International Journal of Control and Automation
Vol. 9, No. 6 (2016), pp.241-250
http//dx.doi.org/10.14257/ijca.2016.9.6.23

The Design of Guaranteed Cost Reliable Controller for T-S Fuzzy Time-Delay Systems with Dynamic Output Feedback

Huang Ling, Sun Min, Wang Shen and Wang Jiexun

School of Automation, Harbin University of Science and Technology, Harbin 150080, China
Email: huangling@hrbust.edu.cn

Abstract

Guaranteed cost reliable controller for T-S uncertain time-delay systems with actuator failure is studied in this paper. For a class of time-delay systems with norm bounded uncertainties, the sufficient existence condition of guaranteed cost reliable controller is deduced with the theory of Lyapunov and linear matrix inequality. Meanwhile, singular value decomposition is used to construct the design method of the controller. Finally, a numerical example is given to show the steps and method of the theories proposed in the paper, also show the correctness of the theories.

Keywords: *output feedback; Fuzzy System; Guaranteed Cost; actuator failures*

1. Introduction

In 1972, the guaranteed cost theory was proposed firstly by Chang and Peng[1]. The quadratic performance index is less than an upper bound for all the uncertainty. There is a close connection among the guaranteed cost control and robust control、h-infinity control、optimal control[2], and a wide attention is received for guaranteed cost control.

Literature [3] proposed a quadratic performance index for a class of linear discrete-time uncertain systems, the design of guaranteed cost controller with norm-bounded time-varying parameter is studied. The optimal guaranteed cost reliable controller is designed with convex optimization method and LMI method. Literature [4] mainly studies on the issue of H_2 / H_∞ guaranteed cost control of a class of uncertainty discrete systems, and the uncertainty depends on bounded delay. Sufficient existence condition has been derived. The feasible solution is discussed with linear matrix inequality technique. Meanwhile, the H_2 / H_∞ optimal guaranteed cost controller is designed. Literature [5] studies a class of discrete uncertain time-delay systems, the guaranteed cost controller is designed with selected Lyapunov Function, and the sufficient existence condition of this controller could be deduced with LMI form. And the effective control methods have pushed the research of guaranteed cost controller for uncertain discrete-time systems to a new level.

Since the fuzzy logic control theory was put forward by Zadeh[6], the research of guaranteed cost control for fuzzy control system becomes one of the most important fields of the study of guaranteed cost control[7-10]. Literature [7] studies a class of uncertain T-S fuzzy time-delay switched systems. Guaranteed cost controller is designed with PDC method, the sufficient existence condition of guaranteed cost control is deduced with single Lyapunov Function and multiple Lyapunov Function, and the switching law is designed, the closed loop control system realizes guaranteed cost control for all uncertainty which is allowed. Literature [8] studies a class of T-S uncertain time-delay systems; the reliable guaranteed cost control method is put forward under the actuator fault. With the construction of the T-S uncertain time-delay system model, a performance index of guaranteed cost control is given by proper Lyapunov Function, a group of LMIs

ISSN: 2005-4297 IJCA

are solved, and the sufficient existence condition of reliable guaranteed cost control is deduced. Literature [9] studies the h-infinity tracking of nonlinear networked control system which is based on T-S model, and the solution has been well used in forced concussion system, then a well tracking is obtained.

This paper studies guaranteed cost control of uncertain time-delay system with actuator failure in the situation that the system state is unavailable. The sufficient existence condition of the guaranteed cost control is constructed by using dynamic output feedback and Lyapunov Function method, and then the dynamic output feedback controller is designed.

2. Problem Descriptions

Considering a nonlinear NCS described by a T-S fuzzy model, the i fuzzy rule is:

R^i : if $X_1(t)$ is M_{i1} , …, $X_g(t)$ is M_{ig} , then

$$\begin{aligned} \dot{x}(t) &= (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\tau) + B_i u_f(t) \\ y(t) &= (C_i + \Delta C_i)x(t) + (C_{di} + \Delta C_{di})x(t-\tau) \end{aligned} \tag{1}$$

In which, $M_{il}, i = 1, 2, \dots, r, l = 1, 2, \dots, g$ are fuzzy sets, r is the number of fuzzy rule, $x(t) \in R^n$ is state vector, $u_f(t) \in R^m$ is input control, $y(t) \in R^p$ is measured output, $\tau > 0$ is time-delay constant, $X_1(t), X_2(t), ..., X_g(t)$ are promise variable, $A_i, A_{di}, C_i, C_{di}, B_i$, are real constant matrices with proper dimensions.

Uncertain matrices are:

$$\begin{bmatrix} \Delta A_i & \Delta A_{di} \\ \Delta C_i & \Delta C_{di} \end{bmatrix} = \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix} F(t) \begin{bmatrix} N_{1i} & N_{di} \end{bmatrix} \tag{2}$$

In which, $M_{1i}, M_{2i}, N_{1i}, N_{di}$, are real constant matrices, $F(t)$ is unknown matrix, and satisfies

$$F^T(t)F(t) \le I$$

Global fuzzy models could be written as:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(X(t)) \left[(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-d) + B_i u_f(t) \right] \\ y(t) &= \sum_{i=1}^{r} h_i(X(t)) \left[(C_i + \Delta C_i)x(t) + (C_{di} + \Delta C_{di})x(t-d) \right] \\ x(t) &= \varphi(t), t \in [-\tau, 0] \end{aligned} \tag{3}$$

In which,

$$\begin{aligned} & X(t) = (X_1(t), X_2(t), \cdots X_g(t)) \\ & h_i(X(t)) = \mu_i(X(t)) \Big/ \sum_{i=1}^{m} \mu_i(X(t)) \ge 0 \\ & \sum_{i=1}^{r} h_i(X(t)) = 1, \mu_i(X(t)) = \prod_{j=1}^{r} M_{ij}(X_j(t)) \end{aligned}$$

$M_{il}(X_l(t))$ represents the promise variables $X_l(t)$ corresponding the fuzzy-valued M_{il}, the membership degree is obtained.

Assuming control input actuator of this system is under fault, the expression is:

$$u_f(t) = Nu(t) \tag{4}$$

In which $N = diag(n_1, n_2, \dots, n_m)$, $0 \le n_i \le 1$. $n_i = 0$ means the signal is interrupted in actuator of channel i; $n_i = 1$ means the actuator works well; $0 < n_i < 1$ means the actuator has local fault, or there is a local dimension reduction of the actuator.

For system (3), we design the following form of output feedback control law:

R^i : if $X_1(t)$ is M_{i1} and, …, and $X_g(t)$ is M_{ig} , then

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{ci}\hat{x}(t) + B_{ci}y(t) \\ u(t) &= C_{ci}\hat{x}(t) \\ \hat{x}(t) &= 0, t \in [-\tau, 0] \end{aligned} \tag{5}$$

In which $\hat{x}(t) \in R^n$ is the state of controller.

The global fuzzy output feedback control law of (5) is:

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(X(t))\left[A_{ci}\hat{x}(t) + B_{ci}y(t)\right] \\ u(t) &= \sum_{i=1}^{r} h_i(X(t))\left[C_{ci}\hat{x}(t)\right] \\ \hat{x}(t) &= 0, t \in [-\tau, 0] \end{aligned} \tag{6}$$

Using the control law (6) to system (3), the following closed loop system state equation is obtained:

$$\begin{aligned} \dot{\xi}(t) &= \sum_{i=1}^{r} h_i(X(t)) \sum_{j=1}^{r} h_j(X(t))\left[\tilde{A}_{ij}\xi(t) + \tilde{B}_{ij}H\xi(t-d)\right] \\ \xi(t) &= \phi(t), t \in [-\tau, 0] \end{aligned} \tag{7}$$

In which,

$$\xi(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}, \quad \tilde{A}_{ij} = \begin{bmatrix} A_i + \Delta A_i & NB_iC_{ci} \\ B_{ci}(C_i + \Delta C_i) & A_{ci} \end{bmatrix}, \tilde{B}_{ij} = \begin{bmatrix} A_{di} + \Delta A_{di} \\ B_{ci}(C_{di} + \Delta C_{di}) \end{bmatrix}$$

$$H = \begin{bmatrix} I & 0 \end{bmatrix}, \quad x(t-d) = H\xi(t-d)$$

From uncertain matrix (2) , (7) could be expressed as:

$$\begin{aligned} \dot{\xi}(t) &= \sum_{i=1}^{r} h_i(X(t)) \sum_{j=1}^{r} h_j(X(t))\left[\left(\bar{A}_{ij} + \bar{M}_{ij}F(t)\bar{N}_{ij}\right)\xi(t) + \left(\bar{A}_{dij} + \bar{M}_{ij}F(t)\bar{N}_{di}\right)x(t-d)\right] \\ \xi(t) &= \phi(t), t \in [-\tau, 0] \end{aligned} \tag{8}$$

In which,

$$\bar{A}_{ij} = \begin{bmatrix} A_i & NB_iC_{ci} \\ B_{ci}C_i & A_{ci} \end{bmatrix}, \bar{A}_{dij} = \begin{bmatrix} A_{di} \\ B_{ci}C_{di} \end{bmatrix}, \bar{M}_{ij} = \begin{bmatrix} M_{1i} \\ B_{ci}M_{2i} \end{bmatrix}$$

$$\bar{N}_{ij} = \begin{bmatrix} N_{1i} & 0 \end{bmatrix}, \bar{N}_{di} = N_{di}$$

A performance index is defined:

$$J = \int_{t_0}^{\infty} \left[x^T(t)Qx(t) + u_f^{\,T}(t)Ru_f(t)\right]dt$$

In which Q, R are given symmetric positive definite matrices.

The performance indicator of closed-loop system is:

$$\begin{aligned} J &= \int_{t_0}^{\infty} \left[x^T(t)Qx(t) + \hat{x}^T(t)NC_{ci}^{\,T}RC_{ci}N\hat{x}(t)\right]dt \\ &= \int_{t_0}^{\infty} \xi^T(t)\bar{C}_j^{\,T}\bar{C}_j\xi(t)\,dt \end{aligned} \tag{9}$$

In which,

$$\bar{C}_j = \begin{bmatrix} Q^{1/2} & 0 \\ 0 & NR^{1/2}C_{ci} \end{bmatrix}$$

The problem researched in this paper can be stated as following, we design an output feedback controller (6) for the fuzzy system (3) such that the performance (9) of closed-loop system is lower than a given level.

Before the main result is given, the following lemma is introduced:

Lemma 1: $Y = Y^T$ is given, D and E are matrices with arbitrary dimensions, and F satisfies $F^TF \le I$, then the necessary and sufficient condition for the following inequality

$$Y + DFE + E^TF^TD^T < 0$$

is there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon DD^T + \varepsilon^{-1}E^TE < 0$$

holds.

4. The Design of Output Feedback Guaranteed Cost Controller

Theorem 1: given fuzzy control system (3) and quadratic performance index (9), if there exist a scalar $\varepsilon_{ij} > 0\left(i, j = 1, 2, \cdots m\right)$, symmetric positive definite matrices $P \in R^{2n\times 2n}, T \in R^{n\times n}$ such that the matrix inequality

$$\begin{bmatrix} \bar{A}_{ij}^{\;T} P + P\bar{A}_{ij} & P\bar{A}_{dij} & H^T & P\bar{M}_{ij} & \bar{N}_{ij}^T & \bar{C}_j^T \\ & -T & 0 & 0 & \bar{N}_{di}^T & 0 \\ & & -T^{-1} & 0 & 0 & 0 \\ & & & -\varepsilon_{ij}^{\;-1} I & 0 & 0 \\ & & & & -\varepsilon_{ij} I & 0 \\ & & & & & -I \end{bmatrix} < 0 \tag{10}$$

holds, then there exists output feedback guaranteed cost controller (6) such that closed-loop system (8) is asymptotic stability, and the quadratic performance index satisfies:

$$J \le J^* = \xi^T\left(t_0\right) P \xi\left(t_0\right) + \int_{t_0 - d}^{t_0} x^T\left(\tau\right) T x\left(\tau\right) d\tau$$

Proof: The Lyapunov Function

$$V\left(t\right) = \xi^T\left(t\right) P \xi\left(t\right) + \int_{t-d}^{t} \xi^T\left(\tau\right) H^T T H \xi\left(\tau\right) d\tau \tag{11}$$

is selected, in which, P, T are symmetric positive definite matrices. Calculate the derivation of $V(t)$ along the system (8), the following arithmetic expression is obtained:

$$\begin{aligned} \dot{V}\left(t\right) &= 2\xi^T\left(t\right) P \dot{\xi}\left(t\right) + x^T\left(t\right) T x\left(t\right) - x^T\left(t-d\right) T x\left(t-d\right) \\ &\le \sum_{i=1}^{r} h_i\left(X\left(t\right)\right) \sum_{j=1}^{r} h_j\left(X\left(t\right)\right) \Big\{ 2\xi^T\left(t\right) P \Big[\left(\bar{A}_{ij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{ij}\right) \xi\left(t\right) \\ &\quad + \left(\bar{A}_{dij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{di}\right) x\left(t-d\right) \Big] + \xi^T\left(t\right) \hat{T} \xi\left(t\right) - x^T\left(t-d\right) T x\left(t-d\right) \\ &\quad + \xi^T\left(t\right) \bar{C}_j^T \bar{C}_j \xi\left(t\right) \Big\} \end{aligned} \tag{12}$$

$$= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i\left(X\left(t\right)\right) h_j\left(X\left(t\right)\right) \begin{bmatrix} \xi\left(t\right) \\ x\left(t-d\right) \end{bmatrix}^T \cdot$$

$$\begin{bmatrix} \left(\bar{A}_{ij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{ij}\right)^T P + P\left(\bar{A}_{ij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{ij}\right) + \hat{T} + \bar{C}_j^{\;T} \bar{C}_j & P\left(\bar{A}_{dij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{di}\right) \\ \left(\bar{A}_{dij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{di}\right)^T P & -T \end{bmatrix}$$

$$\cdot \begin{bmatrix} \xi\left(t\right) \\ x\left(t-d\right) \end{bmatrix}$$

In which， $\hat{T} = diag\left\{T, 0\right\}$.

If the closed-loop system is asymptotic stability, then $\dot{V}\left(t\right) < 0$ must be satisfied, and generally the membership degree function is greater than zero, so the following arithmetic expression is correct:

$$\begin{bmatrix} \left(\bar{A}_{ij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{ij}\right)^T P + P\left(\bar{A}_{ij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{ij}\right) + \hat{T} + \bar{C}_j^{\;T} \bar{C}_j & P\left(\bar{A}_{dij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{di}\right) \\ \left(\bar{A}_{dij} + \bar{M}_{ij} F\left(t\right) \bar{N}_{di}\right)^T P & -T \end{bmatrix} < 0$$

That is to say the following inequality is right

$$\begin{bmatrix} \bar{A}_{ij}{}^{T}P + P\bar{A}_{ij} + \hat{T} + \bar{C}_{j}{}^{T}\bar{C}_{j} & P\bar{A}_{dij} \\ \bar{A}_{dij}{}^{T}P & -T \end{bmatrix} + \begin{bmatrix} P\bar{M}_{ij} \\ 0 \end{bmatrix} F(t) \begin{bmatrix} \bar{N}_{ij} & \bar{N}_{di} \end{bmatrix} + \begin{bmatrix} \bar{N}_{ij} & \bar{N}_{di} \end{bmatrix}^{T} F^{T}(t) \begin{bmatrix} P\bar{M}_{ij} \\ 0 \end{bmatrix}^{T} < 0 \tag{13}$$

According to lemma 1, if and only if the scalar $\varepsilon_{ij} > 0(i, j = 1, 2, \cdots m)$ exists, then:

$$\begin{bmatrix} \bar{A}_{ij}{}^{T}P + P\bar{A}_{ij} + \varepsilon_{ij}P\bar{M}_{ij}\bar{M}_{ij}{}^{T}P + \varepsilon_{ij}{}^{-1}\bar{N}_{ij}{}^{T}\bar{N}_{ij} + \hat{T} + \bar{C}_{j}^{T}\bar{C}_{j} & P\bar{A}_{dij} + \varepsilon_{ij}{}^{-1}\bar{N}_{ij}{}^{T}\bar{N}_{di} \\ \bar{A}_{dij}{}^{T}P + \varepsilon_{ij}{}^{-1}\bar{N}_{di}{}^{T}\bar{N}_{ij} & -T + \varepsilon_{ij}{}^{-1}\bar{N}_{di}{}^{T}\bar{N}_{di} \end{bmatrix} < 0$$

According to Schur compensation lemma, the above expression can be transformed into the arithmetic expression (10).

Meanwhile, to the inequality:

$$\dot{V}(t) \le \sum_{j=1}^{r} h_j\left(X(t)\right)\xi^{T}(t)\bar{C}_j^{T}\bar{C}_j\xi(t)$$

The integral is made on both sides and result in:

$$\int_{t_0}^{\infty}\dot{V}(t)dt \le \sum_{j=1}^{r} h_j\left(X(t)\right)\int_{t_0}^{\infty}\xi^{T}(t)\bar{C}_j^{T}\bar{C}_j\xi(t)dt$$

With the system stability and the initial condition, the following expression is obtained:

$$J \le J^{*} = V(t_0) = \xi^{T}(t_0)P\xi(t_0) + \int_{t_0-d}^{t_0} x^{T}(\tau)Tx(\tau)d\tau$$

Theorem 2: given global fuzzy model (3) and quadratic performance index (9), if there exist matrices $R > 0, S > 0$, scalars $\varepsilon_{ij} > 0(i, j = 1, 2, \cdots m)$, symmetric positive-definite matrices $\tilde{T}, W, T \in R^{n\times n}$, and matrices $\hat{A}_{ij}, r_{i1}, r_{i2}(i, j = 1, 2, \cdots m)$, such that following matrix inequalities

$$\begin{aligned} &\Xi_{ii} < 0 \\ &\Xi_{ij} + \Xi_{ji} < 0 \end{aligned} \tag{14}$$

hold, then the output feedback guaranteed cost control (6) exists.

In (14),

$$\Xi_{ii} = \begin{bmatrix} \Xi_{11ii} & \Xi_{12ii} \\ \Xi_{21ii} & \Xi_{22ii} \end{bmatrix}$$

$$\Xi_{11ij} = \begin{bmatrix} A_iW + WA_i^{T} + B_i\Upsilon_{i1} + \Upsilon_{i1}{}^{T}B_i^{T} & A_i + \hat{A}_{ij}{}^{T} & A_{di} & W \\ * & A_i^{T}\tilde{T} + \tilde{T}A_i + \Upsilon_{i2}C_i + C_i{}^{T}\Upsilon_{i2}{}^{T} & \tilde{T}A_{di} + \Upsilon_{i2}C_{di} & I \\ * & * & -T & 0 \\ * & * & * & -T^{-1} \end{bmatrix}$$

$$\Xi_{12ij} = \begin{bmatrix} M_{1i} & W\tilde{N}_{1i}{}^{T} & WQ^{1/2} & \Upsilon_{i1}{}^{T}R^{1/2} \\ \tilde{T}M_{1i} + \Upsilon_{i2}M_{2i} & \tilde{N}_{1i}{}^{T} & Q^{1/2} & 0 \\ 0 & \tilde{N}_{di}{}^{T} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{22ij} = diag\left\{-\varepsilon_{ij}{}^{-1}I \quad -\varepsilon_{ij}I \quad -I \quad -I\right\}$$

Proof: Decompose the matrix P and its inverse matrix as:

$$P = \begin{bmatrix} \tilde{T} & \tilde{N} \\ \tilde{N}^{T} & U \end{bmatrix}, P^{-1} = \begin{bmatrix} W & M \\ M^{T} & V \end{bmatrix}$$

In which $W, \tilde{T}, M, \tilde{N} \in R^{n\times n}$, the following expression could be got from $PP^{-1} = I$

$$M\tilde{N}^{T} = I - W\tilde{T} \tag{15}$$

Define matrix,

$$X_1 = \begin{bmatrix} W & I \\ M^T & 0 \end{bmatrix}, X_2 = \begin{bmatrix} I & \tilde{T} \\ 0 & \tilde{N}^T \end{bmatrix} \tag{16}$$

then $PX_2 = X_1$。

Making a left multiplication with $diag\{X_2^T \quad I \quad I \quad I \quad I \quad I\}$ to the expression (10), and make a right multiplication to the arithmetic expression (10) with its transposed matrix, and define

$$\begin{aligned} &\Upsilon_{i1} = NC_{ci}M^T, \Upsilon_{i2} = \tilde{N}B_{ci} \\ &\hat{A}_{ij} = \tilde{T}A_iW + \tilde{N}B_{ci}C_iW + N\tilde{T}B_iC_{ci}M^T + \tilde{N}A_{ci}M^T \end{aligned} \tag{17}$$

Then substituting the above variables, the inequality

$$\begin{bmatrix} \Xi_{11ii} & \Xi_{12ii} \\ \Xi_{21ii} & \Xi_{22ii} \end{bmatrix} < 0$$

can be obtained, the arithmetic expression (14) is correct.

If there is a feasible solution of linear matrix inequality (14), the coefficient matrix of output feedback guaranteed cost control and symmetric positive definite matrix P could be obtained with following steps:

(1)Given matrix T and scalar ε_{ij}, by solving linear matrix inequality (14), an upper bound of the closed-loop system quadratic performance index and invertible matrices $W, \tilde{T}$ and $\Upsilon_{i1}, \Upsilon_{i2}, \hat{A}_{ij}$ can be obtained.

(2)With singular value decomposition of the matrix $I - W\tilde{T}$, the matrices $M, \tilde{N}$ can be obtained; matrices X_1, X_2 are defined by (16), the symmetric positive definite matrix P is given from $P = X_1X_2^{-1}$.

(3)From arithmetic expression (17), coefficient matrices A_{cj}, B_{cj}, C_{cj} of the controller are obtained.

5. A Simulation Example

A rigid manipulator connect the substrate with a rotating hinge on one side, the motion equation expression is:

$$G\ddot{\vartheta} = -(0.5mgl + Mgl)\sin\vartheta + L$$

In which ϑ is the hinge rotation speed, $m = 1.5kg$ is load weight, $M = 3kg$ is the manipulator quality, $g = 9.8m/s^2$ is gravity acceleration, $l = 0.5m$ is manipulator length, $G = Ml^2 + 1/3ml^2$ is inertia constant, L is controlling torque, $\vartheta = 0$ is manipulator equilibrium position, and perpendicular to the horizontal plane, the intention of control is to make the manipulator back to the equilibrium position in the range of $\vartheta \in [0, \pi/2]$.

Define the state variables $x_1(t) = \vartheta$, $x_2(t) = \dot{\vartheta}$, constructing T-S fuzzy model:

Rule 1: when $x_1(t) \approx 0$ 时,

$$\begin{aligned} \dot{x}(t) &= (A_1 + \Delta A_1)x(t) + (A_{d1} + \Delta A_{d1})x(t-d) + B_1u(t) \\ y(t) &= (C_1 + \Delta C_1)x(t) + (C_{d1} + \Delta C_{d1})x(t-d) \end{aligned}$$

Rule 2: when $x_1(t) \approx \pi/2$ 时,

$$\begin{aligned} \dot{x}(t) &= (A_2 + \Delta A_2)x(t) + (A_{d2} + \Delta A_{d2})x(t-d) + B_2u(t) \\ y(t) &= (C_2 + \Delta C_2)x(t) + (C_{d2} + \Delta C_{d2})x(t-d) \end{aligned}$$

In which,

$$A_1=\begin{bmatrix}0 & 1\\ -(0.5mgl+Mgl)/G & 0\end{bmatrix}, A_2=\begin{bmatrix}0 & 1\\ -2(0.5mgl+Mgl)/\pi G & 0\end{bmatrix}$$

$$A_{d1}=\begin{bmatrix}0.1 & 0.1\\ -0.1 & 0.1\end{bmatrix}, A_{d2}=\begin{bmatrix}0 & 0.2\\ -0.2 & 0\end{bmatrix}, B_1=B_2=\begin{bmatrix}0\\ 1/G\end{bmatrix}$$

$$C_1=[1.5 \quad 0], C_2=[1 \quad 0] C_{d1}=[0.01 \quad 0], C_{d2}=[-0.02 \quad 0]$$

$$M_{11}=\begin{bmatrix}0.2\\ 0.1\end{bmatrix}. M_{12}=\begin{bmatrix}0.1\\ 0.3\end{bmatrix}, M_{21}=0.2, M_{22}=0.1$$

$$N_{11}=[0 \quad 0.5], N_{12}=[0 \quad -0.5], N_{d1}=N_{d2}=[0 \quad -0.5], N_{21}=N_{22}$$

The membership degree functions are $F_1(x_1)=\sin^2(x_1)$， $F_2(x_1)=1-F_1(x_1)$

When N=1，the actuator works with no failures，the linear matrix inequality (13) is solved with LMI tool box, then,

$$W=\begin{bmatrix}0.211 & -0.0174\\ -0.0174 & 0.3932\end{bmatrix}, \tilde{T}=\begin{bmatrix}186.3010 & -11.0715\\ -11.0715 & 9.9833\end{bmatrix}$$

$$\gamma_{11}=[0.0007 \quad -1.1301], \gamma_{21}=[0.0000 \quad -1.0946], \gamma_{12}=\begin{bmatrix}-186.0108\\ 12.1903\end{bmatrix}$$

$$\gamma_{22}=\begin{bmatrix}-159.3631\\ -44.5003\end{bmatrix}, \hat{A}_{11}=\begin{bmatrix}0.0282 & 21.0693\\ -1.1106 & -0.6234\end{bmatrix}, \hat{A}_{21}=\begin{bmatrix}-0.0879 & 13.5793\\ -0.7665 & -0.0781\end{bmatrix}$$

Solving $I-W\tilde{T}$ singular value decomposition , then:

$$M=\begin{bmatrix}-0.3501 & 0.9367\\ 0.9367 & 0.3501\end{bmatrix}, \tilde{N}=\begin{bmatrix}0.9369 & -0.3495\\ -0.3495 & -0.9369\end{bmatrix}$$

From the arithmetic expression (16), coefficient matrices of controller are obtained:

$$A_{c1}=\begin{bmatrix}-64.4326 & -20.8347\\ 11.8028 & 0.0942\end{bmatrix}, A_{c2}=\begin{bmatrix}-69.8946 & -23.7791\\ 13.4997 & 0.9786\end{bmatrix}, B_{c1}=\begin{bmatrix}-178.5383\\ 53.5977\end{bmatrix}$$

$$B_{c2}=\begin{bmatrix}-133.7557\\ 97.3976\end{bmatrix}, C_{c1}=[-1.0588 \quad -0.3950], C_{c2}=[-1.0253 \quad -0.3832]$$

When N=0.5, there is failure with actuator, and the coefficient matrices of controller are obtained:

$$C_{c1}=[-2.1176 \quad -0.7900], C_{c2}=[-2.0506 \quad -0.7664]$$

The guaranteed cost boundary value of the system is $J\le 0.2739$,the initial state is $x(0)=[\pi/6 \quad 0]^T$， $\hat{x}(0)=[0 \quad 0]^T$. The simulation Figure is as follows:

The simulation Figure of output feedback guaranteed cost controller which is based on the T-S fuzzy networked control systems is given in Figure 4-1 and Figure 4-2, from the Figures we can conclude that the closed-loop system is asymptotic stability.

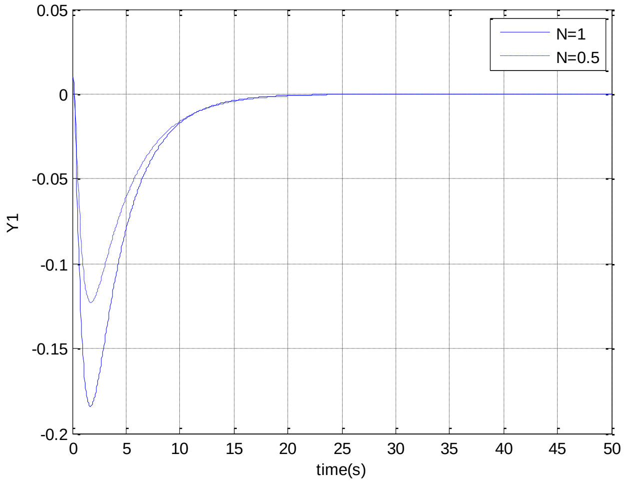

Figure 4-1. The State of System Output y_1

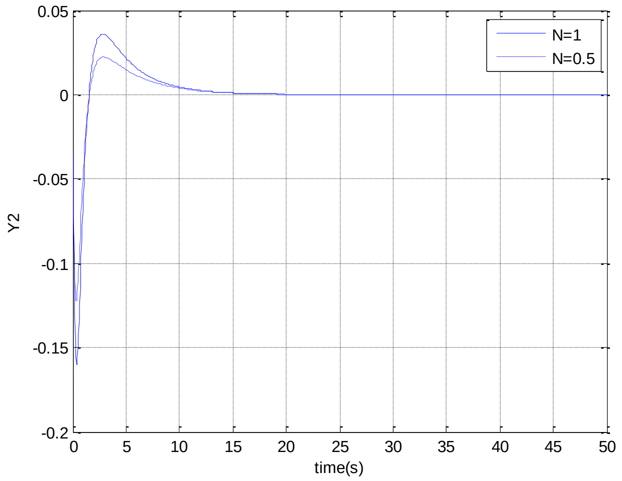

Figure 4-2. The State Of System Output y_2

6. Conclusion

This paper studies guaranteed cost control of T-S uncertain time-delay system with dynamic output feedback control. The sufficient existence condition of controller is deduced with Lyapunov Function method. Meanwhile, the parameters of output feedback controller are designed; finally a numerical example verifies the effectiveness of the theory which is put forward in this paper.

Acknowledgments

This work was supported by Scientific Research Fund of Heilongjiang Provincial Education Department, China (12541118).

References

[1] S．S．L. Chang and Y．K．C．Peng, "Adaptive Guaranteed Cost Control of Systems with Uncertain Parameters", IEEE Translation on Automatic Control. Vol. 17, no. 4, **(1972)**.
[2] H. Q. Xiao, H. E. Yong, W. U. Min and S. P. XIAO, "Improved H-infinity Tracking Control for Nonlinear Networked Control Systems Based on T-S Fuzzy Model", Control Theory and Applications, vol. 29, no. 1, **(2012)**.
[3] L. Yu, "Optimal Guaranteed Cost Control for Uncertain Discrete-Time Linear Systems", Control Theory and Applications, vol. 16, no. 5, **(1999)**.
[4] L. Yu, J. C. Wang and J. Chu, "Guaranteed Cost Control of Uncertain Linear Discrete Time Systems", Proceeding of American Control Conference, vol. 5, **(1997)**.
[5] X. Bing, "Guaranteed Cost Control of Discrete Time-Delay Systems with Uncertain Parameters", The 3rd International Workshop on Intelligent Systems and Applications, vol. 150, no. 5, **(2011)**.
[6] T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control", Transactions Systems Man and Cyber, vol. 15, no. 1, **(1985)**.
[7] Y. Liu and L. Y. Sun, "Guaranteed Cost Control for A Class of Uncertain Switched Fuzzy Time-delay Systems", Control Engineering of China, vol. 20, no. 3, **(2013)**.
[8] X. H. Yuan, Y. Sun, J. C. Ren and W. S. Wang, "Reliable Guaranteed Cost Controller for a Class of Uncertain Fuzzy Control Systems with Time Delay", Journal of Naval Aeronautical and Astronautical University, vol. 23, no. 2, **(2008)**.
[9] X．Jia, D．Zhang, X．Hao and N. Zheng, "Fuzzy H_∞ Tracking Control for Nonlinear Networked Control Systems in T-S Fuzzy Model", IEEE Transactions on Systems, vol. 39, no. 4, **(2009)**.
[10] L. Yao, Y. Liu and L. Sun, "Non-fragile Guaranteed Cost Control for Uncertain Networked Switched Fuzzy Systems", Journal of Harbin University of Science and Technology, vol. 16, no. 4, **(2011)**.

