and compare the LCM gradient with the experimental gradient.

We use the vertical pipe with diameter 75 mm in the experiment. In the first process, we use Eq. 2 to calculate the relative error. The results are shown in Table 2.

From Table 2, it can be seen that nine relative errors exceed 10%. The maximum relative error is 23.37%. The average relative errors are 5.13%, 4.22% and 2.87% for the three known water contents. The average relative error of ninety groups of data is 4.07%. Relatively, the average relative errors of Liao model are 40.64%, 41.54% and 48.11% for the three known water contents. Obviously, LCM model is superior to Liao model.

For the second verification process, we calculate the error as the water content is 60% by using the linear interpolation method described in the previous section. And then we calculate LCM gradient according to Eq. 8. Finally, we use Eq. 3 to calculate the relative error. The comparison results of LCM model and Liao model are shown in Table 3.

Table 2. The Results of LCM Model

Experimental conditions	Liquid flow rate	The predictive relative error
water content: 30%	10m^3/d	*error* =(0.0011, 0.0036, 0.0219, 0.0199, 0.0050)
	15 m^3/d	*error* =(0.0005, 0.0077, 0.0204, 0.0170, 0.0037)
	20 m^3/d	*error* =(0.0194, 0.0537, 0.0065, 0.0567, 0.0375)
	30 m^3/d	*error* =(0.0174, 0.0143, 0.1024, 0.1895, 0.0492)
	40m^3/d	*error* =(0.0527, 0.1282, 0.0017, 0.2036, 0.0554)
	50 m^3/d	*error* =(0.0216, 0.1015, 0.2356, 0.0870, 0.0043)
water content: 60%	10m^3/d	*error* =(0.0079, 0.0332, 0.0336, 0.0095, 0.0012)
	15 m^3/d	*error* =(0.0142, 0.0344, 0.0133, 0.0455, 0.0212)
	20 m^3/d	*error* =(0.0053, 0.0120, 0.0029, 0.0151, 0.0106)
	30 m^3/d	*error* =(0.0162, 0.0023, 0.1324, 0.2337, 0.0645)
	40m^3/d	*error* =(0.0358, 0.1054, 0.0707, 0.0533, 0.0266)
	50 m^3/d	*error* =(0.0304, 0.0958, 0.0134, 0.0976, 0.0268)
water content: 90%	10m^3/d	*error* =(0.0094, 0.0524, 0.0618, 0.0267, 0.0006)
	15 m^3/d	*error* =(0.0159, 0.0700, 0.0718, 0.0321, 0.0011)
	20 m^3/d	*error* =(0.0182, 0.0492, 0.0104, 0.0577, 0.0266)
	30 m^3/d	*error* =(0.0217, 0.0654, 0.0513, 0.0086, 0.0122)
	40m^3/d	*error* =(0.0092, 0.0426, 0.0483, 0.0186, 0.0009)
	50 m^3/d	*error* =(0.0075, 0.0135, 0.0182, 0.0331, 0.0072)

Table 3. The Comparison Results of Liao Gradient and LCM Gradient under Non-Experimental Conditions

Experimental conditions	Liquid flow rate	Gas liquid ratio	Liao relative error	LCM relative error
water content: 60%	$10m^3/d$	50	0.268069	0.075776
		100	0.384154	0.024994
		150	0.514485	0.018170
		200	0.509583	0.006107
		300	0.511128	0.530267
	$15\ m^3/d$	50	0.402709	0.061434
		100	0.484257	0.073648
		150	0.50515	0.003567
		200	0.540135	0.039616
		300	0.362967	0.007330
	$20\ m^3/d$	50	0.340256	0.026706
		100	0.500935	0.048212
		150	0.558296	0.007719
		200	0.535605	0.068362
		300	0.053785	0.182287
	$30\ m^3/d$	50	0.362724	0.026722
		100	0.467135	0.014039
		150	0.532509	0.077931
		200	0.269792	0.259362
		300	0.051973	0.021857
	$40m^3/d$	50	0.45927	0.106270
		100	0.54279	0.047491
		150	0.367887	0.093743
		200	0.255525	0.357062
		300	0.202766	0.090726
	$50\ m^3/d$	50	0.580881	0.019418
		100	0.525889	0.051018
		150	0.467647	0.512477
		200	0.454449	0.169075
		300	0.448991	0.111910
The average relative error			0.415391	0.104443

From Table 3, it can be seen that the precision of LCM model for twenty-six groups of data are higher than Liao model. But the average relative error of LCM model is obvious higher than Liao model.

6. Conclusion

(1) The paper build a quadratic regression model of GLR and predictive deviation, and build a new pressure gradient predictive model by combine the quadratic regression model and predictive model of literature [12], denoted by LCM model.

(2) The LCM model can improve the prediction precision to about 10% when the given data is covered by the experimental conditions.

(3) The precision of LCM model can be expected to improve when the experimental parameters are encrypted, such as water content.

(4) The applicable scope of LCM model is limited by the scope of experimental data. Extending the scope of experimental data will extend the applicable scope of LCM model.

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References

[1] R. Dun and N. C. Ros, "Vertical Flow of Gas and Liquid Mixtures in Wells", Proceedings, Sixth World Pet. Congress, Frankfort, France, **(1963)**.
[2] A. R. Hagedorn and K. E. Brown, "Experimental Study of Pressure Gradients Occurring During Continuous Two-Phase Flow in Small-Diameter Vertical Conduits," Journal of Petroleum Technology, vol.17, no. 4, **(1965)**, pp. 475-484, DOI: 10.2118/940-PA.
[3] J. Orkiszewski, "Predicting Two-Phase Pressure Drops in Vertical Pipe", SPE-1546-PA, Journal of Petroleum Technology, vol.19, no.6, pp. 829-838, **(1967)**, http://dx.doi.org/10.2118/1546-PA.
[4] K. Aziz and G. W. Govier, "Pressure Drop In Wells Producing Oil And Gas", PETSOC-72-03-04, Journal of Canadian Petroleum Technology, vol.11, no.3, **(1972)**, pp. 326-337, http://dx.doi.org/10.2118/72-03-04.
[5] D.H. Beggs and J.P. Brill, "A Study of Two-Phase Flow in Inclined Pipes", SPE-4007-PA, vol.25, no.5, pp. 607-617, 1973. http://dx.doi.org/10.2118/4007-PA.
[6] G.L. Chierici, G.M. Ciucci, G. Sclocchi, "Two-Phase Vertical Flow in Oil Wells - Prediction of Pressure Drop," SPE-4316-PA, Journal of Petroleum Technology, Vol.26, No.8, 927-938, 1974. http://dx.doi.org/10.2118/4316-PA
[7] H.D. Beggs, J.P. Brill, E.A. Proaiio and C Roman-Lazo, "Pressure Drop And Closure Forces In Velocity Type Subsurface Safety", API-77-B001, Annual Meeting Papers, Division of Production, **(1977)**, Texas, Houston. http://dx.doi.org/10.2118/77-B001
[8] H. Mukherjee and K.E. Brown, "Improve Your Gas Lift Design", SPE-14053-MS, International Meeting on Petroleum Engineering, Beijing, China, **(1986)**, http://dx.doi.org/10.2118/14053-MS
[9] A. Rashid Hasan and C. Shah Kabir, "A Study of Multiphase Flow Behavior in Vertical Wells", SPE-15138-PA, vol.3, no.2, **(1988)**, pp. 263-272, http://dx.doi.org/10.2118/15138-PA
[10] A. Rashid Hasan and C. Shah Kabir, "Predicting Multiphase Flow Behavior in a Deviated Well", SPE-15449-PA, vol.3, no.4, **(1988)**, pp. 474-482, http://dx.doi.org/10.2118/15449-PA.
[11] A. M. Ansari, N. D. Sylvester, C. Sarica, O. Shoham and J.P. Brill, "A Comprehensive Mechanistic Model for Upward Two-Phase Flow in Wellbores", Society of Petroleum Engineers, vol.9, no.02, **(1994)**, pp. 143-151, http://dx.doi.org/10.2118/20630-PA.
[12] R. Liao, Q. Wang and B. Zhang, "A Method for Predicting Vertical Multiphase-flow Pressure Gradient in Vertical Wellbore", Journal of Jianghan Petroleum Institute, vol.20, no.1, **(1998)**, pp. 59-63.

Authors

Dong Yong. he received his M.Sc. in Computational Mathematics (2008) from Wuhan University of Technology and PhD in Geodetection and Information Technollogy (2013) from Yangtzeu University. Now he is lecturer of mathematics at School of Information and Mathematics, Yangtzeu University. Since 2014 he engaged in the postdoctoral work. His current research interests include different aspects of Artificial Intelligence and Well Logging Technology.

