

Relative Entropy Evaluation Method for Multi-sensor Target Recognition

Haiping Ren and Xiaohong Qiu

¹*School of Software, Jiangxi University of Science and Technology, Nanchang
330013, China*

¹*chinarhp@163.com, ²jxauqiu@163.com*

Abstract

The aim of this paper is to propose a new multi-sensor target recognition method base on relative entropy evaluation theory. There are several influencing factors in the target recognition problem, which needs several sensors to work together. Then the multi-sensor target recognition problem can be regarded as a multi-attribute decision making problem. Relative entropy measure can depict the closeness of the two systems, and then this paper will use it to develop an improved TOPSIS method for the multi-sensor target recognition problem. A new characteristic index weights method is proposed, which can avoid the subjectivity of the weight of characteristic indexes. Finally, an application example is used to illustrate the effectiveness and feasibility of the proposed method.

Keywords: *multi-sensor, target recognition, relative entropy, TOPSIS*

1. Introduction

In recent years, multi-sensor target recognition technology becomes a hot topic in multi-sensor data fusion field, which defined as the process of integrating information from multi-source to produce the most specific and comprehensive unified data about an entity, activity or event [1]. Compared with single sensor, multi-sensor can provide redundant, complementary information in space or time, which can produce better recognition effect according to certain integrated rule [2]. Multi-sensor target recognition has many applications in pattern recognition, fuzzy control, robotics and medical field *etc.* There are several influencing factors in the target recognition problem, which needs several sensors to work together. Then the multi-sensor target recognition problem can be regarded as a multi-attribute decision making (MADM) problem. There already have been many authors proposed target recognition methods, which are also well known in multi-attribute decision field. For example, Dempster-Shafer evidence theory method [3-6], fuzzy-Bayesian approach [7], vague set method [8-10], variable fuzzy set method [11], extension method [12], VIKOR method [13] and entropy weights method [14].

For above methods, Dempster-Shafer evidence method extensively depends on the selection of the basic probability assignment. Moreover it needs to know the distribution type and the prior probability. However, the determination of prior probability is greatly empirical in practical operation. The Bayes method also has the same shortcoming. Variable fuzzy sets and extension methods are both artificial determined the weights of characteristic indexes, which exist subjective randomness. For multi-sensor target recognition problem, objective weighting methods are more suitable. Coefficient of variation method, proposed in the paper [13] and entropy weight method [14] are two objective weighting methods. Rao and Patel [15] developed an easy weighting method and applied it in material selection problem. This paper will introduced this objective weighting method to multi-sensor target problem. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method is one of the most often used MADM methods, but it encounters the reverse order problem in practical

applications. Moreover, its evaluation value only reflects the relative proximity of each evaluation object inside but not to the degree of closeness to the ideal optimal solution. Relative entropy measure can depict the closeness of two systems. This paper will use the relative entropy measure to improve TOPSIS method, and then a new multi-sensor target recognition method is put forward.

The rest of this paper is organized as follows. Multi-sensor target recognition model is constructed in Section 2. Section 3 put forwards a new multi-sensor target method, which is an improved TOPSIS method using relative entropy measures. Section 4 gives the application through an example. Finally, Section 5 gives a conclusion.

2. Multi-Sensor Target Recognition Model

Suppose there are several standard parts (target categories) in a target recognition database, and noted as $\pi = \{\pi_1, \pi_2, \dots, \pi_m\}$, and each target has a set of n characteristic indexes $o = \{o_1, o_2, \dots, o_n\}$. Set x_{ij} is the characteristic (attribute) value of category π_i with respect to the character o_j . Then system has a characteristic vector matrix

$$X = (x_{ij})_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

When there is an unknown target (part), we use n different sensors to measure it, then we get the observed value $x_{0j} (j=1, 2, \dots, n)$ with respect to the j th characteristic index o_j . In this case, the task of multi-sensor recognition model is to determine which category will be the unknown object belonging to. The following section will give the specific steps of the new multi-sensor target recognition method.

3. Relative Entropy Evaluation for Multi-sensor Target Recognition

TOPSIS is one of the most often used multi-attribute decision making methods[16-21]. It defines the relative closeness coefficient, which can be both near to the positive ideal solution (PIS) and negative ideal solution (NIS). But the TOPSIS method is a static comprehensive evaluation method. It encounters the reverse order problem in practical applications. Moreover, its evaluation value only reflects the relative proximity of each evaluation object inside but not to the degree of closeness to the ideal optimal solution. The evaluation value is also limited to distinguishing between the ranges of merit ranking. Since TOPSIS method has the wide range of applications, it is necessary to overcome the drawbacks of TOPSIS method [22-23]. In this paper, we will use the relative entropy measure to improve the TOPSIS method, and proposed a new multi-sensor target recognition method. The specific calculation steps are given as follows:

Step 1. Transform the original model X to a minimum and maximum membership function recognition matrix $R = (r_{ij})_{m \times n}$, that is

$$R = (r_{ij})_{m \times n} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \quad (2)$$

where

$$r_{ij} = \frac{\min\{x_{0j}, x_{ij}\}}{\max\{x_{0j}, x_{ij}\}} \quad (3)$$

Let $R_i = (r_{i1}, r_{i2}, \dots, r_{in})$ stand for the i th category (part). Eq. (3) shows that r_{ij} is the relative membership degree between measured value and the characteristic values. Then, the task of target recognition is to find the closest to the ideal target category (i.e. the positive ideal solution).

Step 2. Determine the weights of characteristic indexes.

Weighting methods, which try to define the importance of characteristic indexes, are categorized into subjective, objective, and integrated methods. The subjective methods depend on the expert's preference information to determine the weights. Subjective methods are extensively application in MADM problems, but for multi-sensor target recognition problem, there is a need for objective weighting. In this paper, the suggested weighting technique, proposed by Rao and Patel [15], which is an objective method based on standard deviation and given as follows:

$$w_j = \frac{\sigma_j}{\sum_{j=1}^n \sigma_j} \quad (4)$$

Where

$$\sigma_j = \sqrt{\frac{1}{m} \sum_{i=1}^m (r_{ij} - \bar{r}_j)^2}, \quad j = 1, 2, \dots, n \quad (5)$$

It is obvious that w_j satisfies $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$.

This weighting technique given by Eq. (6), can also refer the paper [24].

Step 3. Calculate the weighted minimum and maximum membership function matrix

$$G = (g_{ij})_{m \times n} = (w_j \cdot r_{ij})_{m \times n} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix} \quad (6)$$

Step 4. Define the positive ideal solution (PIS) and negative ideal solution (NIS) as follows.

The PIS is defined as

$$g^* = (g_1^*, g_2^*, \dots, g_n^*) = (\max_i \{g_{i1}\}, \max_i \{g_{i2}\}, \dots, \max_i \{g_{in}\}) \quad (7)$$

The NIS is defined as

$$g^- = (g_1^-, g_2^-, \dots, g_n^-) = (\min_i \{g_{i1}\}, \min_i \{g_{i2}\}, \dots, \min_i \{g_{in}\}) \quad (8)$$

Step 5. Improved TOPSIS method using relative entropy measure.

In probability theory and information theory, the relative entropy, also named Kullback–Leibler divergence, which is defined as follows:

Suppose that A and B are two systems, and they include n states A_i and B_i ($i = 1, 2, \dots, n$) respectively, then the difference of system A and B can be measured by Kullback–Leibler divergence ([25]), and the formula is

$$C = \sum_{i=1}^n \left\{ A_i \log \frac{A_i}{B_i} + (1 - A_i) \log \frac{1 - A_i}{1 - B_i} \right\} \quad (9)$$

Here, the smaller of C the smaller difference of system A and B , and C is called relative entropy.

The paper [26] pointed though the relative entropy is not the real distance measure of system A and B , but it can improve the TOPSIS method when use it to measure the

difference of system A and B . Improved TOPSIS method using relative entropy measure is given as follows:

(i) Calculate the relative entropy measure matrix $S1=(s_{ij}^*)_{m \times n}$ of each characteristic index with PIS, where

$$s_{ij}^* = g_j^* \log \frac{g_j^*}{g_{ij}} + (1 - g_j^*) \log \frac{1 - g_j^*}{1 - g_{ij}} \quad (10)$$

Calculate the relative entropy measure matrix $S2=(s_{ij}^-)_{m \times n}$ of each characteristic index with NIS, where

$$s_{ij}^- = g_j^- \log \frac{g_j^-}{g_{ij}} + (1 - g_j^-) \log \frac{1 - g_j^-}{1 - g_{ij}} \quad (11)$$

(ii) Define the relative entropy measure S_i^* of each category π_i with PIS and the relative entropy measure S_i^- of each category π_i with NIS as follows:

$$S_i^* = \sum_{j=1}^n s_{ij}^* \quad (12)$$

$$S_i^- = \sum_{j=1}^n s_{ij}^- \quad (13)$$

(iii) Define the relative closeness coefficient C_i .

$$C_i = \frac{S_i^-}{S_i^- + S_i^*}, i=1,2,\dots,m \quad (14)$$

The paper [26] have proved that the new relative closeness coefficient C_i is an improve ranking function of ordinal TOPSIS. The larger of C_i is, the closer of category π_i with PIS is.

Step 6. Recognition rule.

From the above analysis, the multi-sensor target recognition rule is given as follows:

If

$$k_0 = \arg \max_{1 \leq i \leq m} \{C_i\} \quad (15)$$

Then the unknown object belongs to the target π_{k_0} .

4. Example Study

To illustrate the effectiveness and feasibility of the new multi-sensor target recognition method, An examples will be given to used the application of the proposed method. This example adopted from the paper [27,11]. In order to realize the automatic recognition and classification for intelligent robots, four independent characteristic indexes are used to demonstrate the work piece (part). The four characteristic indexes are θ_1 (shape factor), θ_2 (the section center moment), θ_3 (surface reflection ability), and θ_4 (surface roughness) of the (part). The weights of characteristic indexes are unknown.

There are four standard parts used to test in the experiment, and four sensors are used to measure the unknown part. The characteristic index values of four standard parts and measurement value of unknown part are shown in Table 1.

Table 1. Characteristic Index Values of Four Standard Parts and Unknown Part

Part	θ_1	θ_2	θ_3	θ_4
1	1.30	1.86	3.07	2.75
2	2.43	3.71	2.28	2.34
3	2.18	1.93	1.37	1.52
4	1.85	2.52	2.97	1.93
Unknown part	2.15	2.30	2.80	2.12

This example presents the use of the proposed method to recognize the unknown part.

Step 1. By Eq. (1) and the data reported in Table 1, the minimum and maximum membership function model $R = (r_{ij})_{m \times n}$ can be obtained as follows:

$$R = \begin{pmatrix} 0.6047 & 0.8087 & 0.9121 & 0.7709 \\ 0.8848 & 0.6199 & 0.8143 & 0.9060 \\ 0.9862 & 0.8391 & 0.4893 & 0.7170 \\ 0.8605 & 0.9127 & 0.9428 & 0.9107 \end{pmatrix} \quad (16)$$

Step 2. According to Eq. (4) and Eq. (5), the weights of characteristic indexes can be obtained as follows:

$$w_1 = 0.2741, w_2 = 0.2106, w_3 = 0.3506, w_4 = 0.1646 \quad (17)$$

Step 3. The weighted minimum and maximum membership function matrix $G = (g_{ij})_{m \times n}$ is given as follows:

$$G = (g_{ij})_{m \times n} = \begin{bmatrix} 0.1658 & 0.1703 & 0.3198 & 0.1269 \\ 0.2426 & 0.1306 & 0.2855 & 0.1491 \\ 0.2407 & 0.1767 & 0.1716 & 0.1180 \\ 0.2359 & 0.1922 & 0.3306 & 0.1499 \end{bmatrix} \quad (18)$$

Step 4. Determine the positive ideal solution and negative ideal solution, and given as follows:

$$g^* = (g_1^*, g_2^*, g_3^*, g_4^*) = (0.2426, 0.1922, 0.3306, 0.1499) \quad (19)$$

$$g^- = (g_1^-, g_2^-, g_3^-, g_4^-) = (0.1658, 0.1306, 0.1716, 0.1180) \quad (20)$$

Step 5. The relative entropy matrix of each characteristic index with PIS and the relative entropy matrix S_i^- of each characteristic index with NIS as follows:

$$S_1 = (s_{ij}^*)_{m \times n} = \begin{bmatrix} 0.0345 & 0.0016 & 0.0003 & 0.0023 \\ 0.0021 & 0.0149 & 0.0048 & 0.0000 \\ 0.0000 & 0.0008 & 0.0071 & 0.0045 \\ 0.0032 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad (21)$$

$$S_2 = (s_{ij}^-)_{m \times n} = \begin{bmatrix} 0.0000 & 0.0060 & 0.0565 & 0.0004 \\ 0.0175 & 0.0000 & 0.0352 & 0.0041 \\ 0.0307 & 0.0079 & 0.0000 & 0.0000 \\ 0.0148 & 0.0135 & 0.0640 & 0.0043 \end{bmatrix} \quad (22)$$

Step 6. The relative entropy measure S_i^* of each category π_i with PIS and the relative entropy measure S_i^- of each category π_i with NIS are botained as follows:

$$S_1^* = 0.0387, S_2^* = 0.0218, S_3^* = 0.0795, S_4^* = 0.0032 \quad (23)$$

$$S_1^- = 0.0628, S_2^- = 0.0567, S_3^- = 0.0386, S_4^- = 0.0965 \quad (24)$$

Step 7. The closeness coefficient of each category is calculated as follows:

$$C_1 = 0.6190, C_2 = 0.7222, C_3 = 0.3267, C_4 = 0.9680 \quad (25)$$

Step8. Due to the maximum $C_4 = 0.9680$, so according to the recognition rule Eq. (15), the unknown part belongs to the fourth kind of work piece. The recognition results are consistent with [11].

In the following discussion, we will make a comparision analysis among our method with other methods, which are the entropy weights method [14], variable fuzzy sets method [11], and double base points method [28]. We take all evaluation value of target value as the support degree. Then despite their recognition results are all the fourth part, but four methods for all kinds of target recognition degree are different. Table 2 shows the four methods the support degree and deviation of each support degree form the maximum support degree (The fourth part's support degree).

Table 2. Comparison Results of Four Methods

Part	1	2	3	4
Entropy weights method[14]	0.7851	0.6973	0.2322	0.9192
	0.1611	0.2219	0.6870	
Variable fuzzy sets method[11]	0.366	0.581	0.556	0.790
	0.424	0.209	0.234	
Double base points method[28]	0.4600	0.4726	0.4662	0.7857
	0.3257	0.3131	0.3195	
The proposed method	0.6190	0.7222	0.3267	0.9680
	0.3490	0.2458	0.6413	

From Table 2, we can see that the proposed method gives the support degree 0.9680 of category 4 which is the largest value of the four methods. If we use of the forth part's support degree minus other parts' support degree, then the total gap of support degree with respect to four methods are respectively: 1.0700, 0.8670, 0.9583 and 1.2361, so the proposed method get support degree always the biggest gap. Obviously, the greater the gap is, the higher level of target recognition results is, and then the higher the credibility is.

5. Conclusions

For the multi-sensor target recognition problem, there are several influencing factors in the target recognition problem, which needs several sensors to work together. Then the multi-sensor target recognition problem can be regarded as a MADM problem. TOPSIS is one of the most often used MADM methods, but it encounters the reverse order problem in practical applications. Moreover, its evaluation value only reflects the relative proximity of each evaluation object inside but not to the degree of closeness to the ideal optimal solution. Relative entropy

measure can depict the closeness of two systems. Thus we use the relative entropy measure to improve the TOPSIS method, and proposed a new multi-sensor target recognition method. A new characteristic index weights method is also be proposed, which can avoid the subjectivity of the weight of characteristic indexes. Finally, an application example is used to analyze the effectiveness and feasibility of the proposed method. Through comparing with other methods, our method is effective and works well in multi-sensor target recognition problem. The new algorithm is simple and easy to use Matlab to solve it. The method can be treated as a multi-attribute decision making method, which can also be applied to many practical problems, such as investment project selection, robot selection and material selection.

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Authors



Haiping Ren is a Lecture of Jiangxi University of Science and Technology. He obtained his bachelor degree of applied mathematics in 2003 from Changsha University Science and Technology, and master degree of probability and statistics in 2005 from Central South University. He obtained doctor degree from Jiangxi University of Finance and Economics of management in 2015. His major researches are Bayes statistics, fuzzy making and information fusion.



Xiaohong Qiu is a professor of Jiangxi University of science and technology. he obtained his bachelor degree of automatic control in 1989 from Beijing University of Aeronautics and Astronautics, master degree and doctor degree of Vehicle Control Guidance and Simulation in 1992 and 1995 respectively from Beijing University of Aeronautics and Astronautics. His major researches are Intelligent computing and multi-attribute decision making.