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Ultra-high Precision Frequency Measurement Research

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Abstract

This paper is focused on the method of ultra-high precision frequency measurement.Based on comparing the phase characteristic of standard frequency and measured frequency, it proposes the relationship between group phase-shift and quantitative phase-shift resolution when existing frequency drift. The relationship among measurement accuracy, trigger pulse position and the pulse number is set up when the measurement gate is formed according to the group period.Through the programmable counters, the pulses of triggering gate are reduced. The application of nonlinear circuit compresses and expandes trigger pulse, which improves the accuracy of triggering gate. Frequency measurement system was designed using FPGA.Experiments showed that measurement accuracy can reach 10^{-12} . The system can be used for ultra-high precision measurement of frequency .

***Keywords**: Frequency drift, Standard frequency,Phase comparision, Phase coincidence point,Quantitative phase-shift resolution*

1. Introduction

High precision measurement of frequency is widely used in communication, measurement, space science, precise positioning and other high-tech fields. The traditional measurement methods of frequency include electronic counting, equal precise measurement of frequency, analog interpolation and vernier method[1-2],etc. The first two methods have ±1 counting error and low accuracy of measurement.The analog interpolation method makes ±1 counting error reduced to one over one thousand through interpolation technology. Through impact oscillators in two different frequencies,the vernier method expands the asynchronous interval between gate and measured frequency by K times,improving the measurement's precision by one in K points.The vernier method requires that two frequencies' stability should be very high. Otherwise,the system can not work due to the interaction of two frequencies[3-6].The accuracy of analog interpolation method and cursor method can reach 10^{-11}/S.But their application are limitted due to the strict demandings and complex circuits. The phase detection broadband frequency measurement technology appears in recent years,which reduces the ±1 counting error in frequency measurement,and the accuracy can reach 10^{-10}~10^{-11}/S.But the accuracy can not be further improved because of the multiple values and randomness of the trigger gate pulse[7-12]. Based on comparing standard frequency and measured frequency phase,this paper set up a new frequency mesearment scheme by changing the amplitude features of phase-coincidence pulse.

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2. Phase Comparasion between Standard Frequency and Measured Frequency

The main factor that influences the precision of frequency measurement is ±1 counting error. If double synchronous gate which is synchronized to standard frequency and measured frequency at the same time is got, the ±1 counting error can be eliminated theoretically.Therefore the phase relationship between signals of two different frequencies need to be studied. The phase relationship between different frequency signal can be expressed by the greatest common factor frequency, the least common multiple period, quantitative phase-shift resolution and group period.It is assumed that the periods of standard frequency f_A and tested frequency f_B are respectively T_A and T_B.If $f_A=Xf_{MAXC}$ and $f_B=Yf_{MAXC}$,f_{MAXC} is called the greatest common factor frequency between the two frequencies ,and its reciprocal is called least common multiple period T_{MINC} .

$$T_{MINC} = \frac{1}{f_{MAXC}} = \frac{X}{f_A} = XT_A \tag{1}$$

$$T_{MINC} = \frac{1}{f_{MAXC}} = \frac{Y}{f_B} = YT_B \tag{2}$$

The phase relationship of the two signal is shown in Figure 1,where f_D is the phase difference's wave form between f_A and f_B, and it is referred to the starting phase of f_A.At the beginning and end of a T_{MINC}'s time, the two signals' starting phase is same,which is called the phase coincidence point. It is found that between the adjacent phase coincidence point,

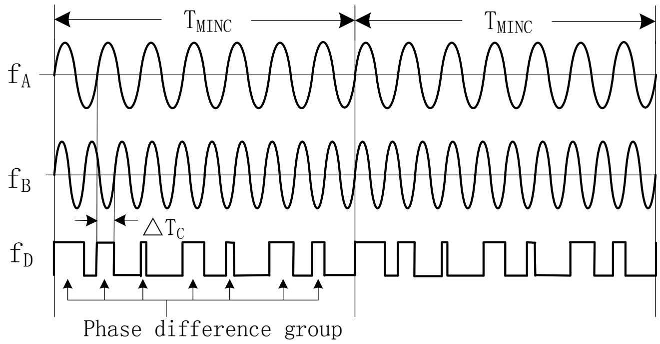

Figure 1. Phase Relationship of Different Frequency Signals

neither is two signals' phase difference continuous,nor is monotonous in most cases.However,if all the phase differences in T_{MINC} are arranged in a group which is called phase difference group,then there is a strict corresponding relation among groups,and the phase differences in corresponding positions of every group are completely equal. If the phase differences of f_B and f_A are represented as ΔT_1、ΔT_2、. . . 、ΔT_X , in a T_{MINC} ,there is such relation:

$$\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \vdots \\ \Delta T_X \end{bmatrix} = T_B \begin{bmatrix} j_1 \\ j_2 \\ \vdots \\ j_X \end{bmatrix} - T_A \begin{bmatrix} 1 \\ 2 \\ \vdots \\ X \end{bmatrix} \tag{3}$$

Where j_1 is a positive integer,j_X=Y-1.The general item of matrix (3) is

$$\Delta T_C = j_C T_B - CT_A \tag{4}$$

Due to $T_A=T_{MINC}/X$ and $T_B=T_{MINC}/Y$,

$$\Delta T_C = j_C T_B - C T_A = \frac{(Xj_C - YC)T_B}{X} \quad (5)$$

It is assumed that $Xj_C-YC=z_C$,then $\Delta T_C=z_C T_B/X$,and (3) becomes

$$\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \vdots \\ \Delta T_X \end{bmatrix} = T_B \begin{bmatrix} \frac{Xj_1 - Y}{X} \\ \frac{Xj_2 - 2Y}{X} \\ \vdots \\ \frac{Xj_X - XY}{X} \end{bmatrix} = T_B \begin{bmatrix} \frac{z_1}{X} \\ \frac{z_2}{X} \\ \vdots \\ \frac{z_X}{X} \end{bmatrix} \quad (6)$$

Owing to （j_C-1） $T_B<CT_A<j_C T_B$,so the phase difference of (5) is

$$0 \le \Delta T_C \le T_B \quad (7)$$

It is impossible to have two or more equal values for ΔT_1、ΔT_2、。。。、ΔT_X in a T_{MINC}.Otherwise, phase coincidence point can appear again between the adjacent phase coincidence points,which is contradictory with the assumption that there is only one T_{MINC}. In addition, X、Y、C、j_C are all positive integers,so z_C must be a positive integer which is less than X,and it ranges from 0,1 to X-1. ΔT_C ranges from 0, T_B/X to $(X-1)T_B/X$.Generally, ΔT_C is not monotonous with signal's periodic expansion.But when arrangig ΔT_C in a T_{MINC}, it is found that the difference between two adjacent phase differences is

$$\Delta T = \frac{T_B}{X} \quad (8)$$

ΔT represents quantitative scale when comparing two signals' phase.It is called quantitative phase-shift resolution. According to equation (1) ,

$$\Delta T = \frac{f_{MAXC}}{f_A f_B} = \frac{1}{XYf_{MAXC}} \quad (9)$$

Assuming $f_{EQU}=XYf_{MAXC}$,it is called equivalent phase-demodulation frequency.

In the actual phase comparision of two signals,there may be phase perturbation and frequency drift due to outside interferences.Changes of frequency must lead to the change of phase difference, which causes group-phase difference's parallel move.It is called group-phase shift.As shown in Figure 2, measured frequency drift is assumed to be $-\Delta f$,then $f_B=Yf_{MAXC}-\Delta f$.The standard frequency f_A does not change.At the end of the first T_{MINC} after phase coincidence point,f_B's phase lags behind f_A's. Lagging amount Δt is called

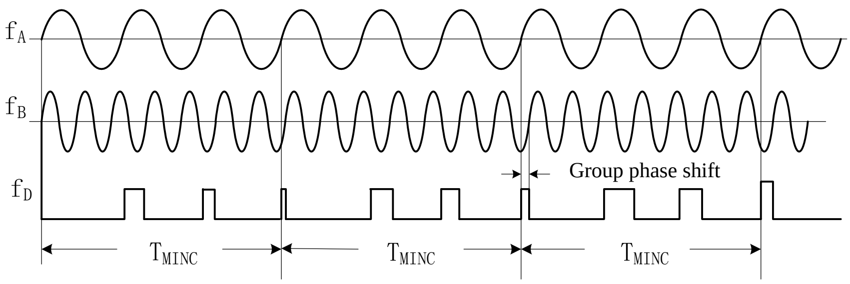

Figure 2.Group Phase Shift Caused By Frequency Drift

$$\Delta t = \frac{Y}{Yf_{MAXC} - \Delta f} - \frac{X}{f_A} \quad (10)$$

group phase-shift quantum.At the end of ith T_{MINC},two signals' phase difference is $i\Delta t$. After a few T_{MINC},there is phase coincidence phenomenon between the two signals.The time between two adjacent phase coincidence points is named group period T_{GP}.In a T_{GP},the difference of phase differences in corresponding positions between two adjacent groups is equal to group phase-shift quantum △t. Group phase-shift's maximum is the phase difference where two signals have full cycle changes.It is quantitative phase-shift resolution △$T=T_B/X$.

3. The Acquisition of High Precision Gate and the Composition of the Measurement System

At the beginning and end of group period,there is phase coincidence phenomenon between measured frequency and standard frequency. With the two adjacent phase coincidence pulses trigger flip-flop, double synchronous gate can be acquired. Phase coincidence signals can be acquired by extracting the overlap of rising edges or falling edges of f_A and f_B.In this way, phase coincidence signals are actually gens pulses. Experiment proves that phase coincidence signal is normal distribution near the phase coincidence points when two signals' frequencies are very close or close to a multiple relationship. As shown in Fig.3, the phase coincidence pulse is named the best phase-coincidence pulses when f_A's and f_B's phase are exactly same.The others are called the false phase coincidence pulses.

Assuming that in the measurement gate, the counter's values for f_A and f_B are respectively N_A and N_B,there are such relation,

Figure 3.Distribution of Phase Coincidence Pulse

$$f_B = \frac{N_B}{N_A} f_A \tag{11}$$

$$\Delta f_B = -\frac{N_B f_A}{N^2_A}\Delta N_A + \frac{f_A}{N_A}\Delta N_B + \frac{N_B}{N_A}\Delta f_A \tag{12}$$

Due to $f_B=N_B/T_{GP}=N_B/(N_A/f_A)$, f_B's relative error is

$$\frac{\Delta f_B}{f_B} = -\frac{\Delta N_A}{N_A} + \frac{\Delta N_B}{N_B} + \frac{\Delta f_A}{f_A} \tag{13}$$

Where f_A is standard frequency, its accuracy is higher than measurement systems.So △f_A/f_A can be neglected.The measuring accuracy is

$$\delta = \left|\frac{\Delta f_B}{f_B}\right| = \left|\frac{\Delta N_A}{N_A}\right| + \left|\frac{\Delta N_B}{N_B}\right| \tag{14}$$

Where △N_A and △N_B are due to the uncertainty of the phase coincidence pulses to trigger gate. Phase coincidence pulses are gens pulses, and their amplitudes are not equal.Some pulses' amplitudes are more than the threshold value of gate.Therefore,phase coincidence pulses' triggering applied to gate has randomness. Assuming the position deviation between actual open-door trigger pulse and the previous best phase coincidence point is R_1, the position deviation between actual close-door trigger pulse and the latter best phase coincidence point is R_2. Relative position deviation is △$R=R_1-R_2$. So △N_A and △N_B of equation (14)are proportional to △R.

$$\delta = \left|\frac{\Delta f_B}{f_B}\right| = \left|\frac{K_A \Delta R}{N_A}\right| + \left|\frac{K_B \Delta R}{N_B}\right| \tag{15}$$

Evidently,the less the number of trigger gate's pulses is,the smaller trigger's contingency and relative position deviation △R are,and thus the higher measurement accuracy is.The larger amplitude difference of phase coincidence pulse is,the smaller trigger's randomness,and the smaller measurement error is.

In order to improve the measurement accuracy,literature [13] puts forward a method that using an adjustable delay circuit reduces the trigger pulse's number and amplitude.In fact, the amount of delay is hard to control,so the application of this method is limited.Literature [14] puts forward that using two-stage-length vernier method reduces the width of the trigger pulse's envelope.But this method is only suitable for the occasion when two frequencies are very close.It can not reduce the width of the trigger pulse's envelope effectively when there is a certain interval between two frequencies.Through the research, as shown in Figure 4,this paper puts forward a new gate control circuit. Phase coincidence detection circuit makes up of G_1~G_3. Programmable counter is for reducing the number of trigger pulses. R_1~R_5,diode D and operational amplifiers A_1~A_2 compose the compression-extension Circuit of phase

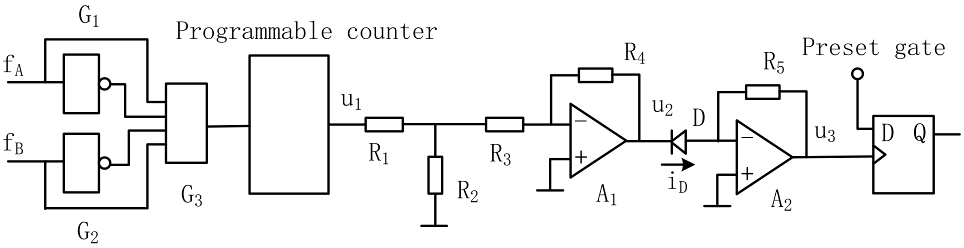

Figure 4. Gate Control Circuit.

coincidence pulse.It is for changing the slope of phase coincidence pulse's envelope.With the nonlinear of diode,when pulse amplitude is small, it is compressed, and when pulse amplitude is large, it is expand.R_1~R_4 and A_1 make u_1(0~3.6V) become u_2(0~ -0.9V).When u_2=-0.5~ -0.7V, i_D changes slowly with u_2 and is compressed.When u_2＜-0.7V, i_D changes quickly with u_2 and is expanded linearly.When u_2 changes from 0V to -0.9V linearly, u_3 changes from 0V to 3V nonlinearly.As shown in Figure 5,it is i_D's nonlinear change that increases the differences among trigger pulses' amplitudes and reduces the number of effective trigger pulses,which greatly reduces the randomness of the trigger gate.

Figure 5. Amplitude Compression and Extension For Phase Coincidence Pulse

As shown Figure 6,to improve the cost performance of frequency measurement system,the whole circuit adopts FPGA except analog circuit. Controller CON is built by NIOSⅡ soft core.CON mainly completes the following functions:to set the gate preset time,to analyse and handle the counting value,to calculate and display the measured frequency and accuracy.

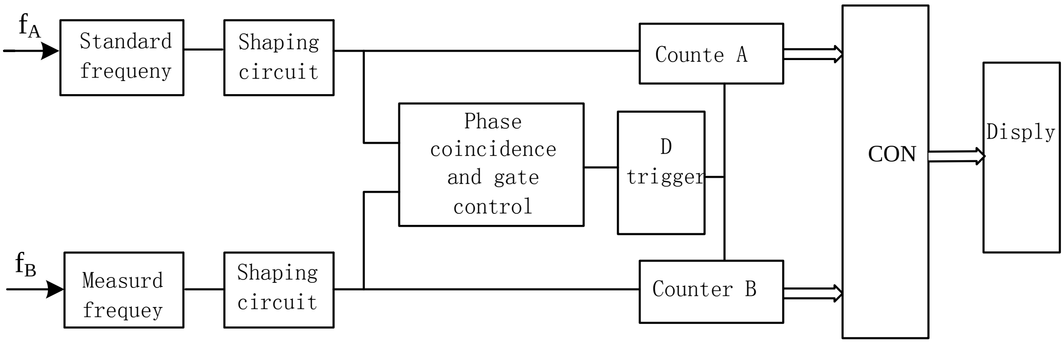

Figure 6. Composition of Measuring System

4. Experimental Results

Equipments used in the experiment include constant temperature crystal oscillator MV83M and frequency synthesizer SSX.MV83M is to produce the standard frequency and SSX's outside standard frequency.The standard frequency is set to 10MHz.SSX is to produce the measured frequency.There are three measured frequencies,which are 10MHz,12.0000001MHz and 23.416MHz. Three groups of measuring data are shown in table 1. According to the measured data, the Allan variance which represents the accuracy of measurement can be calculated .

Table 1. Frequency Measurement Data

First group/MHz	Second group/MHz	Third group/ MHz
10.0000000007	12.0000001003	23.4159999996
10.0000000001	12.0000001002	23.4159999995
10.0000000004	12.0000001006	23.4159999996
10.0000000006	12.0000001005	23.4159999998
10.0000000009	12.0000001003	23.4159999998
10.0000000012	12.0000001008	23.4159999996
10.0000000002	12.0000001007	23.4159999997
10.0000000005	12.0000001004	23.4159999997
10.0000000007	12.0000001003	23.4159999999
10.0000000006	12.0000001006	23.4159999996
10.0000000003	12.0000001004	23.4159999997
10.0000000009	12.0000001007	23.4159999998
10.0000000003	12.0000001011	23.4159999996
10.0000000003	12.0000001002	23.4159999995
10.0000000005	12.0000001005	23.4159999997

$$\sigma_a = \frac{1}{f_0}\sqrt{\frac{\sum_{k=1}^{m-1}(f_{k+1}-f_k)^2}{2(m-1)}} \qquad (16)$$

Where f_0 is frequency nominal value,f_k is measured frequency and m is the number of continuous measurements.The Allan variances calculated from group 1,2and 3 are respectively $\sigma_{a1}=7.628\times10^{-12}$,$\sigma_{a2}=6.237\times10^{-12}$, $\sigma_{a3}=2.317\times10^{-12}$.It can be seen that the larger measured frequency is than standard frequency ,the higher accuracy of measurement is.The reason is that under the condition that the preset gate is invariable, the width of actual double synchronous gate basically remains unchanged,although the number of group periods within the preset gate changes due to f_B to change Δf.Therefore,with the increase of f_B, count quantization error $\Delta N_B/N_B$ decreases.According to equation (14),the accuracy of measurement is improved.

5. Conclusion

Based on comparing standard frequency and measured frequency phase, this paper has analysised the relationship between group phase shift and quantitative phase-shift resolution.It has discussed the relationship between accuracy of measurement and actual trigger pulse position deviation. Applying FPGA, the frequency measurement system was implemented . Through the programmable counters, trigger gate's pulse number has been reduced.Through the application of nonlinear circuit, the amplitude of triggering pulses have been compressed and extened,which improve the accuracy of the triggered gate. Experiments show that accuracy of measurement can reach 10^{-12} ,which is higher than traditional frequency measuring equipment.And with the improvement of the accuracy of electronic components and electronic technology,the accuracy of this method can be further improved.

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