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Robust H_∞ Control for Bilinear Systems Using the Dynamic Takagi-Sugeno Fuzzy Models Based on Linear Matrix Inequalities

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Abstract

The robust H_∞ control design for bilinear systems with multi inputs is presented in this paper. First, the bilinear system is represented as a dynamic Takagi-Sugeno (TS) fuzzy system by using sector nonlinearity approach. The dynamic TS fuzzy system is a convex combination of local linear systems. The local robust H_∞ controller is designed for each local linear system. The controller synthesis for the local linear systems is then formulated in the bilinear matrix inequalities (BMIs) problem. After that, the BMIs problem is reduced to an equivalent parameter of linear matrix inequalities (LMIs) problem which has a feasible solution. The robust H_∞ controller for bilinear systems as a convex combination of the local robust H_∞ controllers is obtained by using defuzzyfication. The existence condition of the robust H_∞ controller for the bilinear systems is also presented. The simulation results are given to clarify the proposed method for the robust H_∞ control design of the bilinear systems.

***Keywords**: robust H_∞ controller, bilinear systems, dynamic TS fuzzy system, local linear systems, linear matrix inequalities*

1. Introduction

The robust H_∞ controller for bilinear systems [1-3] with multi inputs is investigated in this paper. The bilinear system with uncertainties that involves the exogenous inputs is represented as a dynamic Takagi-Sugeno (TS) fuzzy system. These problems have been considered in references [4-5] because the dynamic TS fuzzy system can describe the bilinear term. A robust H_∞ fuzzy control for a class of bilinear systems has been discussed in [6-7] using the state feedback controller, but it is only for the bilinear systems with single control input. The paper investigates the robust H_∞ controller which has own dynamics for the bilinear system with multi inputs. In the TS fuzzy system approach, the robust H_∞ controller for the bilinear system with multi inputs is a convex combination of the local robust H_∞ controllers. Based on this approach, the bilinear system is represented as a convex combination of the local linear systems.

The novelty of this paper is the existence of the robust H_∞ controller for the bilinear systems with multi inputs that guarantees the closed loop system is asymptotically stable and has L_2-gain $\leq \gamma, \gamma > 0$. The robust H_∞ controller for the bilinear systems is designed on each subsystem called local linear systems of the bilinear systems. The robust H_∞-performance of the local linear systems is formulated as Bilinear Matrix Inequality (BMI). The BMI is represented as a Riccati inequality which can be used to characterize the behaviors of the local linear systems. By parameterization (change of variables), the BMI

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is converted into an equivalent set of Linear Matrix Inequalities (LMIs) which have feasible solutions. Furthermore, the solution of the set of LMIs is used to design the suboptimal solution for the robust H_∞ control design problem.

The bilinear system represents a simple model of nonlinear systems which is linear in inputs and states but it is not linear in both. The bilinear systems appear naturally in science and technology problems such as power systems [8], suspension systems [9], electrical circuits [10], quantum mechanics [11-12], paper making machines [2], immune systems [1] and biomedicine systems [1]. The bilinear system is usually a high order system, so that the reduced order model is an important part in control system design. There are many methods to reduce the order of the bilinear system [8,10,13]. The bilinear systems give a lot of theoretical knowledge because they form an intermediary class between the linear and the general nonlinear systems.

Theory and control design of the bilinear systems have been investigated by many researchers. The nonlinear state feedback H_∞ control of nonlinear system has been discussed by using an approach based on Hamilton-Jacobi equations and inequalities [14]. The nonlinear H_∞ control of the nonlinear systems is characterized in term of continuous positive definite solutions of algebraic nonlinear matrix inequalities [15]. The robust H_∞ control design for the bilinear systems is solved via algebraic H_∞ Riccati equations [16-17]. In [18], the H_∞ suboptimal control problem of nonlinear system with disturbance attenuation level $\gamma > 0$, is solved by the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equations or inequalities.

Recently, analysis and synthesis to design the controller of the linear and the nonlinear systems are formulated in LMIs problem [19-33]. To illustrate, the H_∞ optimal control design problems involve Riccati inequality which can be solved by basic manipulation on LMIs [19]. The LMI can be described as a convex optimization which can be solved by an efficient algorithm [20]. The solvability condition of regular and singular H_∞ control problems for the linear systems can be presented in LMIs [21]. The stabilization of the bilinear systems via linear state-feedback control for a certain domain of state space by using linear matrix inequalities (LMIs) has also been discussed in [22]. Moreover, the estimation of stability regions for the bilinear systems has been considered in [28-29].

The paper is organized as follow. The representation of the bilinear systems in the dynamic TS fuzzy systems and the definition of robust H_∞-performance are presented on Section 2.1. and Section 2.2.. Respectively, in Section 3, it is represented the existence of the robust H_∞ controller for the bilinear system which guarantees the closed loop system is asymptotically stable and has L_2-gain $\leq \gamma, \gamma > 0$. The other main result is the formulation of robust H_∞ control design for the bilinear systems in LMIs. Moreover, the algorithm to design the robust H_∞ controller of the bilinear systems is presented. Section 4 shows the simulation results which clarify the proposed methods and Section 5 gives the conclusions.

2. Representation for Bilinear Systems

In this section, the bilinear systems are represented in the dynamic TS fuzzy systems by nonlinearity approach. The definition of robust H_∞-performance for linear system is presented.

2.1. Representation of Bilinear Systems in Takagi-Sugeno Fuzzy Systems

Consider the bilinear system G with uncertainty is described as follows

$$G:\begin{aligned} \dot{x}(t) &= Ax(t) + B_1\widehat{w}(t) + B_2(x)u(t),\\ q(t) &= C_1x(t) + D_{11}\widehat{w}(t) + D_{12}u(t),\\ y(t) &= C_2x(t) + D_{21}\widehat{w}(t) + D_{22}u(t), \end{aligned} \tag{1}$$

where $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$ is the state vector, $\hat{w} \in \mathbb{R}^q$ is the exogenous inputs (uncertainty), $y \in \mathbb{R}^h$ is the measured outputs, $u \in \mathbb{R}^m$ is the control inputs, and $q \in \mathbb{R}^r$ is the controlled outputs. While $A, B_1, B_2(x), C_1, C_2, D_{11}, D_{12}, D_{21}$ are matrices with suitable dimensions, where element of B_2 is a linear function of x. The system is assumed to be strictly proper from u to y, $i.e.$ $D_{22} = 0$.

The robust H_∞ controller for the bilinear system G is difficult to obtain directly. Therefore, the bilinear system will be represented as the generalized TS fuzzy system. Consider the bilinear system (1). Denote

$$B_2(x) = \begin{bmatrix} f_{11}(x) & \cdots & f_{1m}(x) \\ \vdots & \ddots & \vdots \\ f_{n1}(x) & \cdots & f_{nm}(x) \end{bmatrix},$$

f_{ij} is a linear function of x for each $i = 1,2,\dots,n, j = 1,2,\dots,m, x \in \mathbb{R}^n$. Then, set a new variables as $z_1 = f_{11}(x), z_2 = f_{12}(x), \dots, z_p = f_{nm}(x), p = nm$. Furthermore, consider a polytope $\mathcal{P} \subset \mathbb{R}^n$ which is described as follows

$$\mathcal{P} = \text{conv}\{\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_k\} \tag{2}$$

where $k = 2^n$ is an integer number, $\tilde{x}_l$ denotes the l-th vertex of polytope $\mathcal{P}, l = 1,2,\dots,k$ and conv{} denotes the operation of taking the convex hull of arguments. For example, the box in $\mathbb{R}^2$ is given by

$$\mathcal{P} \coloneqq [-3,3] \times [-2,2].$$

It can be described as convex hull of the vertices as $\mathcal{P} = \text{conv}\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\}$ where $\tilde{x}_1 = (-3,-2)^T, \tilde{x}_2 = (-3,2)^T, \tilde{x}_3 = (3,-2)^T$ and $\tilde{x}_4 = (3,2)^T$.

The generalized bilinear system can be represented as the generalized TS fuzzy system by using the sector nonlinearity approach [5]. The TS fuzzy model consists of an if-then rule base. The partition of a subset of the new variables is carried out into fuzzy sets as the rule antecedents and the simple functional expression as the sequent of each rule. The form of the i-th rules are as follows

Model rule i:
If z_1 is Z_1^i and ... and z_p is Z_p^i then

$$\dot{x}(t) = Ax(t) + B_1\hat{w}(t) + B_{2i}u(t),$$
$$q(t) = C_1x(t) + D_{11}\hat{w}(t) + D_{12}u(t),$$
$$y(t) = C_2x(t) + D_{21}\hat{w}(t),$$

where $B_{2i} = B_{2i}(\tilde{x}_l)$, $\tilde{x}_l$ denotes the l-th vertex of polytope $\mathcal{P}, l = 1,2,\dots,k$, $z_j, j = 1,2,\dots,p$ represent the scheduling variables and $Z_j^i, i = 1,2,\dots,s$, are fuzzy sets, with the s is number of rules. The value of z_j belongs to Z_j^i with a truth value given by the membership function $w_{ij}(z_j): R \to [0,1]$. The fuzzy set Z_j^i can be either $\bar{Z}_j^0$ or $\bar{Z}_j^1$ [5]. Consequenly, the TS fuzzy rules consist of $s = 2^p$ rules. The scheduling variables are chosen as

$$z_j(.) \in [\underline{z}_j,\ \overline{z}_j], j = 1,2,\dots,p,$$

where $\underline{z}_j$ and $\overline{z}_j$ are the minimum and maximum of z_j, respectively. The scheduling variables are usually selected as a subset of the state, input, output or other exogenous variables in the system or functions of the states, inputs, outputs or other exogenous variables. The weighting functions can be constructed as

$$\beta_j^0(.) = \frac{\overline{z}_j - z_j(.)}{\overline{z}_j - \underline{z}_j},$$
$$\beta_j^1(.) = 1 - \beta_j^0(.),\ j = 1,2,\dots,p.$$

Moreover z_j can be presented as $z_j = \underline{z}_j\beta_j^0(z_j) + \overline{z}_j\beta_j^1(z_j)$. The fuzzy sets corresponding the both weighting functions are defined on $[\underline{z}_j,\ \overline{z}_j]$ which denoted in the sequal by $\bar{Z}_j^0$ and $\bar{Z}_j^1$.

The membership function of rule i is computed as the product of the weighting functions [5] that correspond to the fuzzy sets in the rule that is

$$w_i(z) = \prod_{i=1}^{s} w_{ij}(z_j), \tag{3}$$

where $w_{ij}(z_j)$ is either $\beta_j^0(z_j)$ or $\beta_j^1(z_j)$ depending on which weighting function is used in the rule. If the scheduling variables are taken as a subset of the state, then $\beta_j^0(z_j)$ or $\beta_j^1(z_j)$ is an affine function of $x, x \in \mathbb{R}^n$ [5]. Therefore, the subset of the states will perform as a polytope for example box in $\mathbb{R}^n$. Clearly that

$$w_i(z) \geq 0 \text{ and } \sum_{i=1}^{s} w_i(z) = 1.$$

The generalized TS fuzzy system by using the sector nonlinearity approach can be presented as follows

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1\widehat{w}(t) + \textstyle\sum_{i=1}^{s} w_i(z)\, B_{2i}u(t), \\ q(t) &= C_1x(t) + D_{11i}\widehat{w}(t) + D_{12i}u(t), \\ y(t) &= C_2x(t) + D_{21i}\widehat{w}(t), \end{aligned} \tag{4}$$

where $w_i(z)$ is weighting function of rule i. Representation (4) is not unique. Therefore the TS fuzzy system representation of the bilinear system which is obtained by the sector nonlinearity approach is not unique. Hence, it will be chosen $w_i(z), i = 1,2, \dots, p$ such that the error between the impulse response of the bilinear system and the TS fuzzy system is as small as possible. Denote the local linear system with uncertainty of (4) by

$$G_i = \{A, B_1, B_{2i}, C_1, D_{11}, D_{12}, C_2, D_{21}\}, i = 1,2, \dots, s. \tag{5}$$

Then, the form of the fuzzy *i*-th controller is given by

Controller rule i:
If z_1 is Z_1^i and ... and z_p is Z_p^i then

$$\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \widehat{D}_i\}, i = 1,2, \dots, s.$$

The *i*-th controller is presented as a state space realization as follows

$$\begin{aligned} \dot{\xi}(t) &= \hat{A}_i\xi(t) + \hat{B}_iy(t), \\ u(t) &= \hat{C}_i\xi(t) + \widehat{D}_iy(t), i = 1,2, \dots, s, \end{aligned} \tag{6}$$

where $\xi \in \mathbb{R}^k, k = n$ is a compact open set which contains the origin. From (5) and (6), it will be obtained the local closed loop system for each the local linear system as follows

$$\begin{aligned} \dot{x}_c(t) &= A_{ci}x_c(t) + B_{ci}\widehat{w}(t), \\ q(t) &= C_{ci}x_c(t) + D_{ci}\widehat{w}(t), i = 1,2, \dots, s, \end{aligned} \tag{7}$$

where $x_c = \begin{bmatrix} x \\ \xi \end{bmatrix}, A_{ci} = \begin{bmatrix} A + B_{2i}\widehat{D}_iC_2 & B_{2i}\hat{C}_i \\ \hat{B}_iC_2 & \hat{A}_i \end{bmatrix}, B_{ci} = \begin{bmatrix} B_1 + B_{2i}\widehat{D}_iD_{21} \\ \hat{B}_iD_{21} \end{bmatrix},$

$C_{ci} = [C_1 + D_{12}\widehat{D}_iC_2 \quad D_{12}\hat{C}_i]$, and $D_{ci} = [D_{11} + D_{12}\widehat{D}_iD_{21}]$. The robust H_∞ control design problem for the systems (5) is to find a controller (6) $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \widehat{D}_i\}, i = 1,2, \dots, s$ on each the local linear system of the generalized TS fuzzy system G_i such that the closed loop system (7) is asymptotically stable and has L_2-gain $\leq \gamma_i, \gamma_i > 0$ for each $i = 1,2, \dots, s$.

2.2. Definition of Robust H_∞-Performance

Definition 1 is used to define the asymptotically stable condition and the L_2- gain of a system.

Definition 1 [14-15, 31-33] Consider the closed loop system (7) where initial condition $x_c(0) = x_0$. System (7) is called has L_2- gain $\leq \gamma_i, \gamma_i > 0$ if

$$\int_0^T \|q(t)\|^2 dt \leq {\gamma_i}^2 \int_0^T \|\hat{w}(t)\|^2 dt, \forall T \geq 0, \hat{w} \in L_2[0,T],$$

for each $i = 1,2,\ldots,s$, where $L_2[0,T] := \left\{\hat{w} \middle| \int_0^T \|\hat{w}(t)\|^2 dt < \infty\right\}$ and $\|.\|$ denotes the Euclidian norm. System (7) is called asymptotically stable if $\lim_{t\to\infty} \phi(t, x_0, 0) = 0$, for any initial state $x_0 \in \mathbb{R}^{2n}$, where ϕ is the transition matrix.

Lemma 1 is known as Complement Schur. The property is used in order to change nonlinear matrix inequality into linear matrix inequality.

Lemma 1 [20] Consider matrix-valued function $S(x) \in \mathbb{R}^{n\times n}$ and symmetric matrix-valued functions $Q(x), R(x) \in \mathbb{R}^{n\times n}$ that depend affinely on x. Then

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^t & R(x) \end{bmatrix} \succcurlyeq 0, \text{ if only if } R(x) \succ 0 \text{ and } Q(x) - S(x)R(x)^{-1}S(x)^t \succcurlyeq 0.$$

According to the bounded real lemma for linear systems [19], the closed loop system (7) is asymptotically stable and has L_2-gain $\leq \gamma_i, \gamma_i > 0$ if only if there exists a positive definite matrix P_c such that

$$\begin{bmatrix} A_{ci}^t P_{ci} + P_{ci}A_{ci} + C_{ci}^t C_{ci} & P_{ci}B_{ci} + {C_{ci}}^t D_{ci} \\ B_{ci}^t P_{ci} + D_{ci}^t C_{ci} & D_{ci}^t D_{ci} - {\gamma_i}^2 I \end{bmatrix} \preccurlyeq 0, i = 1,2,\ldots,s. \tag{8}$$

The linear matrix inequality (8) can be rewritten as

$$\begin{bmatrix} A_{ci}^t P_{ci} + P_{ci}A_{ci} & P_{ci}B_{ci} \\ B_{ci}^t P_{ci} & -{\gamma_i}^2 I \end{bmatrix} + \begin{bmatrix} {C_{ci}}^t \\ {D_{ci}}^t \end{bmatrix} I [C_{ci} \quad D_{ci}] \preccurlyeq 0, i = 1,2,\ldots,s. \tag{9}$$

By multiplying on each side of the inequality (9) by ${\gamma_i}^{-1}$ and let $P_{1i} = {\gamma_i}^{-1} P_{ci}$, then inequality (10) can be obtained

$$\begin{bmatrix} P_{1i}A_{ci} + A_{ci}^t P_{1i} & P_{1i}B_{ci} \\ B_{ci}^t P_{1i} & -\gamma_i I \end{bmatrix} + \begin{bmatrix} {C_{ci}}^t \\ {D_{ci}}^t \end{bmatrix} {\gamma_i}^{-1} I [C_{ci} \quad D_{ci}] \preccurlyeq 0, i = 1,2,\ldots,s. \tag{10}$$

By Schur complement and replace P_{1i} by P_{ci}, inequality (10) can be written as

$$\begin{bmatrix} A_{ci}^t P_{ci} + P_{ci}A_{ci} & P_{ci}B_{ci} & C_{ci}^t \\ B_{ci}^t P_{ci} & -\gamma_i I & D_{ci}^t \\ C_{ci} & D_{ci} & -\gamma_i I \end{bmatrix} \preccurlyeq 0, i = 1,2,\ldots,s. \tag{11}$$

Thus, finding a matrix P_{ci} such that the inequality in (8) is satisfied is equivalent to finding a matrix P_{ci} such that the inequality in (11) is satisfied. Therefore, the system (7) is said to have robust H_∞-performance if there exists $P_{ci} \succ 0$ such that satisfies (11).

3. Robust H_∞ Control for Bilinear Systems Using the Dynamic Takagi-Sugeno Fuzzy Models Based on Linear Matrix Inequalities

In this section, the main result is presented that is the formulation of the robust H_∞ control design for the bilinear systems in LMIs. The bilinear system with uncertainty is represented as a convex combination of the local linear systems with uncertainty. The local robust H_∞ controller of the local linear systems with uncertainty is then designed on a polytope for each the local linear system, given the generalized TS fuzzy system in (4). In general, the block diagram of the bilinear control systems is depicted by Figure 1. The bilinear system is represented as the generalized TS fuzzy system. The design of the controller for bilinear system is performed through the dynamic parallel distributed compensation (DPDC) [4-5]. The main idea of the DPDC is to obtain each control rule so as to compensate each rule of the TS fuzzy system. The local robust H_∞ controller is then designed for each local linear system. The total robust H_∞ controller is then obtained by a fuzzy blending of each local robust H_∞ controller.

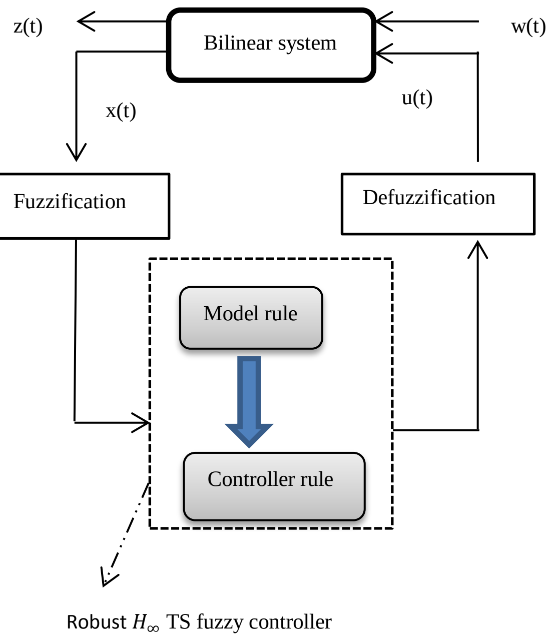

Figure 1. Scheme of Robust H_∞ Fuzzy Controller

Formulation (11) is a bilinear matrix inequality (BMI) of the variables P_{ci} and $\mathcal{K}_i, i = 1,2,\dots,s$. The condition L_2-gain of the closed loop system less than γ is the robustness problem of bilinear systems. Therefore, the robust H_∞ control synthesis problem is to minimize γ such that inequalities (11) are satisfied. Because (11) is a bilinear matrix inequality, then it is a difficult problem to solve it. Furthermore, it will be derived a formulation which is equivalent to (11). The $P_{ci} \succ 0$ that satisfies (11) is equivalent to the existence of $P_{ci} \succ 0$ that satisfies the following BMI:

$$\begin{bmatrix} -A_{ci}^t P_{ci} - P_{ci} A_{ci} & P_{ci} B_{ci} & C_{ci}^t \\ B_{ci}^t P_{ci} & \gamma_i I & -D_{ci}^t \\ C_{ci} & -D_{ci} & \gamma_i I \end{bmatrix} \succcurlyeq 0, i = 1,2,\dots,s. \qquad (12)$$

It is difficult to obtain the solution of the BMI (12). By the Theorem 1 on [23], there exist a certain LMIs whose solvability is equivalent to the BMI (12).

The main results of the paper are Theorem 1 and Corollary 1. The necessary and sufficient conditions of the local robust H_∞ controller on each the local linear system of the generalized TS fuzzy system are given in the following theorem.

Theorem 1. Consider the local linear system of the generalized TS fuzzy system on polytope $\mathcal{P} \subset \mathbb{R}^n$ that is

$G_i = \{A, B_1, B_{2i}, C_1, D_{11}, D_{12}, C_2, D_{21}\}, i = 1,\dots,s.$

Consider the local closed loop system (7) that has robust H_∞-performance by the local robust H_∞controller $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i\}$ for each $i = 1,\dots,s$. The inequality (12) holds for some $(P_{ci}, \mathcal{K}_i)$, if only if LMIs (13-14) hold for some $P_i = \{V_i, W_i, F_i, G_i, H_i, L_i\}$,

$$\begin{bmatrix} V_i & I \\ I & W_i \end{bmatrix} \succ 0, V_i, W_i \succ 0, \qquad (13)$$

$$\begin{bmatrix} \varphi_{11} & * & * & * \\ \varphi_{21} & \varphi_{22} & * & * \\ \varphi_{31} & \varphi_{32} & \gamma_i I & * \\ \varphi_{41} & \varphi_{42} & \varphi_{43} & \gamma_i I \end{bmatrix} \geqslant 0, \tag{14}$$

where * denotes the symmetric matrix, $\varphi_{11} = -(AV_i + B_{2i}F_i) - (AV_i + B_{2i}F_i)^T$, $\varphi_{21} = -L_i - (A + B_{2i}H_iC_2)^T$, $\varphi_{22} = -(W_iA + G_iC_2) - (W_iA + G_iC_2)^T$, $\varphi_{31} = (B_1 + B_{2i}H_iD_{21})^T$, $\varphi_{32} = (W_iB_1 + G_iD_{21})^T$, $\varphi_{41} = C_1V_i + D_{12}F_i$, $\varphi_{42} = C_1 + D_{12}H_iC_2$, and $\varphi_{43} = -(D_{11} + D_{12}H_iD_{21})$, $i = 1,2,\dots,s$.

If the LMIs (13-14) have a solution P_i, one of the solutions to the BMIs (12) is given by

$\hat{A}_i = {W_i}^{-1}G_iC_{2i}V_iS_i^{-1} - B_{2i}H_iC_{2i}V_iS_i^{-1} + B_{2i}F_iS_i^{-1} - {W_i}^{-1}L_iS_i^{-1} + A_iV_iS_i^{-1}$,

$\hat{B}_i = B_{2i}H_i - {W_i}^{-1}G_i$,

$\hat{C}_i = F_iS_i^{-1} - H_iC_{2i}V_iS_i^{-1}$, and

$\hat{D}_i = H_i, i = 1,2,\dots,s$.

Proof. Procedure of proof abreast of Theorem 1 on [23] by parameterization of matrix variables. Without loss of generality, P_{ci} is assumed to have the following structure:

$P_{c1i} := \begin{bmatrix} V_i & S_i \\ S_i & S_i \end{bmatrix}, P_{c2i} := P_{c1i}^{-1} = \begin{bmatrix} T_{1i} & T_{2i} \\ T_{3i} & T_{4i} \end{bmatrix}$, and $U_{1i} := \begin{bmatrix} I & 0 \\ W_i & -W_i \end{bmatrix}$,

where $T_{1i} = {V_i}^{-1} + {V_i}^{-1}S_i(S_i - S_i{V_i}^{-1}S_i)^{-1}S_i{V_i}^{-1}$, $T_{2i} = -{V_i}^{-1}S_i(S_i - S_i{V_i}^{-1}S_i)^{-1}$, $T_{3i} = -S_i\left(S_i - S_i{V_i}^{-1}S_i\right)^{-1}S_i{V_i}^{-1}$, and $T_{4i} = S_i - S_i{V_i}^{-1}S_i, i = 1,2,\dots,s$. Setting a regular matrix as follows

$$U_{2i} := P_{c1i}U_{1i}^t = \begin{bmatrix} V_i & S_i \\ S_i & S_i \end{bmatrix}\begin{bmatrix} I & W_i \\ 0 & -W_i \end{bmatrix} = \begin{bmatrix} V_i & I \\ S_i & 0 \end{bmatrix},$$

where $V_iW_i - S_iW_i = I, i = 1,2,\dots,s$. Define the matrix valued affine functions as follow

$M_P(P_{ci}) := U_{2i}^tP_{c2i}U_{2i} = \begin{bmatrix} V_i & I \\ I & W_i \end{bmatrix}$,

And

$$\begin{bmatrix} M_A(P_{ci}) & M_B(P_{ci}) \\ M_C(P_{ci}) & M_D(P_{ci}) \end{bmatrix} = \begin{bmatrix} U_{1i} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} A_{ci}P_{c1i} & B_{ci} \\ C_{ci}P_{c1i} & D_{ci} \end{bmatrix}\begin{bmatrix} U_{1i}^t & 0 \\ 0 & I \end{bmatrix},$$
$$= \left[\begin{array}{cc|c} \Lambda_{11} & A + B_{2i}H_iC_2 & B_1 + B_{2i}H_iD_{21} \\ L_i & W_iA + G_iC_2 & W_iB_1 + G_iD_{21} \\ \hline \Lambda_{31} & C_1 + D_{12}H_iC_2 & D_{11} + D_{12}H_iD_{21} \end{array}\right],$$

where $V_i, W_i \succ 0, V_i \succ W_i$, $\Lambda_{11} = AV_i + B_{2i}F_i$ and $\Lambda_{31} = C_1V_i + D_{12}F_i$, $i = 1,2,\dots,s$. According to [23], inequality (13) is obtained because $P_{ci} \succ 0$. BMI (12) equivalent to the following inequality (14)

$$\begin{bmatrix} -M_A(P_{ci}) - M_A(P_{ci})^t & M_B(P_{ci}) & M_C(P_{ci})^t \\ M_B(P_{ci})^t & \gamma_i I & -M_D(P_{ci})^t \\ M_C(P_{ci}) & -M_D(P_{ci}) & \gamma_i I \end{bmatrix} = \begin{bmatrix} \varphi_{11} & * & * & * \\ \varphi_{21} & \varphi_{22} & * & * \\ \varphi_{31} & \varphi_{32} & \gamma_i I & * \\ \varphi_{41} & \varphi_{42} & \varphi_{43} & \gamma_i I \end{bmatrix} \geqslant 0,$$

where * present this matrix is symmetric and φ_{11}, φ_{21}, φ_{22}, φ_{32}, φ_{41}, φ_{42}, and φ_{43} as in Theorem 1. Hence, the local robust H_∞controllers $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i\}$, $i = 1,2,\dots,s$ are obtained from the relation

$$\begin{bmatrix} H_i & F_i \\ G_i & L_i \end{bmatrix} = \begin{bmatrix} I & 0 \\ W_i B_{2i} & -W_i \end{bmatrix} \begin{bmatrix} \widehat{D}_i & \hat{C}_i \\ \widehat{B}_i & \hat{A}_i - A V_i S_i^{-1} \end{bmatrix} \begin{bmatrix} I & -C_2 V_i \\ 0 & S_i \end{bmatrix},$$

where W_i and S_i are invertible matrices

The robust H_∞ control problem for the TS fuzzy system is finding the solutions $P_{ci} \succ 0$ and $\mathcal{K}_i, i = 1,2,\dots,s$ of the BMI (12) which equivalent to find $P_i = \{V_i, W_i, F_i, G_i, H_i, L_i\}$ satisfy (13) and (14). Corollary 1 states that the robust H_∞ controller of bilinear systems is obtained on a subset of the polytope. The robust H_∞ controller of the bilinear system is defined as a convex linear combination of the local H_∞ robust controllers. Furthermore, the subset of the polytope is called a basin of attraction of TS fuzzy system.

Corollary 1. Consider the generalized bilinear system (1) where initial condition $x(0) = x_0$. Suppose a polytope $\mathcal{P}$ is defined on (2), where $x_0 \in \mathcal{P}$. Let the bilinear system (1) is approximated by the generalized TS fuzzy system (4) on polytope $\mathcal{P} \subset \mathbb{R}^n$ where $w_i(z)$ is a membership function on (3). There exists a H_∞ robust controller $\mathcal{K} = \{\hat{A}, \hat{B}, \hat{C}, \widehat{D}\}$ on $\mathfrak{D} \subseteq \mathcal{P} \subset \mathbb{R}^n$ with $\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \widehat{D} \end{bmatrix} := \sum_{i=1}^{s} w_i(z) \begin{bmatrix} \hat{A}_i & \hat{B}_i \\ \hat{C}_i & \widehat{D}_i \end{bmatrix}$, $w_i(z) \geq 0, \sum_{i=1}^{s} w_i(z) = 1$, where $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \widehat{D}_i\}, i = 1,2,\dots,s$ are solution of the BMI (12), such that closed loop system

$$\dot{x}_c(t) = \sum_{i=1}^{s} w_i(z)\,(A_{ci} x_c(t) + B_{ci} \widehat{w}(t)),$$
$$q(t) = \sum_{i=1}^{s} w_i(z)\,(C_{ci} x_c(t) + D_{ci} \widehat{w}(t)), \tag{15}$$

is asymptotically stable and has L_2-gain $\leq \gamma_i, \gamma_i > 0$, where

$$A_{ci} = \begin{bmatrix} A + B_{2i}\widehat{D}_i C_2 & B_{2i}\hat{C}_i \\ \hat{B}_i C_2 & \hat{A}_i \end{bmatrix}, B_{ci} = \begin{bmatrix} B_1 + B_{2i}\widehat{D}_i D_{21} \\ \hat{B}_i D_{21} \end{bmatrix}, C_{ci} = [C_1 + D_{12}\widehat{D}_i C_2 \quad D_{12}\hat{C}_i],$$

and $D_{ci} = [D_{11} + D_{12}\widehat{D}_i D_{21}]$.

Proof. Because $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \widehat{D}_i\}, i = 1,2,\dots,s$ are solution of BMI (12), then according to Definition 2 and Theorem 1, the local H_∞ robust controllers $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \widehat{D}_i\}$ lead to the closed loop systems (7) is asymptotically stable and has L_2-gain $\leq \gamma_i$ for each $i = 1,2,\dots,s$ on polytope $\mathcal{P} \subset \mathbb{R}^n$. Hence $Re(\lambda_k(A_{ci})) < 0$, for each i, where $\lambda_k(.)$ denote eigenvalues k-th of $(.)$ matrix. Because of the bilinear system (1) is approximated by the generalized TS fuzzy system (4) on polytope $\mathcal{P} \subset \mathbb{R}^n$, where $w_i(z)$ is membership function on (3) then

$w_i(z) = h(x_1, x_2, \dots, x_n) = h(x)$

for some function h. Choose $\bar{x} \in \mathcal{P}$ such that

$Re(\lambda_k(\sum_{i=1}^{s} w_i(z)\,A_{ci})) < 0$.

Suppose $\mathfrak{D} = \{\bar{x}\} \subset \mathcal{P}$ then $\mathfrak{D}$ is attraction domain of closed loop system (15) where initial condition $x(0) = x_0$. Because $\mathcal{K} = \{\hat{A}, \hat{B}, \hat{C}, \widehat{D}\}$ is a convex linear combination of $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \widehat{D}_i\}, i = 1,2,..,s$ where $w_i(z) = h(\bar{x}), \bar{x} \in \mathfrak{D}$ then $Re(\lambda_k(\sum_{i=1}^{s} w_i(z)\,A_{ci})) < 0$. Another word, the closed loop system (15) is asymptotically stable and has L_2-gain $\leq \gamma, \gamma > 0$ for some $w_i(z) \geq 0, \sum_{i=1}^{s} w_i(z) = 1$ on $\mathfrak{D}$, and $\gamma = \min_{i=1,2,3,..,s}\{\gamma_i\}$.

Furthermore, the following algorithm is proposed to obtain the robust H_∞ controller for bilinear systems.

Input: Generalized bilinear system consist of $A, B_1, B_2(x), C_1, D_{11}, D_{12}, C_2, D_{21}$.

Process:

Construct the TS fuzzy system by the sector nonlinearity approach.

Choose $w_i(z) \geq 0,\ \sum_{i=1}^{s} w_i(z) = 1$, such that the error of impulse response between the bilinear system and TS fuzzy system as small as possible.

Design the local robust H_∞ controller on each the local linear system by using Theorem 1.

Output: The total controller is stated by Corollary 1.

The process is repeated until the robust H_∞ controller lead to the closed-loop system (15) which is asymptotically stable. While the L_2-gain of the closed-loop system (15) is the minimum of L_2-gains at the local closed-loop systems.

4. Numerical Simulations

The examples of the robust H_∞ control design for the bilinear systems are presented.

Consider the stable bilinear system of chemical reactor [2,16] with uncertainty $\widehat{w}$ as follows

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}\frac{3}{16} & \frac{5}{12}\\ -\frac{50}{3} & -\frac{8}{3}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0.1\\ 0\end{bmatrix}\widehat{w}+\begin{bmatrix}-\frac{1}{8}-x_1\\ x_2\end{bmatrix}u,$$

$$q=\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+0.5\widehat{w}+0.02u, \tag{16}$$

$$y=\begin{bmatrix}3 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\widehat{w},$$

where x_1 and x_2 represent the temperature and the concentration of the initial product of the chemical reactor, while u represents the cooling flow rate in a jacket around the reactor.

The scheduling variables of the nonconstant elements in the matrix function $\begin{bmatrix}-\frac{1}{8}-x_1\\ x_2\end{bmatrix}$ are $z_1=-\frac{1}{8}-x_1$ and $z_2=x_2$. Suppose $x_1\in[-3,3]$ and $x_2\in[-1,4]$. Two conditions will perform the polytope $\mathcal{P}$ that is

$$\mathcal{P}=[-3,3]\times[-1,4].$$

For each of these two terms, the two weighting functions and the corresponding matrix elements are computed as follows:

1. $z_1=-\frac{1}{8}-x_1\in[-3.125,2.875]$.

The first weighting function is $\beta_1^0=\frac{2.875-(-\frac{1}{8}-x_1)}{2.875-(-3.125)}=\frac{1}{2}+\frac{x_1}{6}$, and $\underline{z}_1=-3.125$. The second weighting function is $\beta_1^1=1-\beta_1^0=\frac{1}{2}-\frac{x_1}{6}$, and $\overline{z}_1=2.875$. Then, the scheduling variable z_1 is represented as the weighted sum

$$z_1=\underline{z}_1\beta_1^0(z_1)+\overline{z}_1\beta_1^1(z_1).$$

2. $z_2=x_2\in[-1,4]$.

The first weighting function is $\beta_2^0=\frac{2-x_2}{4-(-1)}=\frac{2}{5}-\frac{x_2}{5}$, and $\underline{z}_2=-1$. The second weighting function is $\beta_2^1=1-\beta_2^0=\frac{3}{5}+\frac{x_2}{5}$, and $\overline{z}_2=4$. Then, the scheduling variable z_2 is represented as the weighted sum

$$z_2=\underline{z}_2\beta_2^0(z_2)+\overline{z}_2\beta_2^1(z_2).$$

For each weighting function, denote the corresponding fuzzy set by $\bar{Z}_j^i, i=0,1, j=1,2$. The fuzzy set corresponding to $\beta_1^0,\beta_1^1,\beta_2^0,\beta_2^1$ is denoted by $\bar{Z}_1^0$, $\bar{Z}_1^1$, $\bar{Z}_2^0$, $\bar{Z}_2^1$, respectively. Therefore, the TS fuzzy model having $s=2^2=4$ rules can be written as

Model rule 1:

If z_1 is about -3.125 and z_2 is about -1 then

$$\dot{x}(t)=Ax(t)+B_1\widehat{w}(t)+B_{21}u(t),$$
$$q(t)=C_1x(t)+D_{11}\widehat{w}(t)+D_{12}u(t),$$
$$y(t)=C_2x(t)+D_{21}\widehat{w}(t),$$

and the membership function of the rule is computed as $w_1(z)=\beta_1^0\beta_2^0$.

Model rule 2:

If z_1 is about -3.125 and z_2 is about 4 then

$\dot{x}(t) = Ax(t) + B_1\widehat{w}(t) + B_{22}u(t),$

$q(t) = C_1x(t) + D_{11}\widehat{w}(t) + D_{12}u(t),$

$y(t) = C_2x(t) + D_{21}\widehat{w}(t),$

and the membership function of the rule is computed as $w_2(z) = \beta_1^0\beta_2^1$.

Model rule 3:

If z_1 is about 2.875 and z_2 is about -1 then

$\dot{x}(t) = Ax(t) + B_1\widehat{w}(t) + B_{23}u(t),$

$q(t) = C_1x(t) + D_{11}\widehat{w}(t) + D_{12}u(t),$

$y(t) = C_2x(t) + D_{21}\widehat{w}(t),$

and the membership function of the rule is computed as $w_3(z) = \beta_1^1\beta_2^0$.

Model rule 4:

If z_1 is about 2.875 and z_2 is about 4 then

$\dot{x}(t) = Ax(t) + B_1\widehat{w}(t) + B_{24}u(t),$

$q(t) = C_1x(t) + D_{11}\widehat{w}(t) + D_{12}u(t),$

$y(t) = C_2x(t) + D_{21}\widehat{w}(t),$

and the membership function of the rule is computed as $w_4(z) = \beta_1^1\beta_2^1$.

Let $x_1 = -0.5$ and $x_2 = 0.01$ then $\beta_1^0 = 0.4167$, $\beta_1^1 = 0.5833$, $\beta_2^0 = 0.7980$, and $\beta_2^1 = 0.2020$. Hence, the membership functions of the rules are

$w_1(z) = 0.3325, w_2(z) = 0.0842, w_3(z) = 0.4655,$

and $w_4(z) = 0.1178$. These membership functions lead to the error of the impulse response between the bilinear system and the TS fuzzy system as small as possible. The bilinear system can be represented as a convex linear combination of the generalized linear system as on (4) where

$A = \begin{bmatrix} 0.1875 & 0.4167 \\ -16.667 & -2.667 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, B_{21} = \begin{bmatrix} -3.125 \\ -1 \end{bmatrix}, B_{22} = \begin{bmatrix} -3.125 \\ 4 \end{bmatrix}, B_{23} = \begin{bmatrix} 2.875 \\ -1 \end{bmatrix}, B_{24} = \begin{bmatrix} 2.875 \\ 4 \end{bmatrix}, C_1 = [1 \quad 2], C_2 = [3 \quad 1], D_{11} = 0.5,$ $D_{12} = 0.02$, and $D_{21} = 1$.

By using the proposed algorithm and initial state $x_0 = \begin{bmatrix} -0.4 \\ 0.02 \end{bmatrix}$ then it can be obtained the local robust H_∞ controller $\mathcal{K}_i = \{\hat{A}_i, \hat{B}_i, \hat{C}_i, \widehat{D}_i\}$, $i = 1,2,3,4$ as follow

$\hat{A}_1 = \begin{bmatrix} -20.0672 & 15.2638 \\ -26.3669 & 1.0134 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} 3.3446 \\ 2.1438 \end{bmatrix}, \hat{C}_1 = [6.4074 \quad -4.7758],$

$\widehat{D}_1 = [-1.0456],$

$\hat{A}_2 = \begin{bmatrix} -6.4453 & 5.1301 \\ -10.1342 & -9.3521 \end{bmatrix}, \hat{B}_2 = \begin{bmatrix} 2.1059 \\ -2.0419 \end{bmatrix}, \hat{C}_2 = [2.0496 \quad -1.5326],$

$\widehat{D}_2 = [-0.6496],$

$\hat{A}_3 = \begin{bmatrix} 29.8193 & 47.4821 \\ -30.9622 & -20.3664 \end{bmatrix}, \hat{B}_3 = \begin{bmatrix} -6.1805 \\ 3.4791 \end{bmatrix}, \hat{C}_3 = [10.3796 \quad 16.3949],$

$\widehat{D}_3[-2.1740]$

and

$\hat{A}_4 = \begin{bmatrix} 16.1321 & 18.0186 \\ 6.5955 & 22.1836 \end{bmatrix}, \hat{B}_4 = \begin{bmatrix} -11.1588 \\ -15.8856 \end{bmatrix}, \hat{C}_4 = [5.6118 \quad 6.1443],$

$\widehat{D}_4 = [-3.9033].$

According to Corollary 1, the robust H_∞ controller for bilinear system is $\mathcal{K} = \{\hat{A}, \hat{B}, \hat{C}, \widehat{D}\}$ given by

$\hat{A} = \begin{bmatrix} 8.5670 & 29.7331 \\ -23.2557 & -7.3168 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} -2.9026 \\ 0.2886 \end{bmatrix}, \quad \hat{C} = [7.7959 \quad 6.6389],$

$\widehat{D} = [-1.8743].$

The L_2-gains of the local closed-loop systems are $\gamma_1 = 1.0014, \gamma_2 = 0.0161, \gamma_3 = 1.0019,$

and $\gamma_4 = 0.9497$. Hence the closed-loop system (15) has L_2-gain $\leq \min_{i=1,2,3,4}\{\gamma_i\} = 0.0161$.

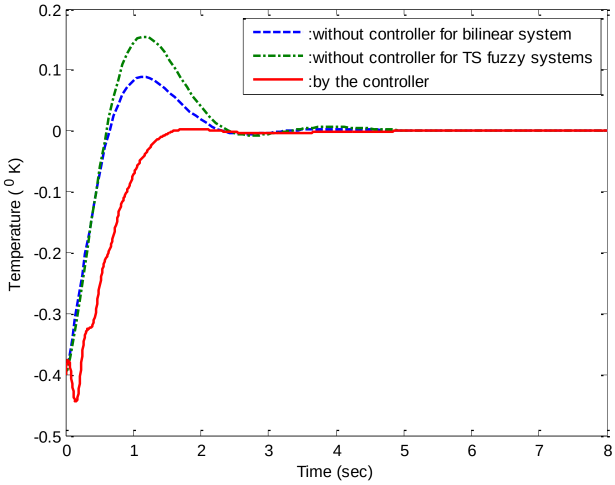

Figure 2. The Temperature of the Initial Product of the Chemical Reaction

The 1st and 2nd states of the bilinear system before and after the controller are given in Figure 2 and Figure 3, respectively. Figure 2 and Figure 3 show that the behavior of state variables of the temperature and the concentration of the initial product of the chemical reactor before and after the system is given by the controller. The temperature of the initial product by the controller is smaller than without the controller. The concentration of the initial product by the controller is more concentrated than without the controller. The temperature and the concentration of the initial product converge to a steady state asymptotically. The basin of attraction of TS fuzzy system (16) is polytope $\mathcal{P} = [-3,3] \times [-1,4]$.

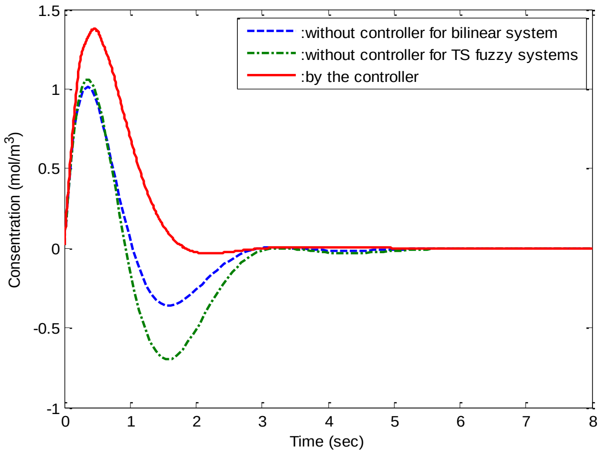

Figure 3. The Concentration of the Initial Product of the Chemical Reaction

2. Consider the bilinear system with multi inputs as follow

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{3}{16} & \frac{5}{12}\\ -\frac{50}{3} & -\frac{8}{3}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0.1\\ 0\end{bmatrix}\hat{w} + \begin{bmatrix}-\frac{1}{8} - x_1 & 0\\ x_2 & 0.15x_1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix},$$

$$q = \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + 0.5\widehat{w} + \begin{bmatrix}0.02 & 0.01\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}, \quad (17)$$
$$y = \begin{bmatrix}3 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \widehat{w},$$

where x_1 and x_2 represent the temperature and the concentration of the initial product of the chemical reactor, while u_1 represents the cooling flow rate in a jacket around the reactor and u_2 represents the velocity of the impeller agitator. The impeller stirs the reagents to ensure proper mixing.

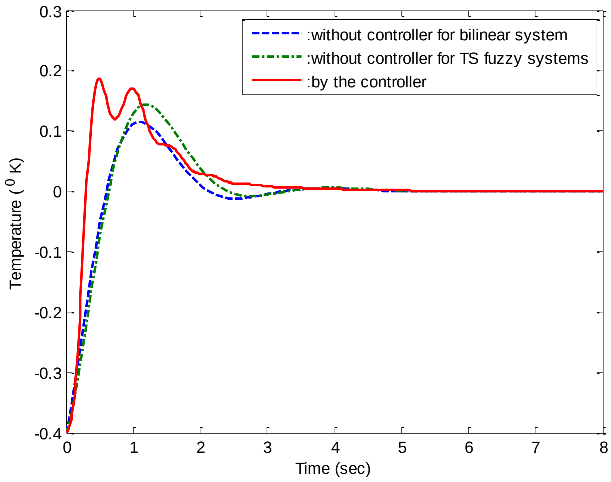

Figure 4. The Temperature of the Initial Product of the Chemical Reaction

The scheduling variables of the nonconstant elements in the matrix function $\begin{bmatrix}-\frac{1}{8}-x_1 & 0\\ x_2 & 0.15x_1\end{bmatrix}$ are $z_1 = -\frac{1}{8} - x_1$, $z_2 = x_2$ and $z_3 = 0.15x_1$. Therefore, the TS fuzzy model having $s = 2^3 = 8$ rules. Suppose $x_1 \in [-3,3]$ and $x_2 \in [-1,4]$. Two conditions will perform the polytope $\mathcal{P}$ that is $\mathcal{P} = [-3,3] \times [-1,4]$. By the same procedure, where initial state $x_0 = \begin{bmatrix}-0.4\\0.02\end{bmatrix}$ and $w_1 = 0.2706$, $w_2 = 0.0685$, $w_3 = 0.0685$, $w_4 = 0.0173$, $w_5 = 0.3662$, $w_6 = 0.0927$, $w_7 = 0.09$ and $w_8 = 0.0235$, the robust H_∞ controller for bilinear system is $\mathcal{K} = \{\hat{A}, \hat{B}, \hat{C}, \widehat{D}\}$ given by

$$\hat{A} = \begin{bmatrix}-3.5208 & 15.9175\\ -13.2244 & -5.7906\end{bmatrix}, \widehat{B} = \begin{bmatrix}1.1297\\2.1170\end{bmatrix}, \hat{C} = \begin{bmatrix}-1.5426 & -1.4109\\ -102.5160 & -60.0878\end{bmatrix},$$

and

$$\widehat{D} = \begin{bmatrix}0.4788\\5.2459\end{bmatrix}.$$

The L_2-gains of the local closed-loop systems are $\gamma_1 = 0.0951$, $\gamma_2 = 0.08$, $\gamma_3 = 0.0442$, $\gamma_4 = 0.0151$, $\gamma_5 = 0.0426$, $\gamma_6 = 0.0671$, $\gamma_7 = 0.0379$, and $\gamma_8 = 0.0347$. Hence the closed-loop system (15) has L_2-gain $\leq \min_{i=1,2,\ldots,8}\{\gamma_i\} = 0.0151$. The 1st and 2nd states of the bilinear system before and after the controller are given in Figure 4 and Figure 5, respectively.

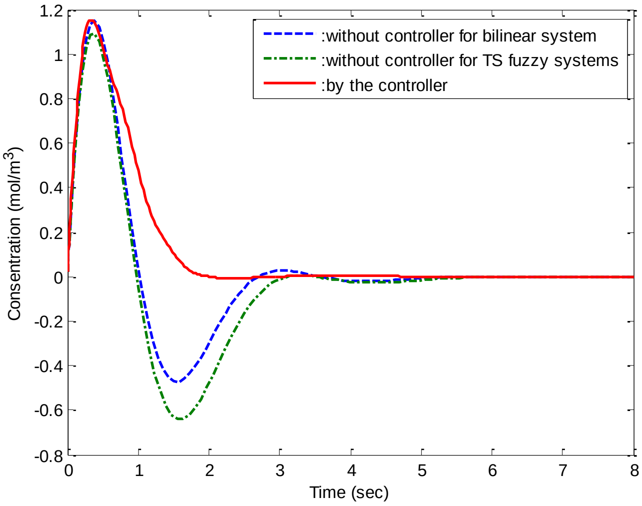

Figure 5. The Concentration of the Initial Product of the Chemical Reaction

x_2

From the numerical experiment, there are some points which closed loop system is not stable (horizontal and vertical axis are x_1 and x_2, respectively). For example, the black area in Figure 6 is a region which the closed loop system is not stable.

5. Conclusion

The sector nonlinearity approach for the TS fuzzy system provided complementary and advantage in control design because TS fuzzy systems can describe the nonlinear phenomena. The formulation of the robust H_∞ control design in LMIs is way towards the numerical solution. The robust H_∞ controller for the bilinear system can be obtained by designing the local controllers for each the local linear systems on a polytope. The local controllers were obtained by solving the set of LMIs on the polytope. Furthermore, the robust H_∞ controller for the bilinear system was obtained on a subset of the polytope which a convex linear combination of the local robust H_∞ controllers. A numerical example confirmed the proposed method for designing the robust H_∞ controller of the bilinear system.

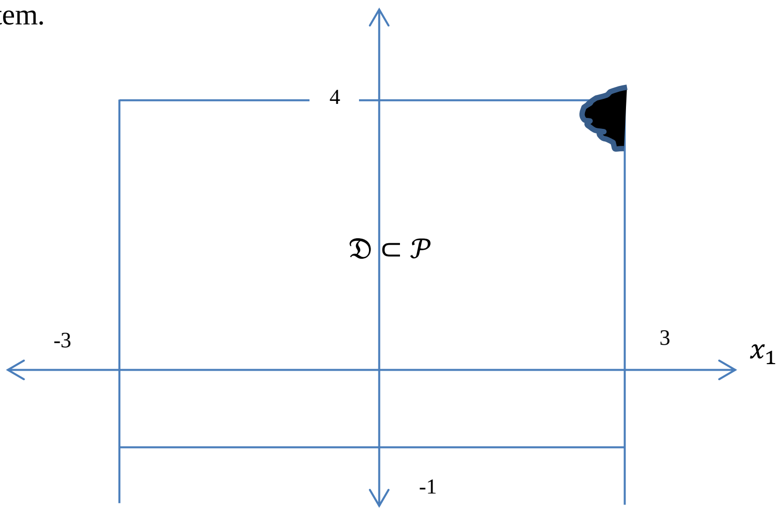

Figure 6. Region which the Closed Loop System is not Stable

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