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Band Gaps off In-Plane Waves Propagating Vertically Through Piezoelectric Phononic Crystal With Initial Stresses

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Abstract

Dispersive characteristics of in-plane waves propagating vertically through one-dimensional elastic/piezoelectric phononic crystal with initial stresses are studied in this paper. Firstly, in-plane waves are studied in this paper. In-plane waves are coupled by P-wave (or primary wave) and SV-wave (Secondary wave in the vertical plane). Secondly, the transfer matrix method is used to derive the dispersion equations. The solving process is as follow. The state vectors on both sides of the sub-layer in a unit cell are connected by a matrix, and those on both sides of the interface between the adjacent sub-layers are connected by an identical matrix. Thus, a matrix connecting the state vectors of both sides of the unit cell can be gotten, and the dispersion equation will be derived with the Bloch Theory. Finally, the effect of the initial stresses is considered. For in-plane waves both the normal initial stresses and the shear initial stresses are taken effect. So they are calculated respectively in this paper.

Keywords: *In-plane wave, transfer matrix method, initial stress, phononic crystal*

1. Introduction

Phononic crystal(PC) is a sort of artificial periodical structure composed of different materials. Wave propagation in PC has received considerable attention[1-3] due to the existence of complete acoustic band gaps within which elastic waves can't propagate while in other frequencies range they can transmit without amplitude fading. Because of the presence of band gaps, PC has a broad application prospect in many fields[4,5], such as vibration isolation and noise reduction. Recently, piezoelectric PCs have evolved into a significant research[6-8] owing to the increasing use of piezoelectric transducers in acoustics and ultrasonics. Additionally, transmission and reflection coefficients are also an essential part for finite-layer structure. Rodríguez-Ramos et al.[9] applied the global matrix method to find the behavior of transmission coefficients for shear horizontal wave propagation with oblique incidence in composite layered systems. Voronov[10] studied the propagation of electro-magnetic waves in 1D quasi-periodic media and deterministic aperiodic structures in quasicrystals. Li and Wang[11] studied the wave localization in disordered layer piezoelectric composites. Furthermore, the propagation of electromagnetic and elastic waves in the piezoelectric/piezomagnetic PCs has been investigated. Zhao et al.[12] investigated propagation of electromagnetic waves in a piezoelectric/piezomagnetic superlattice within which the electric and magnetic vectors could simultaneously couple with the identical superlattice vibration. Sabina and Movchan[13] discuss the interfacial effects in electromagnetic coupling within piezoelectric phononic crystals. Those papers are about effects of many factors on band gaps, but all these researches are based on wave propagating through PCs without initial stresses.

However, initial stresses are inevitable in the composites because of the limitation of the manufacturing technology. PC is a kind of composites, so material properties between

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adjacent sub-layers are different. Meanwhile, initial stresses are always existing during wave production to prevent the brittle fracture of piezoelectric materials. The existence of initial stresses make great influences on band gaps when wave propagating through PCs. There have been many researches on the effect of initial stresses. Liu et al. [14] have observed the effect of initial stresses on Love waves propagating through the elastic/piezoelectric structure. Yu and Wang et al.[15] studied the stop band properties of elastic waves in pre-stressed three-dimensional piezoelectric phononic crystals in which the mechanical and electrical coupling are taken into account. There are also some researches about the effect of initial stresses on wave propagating on layered structure without piezoelectric properties. For example, Zhang[16] investigated the guided wave propagation in FGM plates under gravity, homogeneous initial stress in the thickness direction and inhomogeneous initial stress in the wave propagation direction. The reflection and refraction at the interface of two dissimilar half-spaces with initial stress attracted many attentions, too. Guo[17] studied the influences of mechanically and dielectrically imperfect interfaces on the reflection and transmission waves between two piezoelectric half spaces. Singh and Chakraborty[18] investigated the problem of reflection and refraction of thermoelastic wave at a solid-liquid interface in presence of initial stress. Most of these researches didn't focus on in-plane waves propagating vertically through periodic layered structure. In this paper we are concerned with wave propagation in PCs with initial stresses by the method of transfer matrix.

2. Constitutive and Governing Equations for Pre-Stressed PC

The one-dimensional (1-D) elastic/piezoelectric periodic laminated structure consists of elastic layers and piezoelectric layers alternately. The lattice constant is defined as $a = a_1 + a_2$. The local orthogonal Cartesian coordinates system, namely (x_k, y, z_k), are established with the origin at the left boundary of each sub-layer. z^kaxis is perpendicular to the planar interface in K-th unit cell. This model system is illustrated in Figure 1.

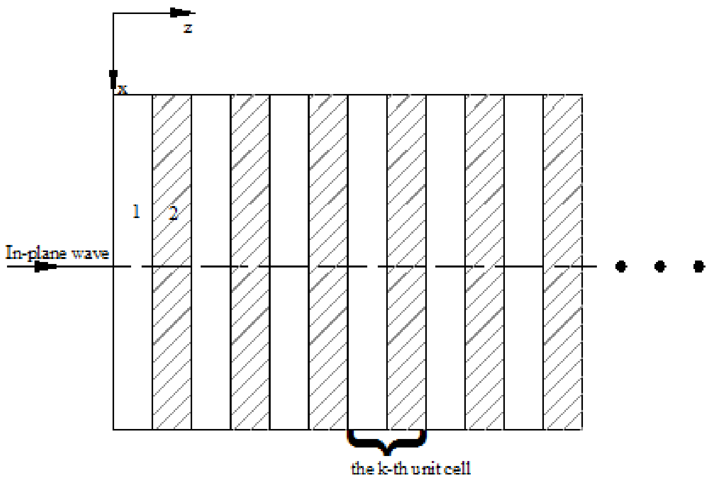

Figure 1. Schematic Illustration of One 1D Elastic/Piezoelectric PC

The polarization axis and the direction of waves propagating are both parallel to the z-direction. Then, the mechanical displacements and the electric potential can be expressed in the form

$$u_x = u(z,t), u_y = 0, u_z = w(z,t), \varphi = \varphi(z,t)$$
(1)

where $u_i(i = x, y)$ is mechanical displacement, and φ is the electric potential. The stress equations of motion and the Gaussian equation with initial stress considered can be given by[19]

$$\begin{cases}\sigma_{ij,i} + (u_{j,k}\sigma_{ki}^0)_{,i} = \rho\ddot{u}_j \\ D_{i,i} + (u_{i,j}D_j^0)_{,i} = 0\end{cases}$$

(2)

The constitutive equations can be expressed as

$$\begin{cases}\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k \\ D_i = e_{ikl}\varepsilon_{kl} + \eta_{ik}E_k\end{cases}$$

(3)

where σ_{ij},σ_{ij}^0,ε_{kl},D_i andE_k are the stress tensor, initial stress, strain tensor, electric displacement and electric field intensity; c_{ijkl}, e_{kij} and η_{ij} are the stiffness coefficient, piezoelectric coefficient and dielectric coefficient.

Substitution of Eq. 1 in Eq.2 yields the governing equations

$$\begin{cases}\frac{\partial\tau_{zx}}{\partial z} + \tau_{zx}^0\frac{\partial^2 u}{\partial z^2} = \rho\ddot{u} \\ \frac{\partial\tau_{zx}}{\partial z} + \sigma_z^0\frac{\partial^2 w}{\partial z^2} = \rho\ddot{w} \\ \frac{\partial D_z}{\partial z} = 0\end{cases}$$

(4)

It can be observed from Eq. 4 that both the initial normal stress σ_z^0 and the initial shear stress τ_{zx}^0 have influences on the governing equation. Let $\sigma_z^0 = \tau_{zx}^0 = 0$, Eq. 4 will be simplified to the governing equations without initial stresses. Furthermore, substituting Eq. 3 into Eq. 4 leads to

$$\begin{cases}(c_{44} + \tau_{zx}^0)\frac{\partial^2 u_1}{\partial z_1^2} = \rho\frac{\partial^2 u_1}{\partial t^2} \\ (c_{11} + \sigma_z^0)\frac{\partial^2 w_1}{\partial z_1^2} = \rho\frac{\partial^2 w_1}{\partial t^2} \\ -\eta_{11}\frac{\partial^2\varphi_1}{\partial z_1^2} = 0\end{cases}$$

(5a)

for the isotropic medium (Material A), and

$$\begin{cases}(c'_{44} + \tau_{zx}^0)\frac{\partial^2 u_2}{\partial z_2^2} = \rho'\frac{\partial^2 u_2}{\partial t^2} \\ (c'_{33} + \sigma_z^0)\frac{\partial^2 w_2}{\partial z_2^2} + e'_{33}\frac{\partial^2\varphi_2}{\partial z_2^2} = \rho'\frac{\partial^2 w_2}{\partial t^2} \\ e'_{33}\frac{\partial^2 w_2}{\partial z_2^2} - \eta'_{33}\frac{\partial^2\varphi_2}{\partial z_2^2} = 0\end{cases}$$

(5b)

for the transverse isotropic medium (Material B), where the subscripts “1” and “2” represent material A and B respectively, the superscript “$'$” represents the parameter of material B. From Eqs. 4a and 4b it can be found that the initial normal stress “σ_z^0” and shear stress “τ_{zx}^0” will affect the dispersive relation of in-plane wave propagating perpendicular to laminated structure.

Assumed that the solutions of Eqs. 5a and 5b are expressed as

$$\{u_i, w_i, \varphi_i\}(z_i, \mathrm{t}) = \{U_i, W_i, \Phi_i\}(z_i)e^{-i\omega t}$$

(6)

where $i = 1,2$, $\omega = kc$, $k = (2\pi/\lambda)$ is wave number, λ is wave length, c is wave velocity, $U_i(z_i), W_i(z_i), \Phi_i(z_i)$ are undetermined function. Substituting Eq. 6 into Eqs. 5a and 5b yields the following differential equations:

$$\begin{cases}(c_{44}+\tau_{zx}^{0})U_1''+\rho\omega^2U_1=0\\(c_{11}+\sigma_z^0)W_1''+\rho\omega^2W_1=0\\-\eta_{11}\Phi_1''=0\end{cases}$$
(7a)

for the isotropic medium, and

$$\begin{cases}(c_{44}'+\tau_{zx}^{0})U_2''+\rho'\omega^2U_2=0\\(c_{33}'+\sigma_z^0)W_2''+e_{33}'\Phi_2''+\rho'\omega^2W_2=0\\e_{33}'W_2''-\eta_{33}'\Phi_2''=0\end{cases}$$
(7b)

for the transverse isotropic medium. Equations 6, 7a and 7b yield the expressions of the mechanical displacements and electric potentials in materials A and B

$$\begin{cases}u_1(z_1,t)=(A_1e^{i\omega/M_1z_1}+B_1e^{-i\omega/M_1z_1})e^{-i\omega t}\\w_1(z_1,t)=(C_1e^{i\omega/N_1z_1}+D_1e^{-i\omega/N_1z_1})e^{-i\omega t}\\\varphi_1(z_1,t)=(E_1z_1+F_1)e^{-i\omega t}\end{cases}$$
(8a)

$$\begin{cases}u_2(z_2,t)=\left(A_2e^{i\omega/M_2z_2}+B_2e^{-i\omega/M_2z_2}\right)e^{-i\omega t}\\w_2(z_2,t)=\left(C_2e^{i\omega/N_2z_2}+D_2e^{-i\omega/N_2z_2}\right)e^{-i\omega t}\\\varphi_2(z_2,t)=[E_2z_2+F_2+p\left(C_2e^{i\omega/N_2z_2}+D_2e^{-i\omega/N_2z_2}\right)]e^{-i\omega t}\end{cases}$$
(8b)

where,$p=e_{33}'/\eta_{33}'$, $M_1=\sqrt{\tilde{c}_{44}/\rho}$, $M_2=\sqrt{\tilde{c}_{44}'/\rho'}$, $N_1=\sqrt{\tilde{c}_{11}/\rho}$, $N_2=\sqrt{\tilde{c}_{33}'/\rho'}$, $\tilde{c}_{44}=c_{44}+\tau_{zx}^0$, $\tilde{c}_{11}=c_{11}+\sigma_z^0$, $\tilde{c}_{44}'=c_{44}'+\tau_{zx}^0$, $\tilde{c}_{33}'=\hat{c}_{33}'+\sigma_z^0$, $\hat{c}_{33}'=c_{33}'+e_{33}'^2/\eta_{33}'$. Furthermore, substituting Eqs. 8a and 8b into Eq. 3, the expressions of stresses and electric displacements will be given,

$$\begin{cases}\tau_{zx1}(z_1,t)=i\omega c_{44}/M_1(A_1e^{i\omega/M_1z_1}-B_1e^{-i\omega/M_1z_1})e^{-i\omega t}\\\sigma_{z1}(z_1,t)=i\omega c_{11}/N_1\left(C_1e^{i\omega/N_1z_1}-D_1e^{-i\omega/N_1z_1}\right)e^{-i\omega t}\\D_{z1}(z_1,t)=-\eta_{11}E_1e^{-i\omega t}\end{cases}$$
(9a)

$$\begin{cases}\tau_{zx2}(z_2,t)=i\omega c_{44}'/M_2\left(A_2e^{i\omega/M_2z_2}-B_2e^{-i\omega/M_2z_2}\right)e^{-i\omega t}\\\sigma_{z2}(z_2,t)=[e_{33}'E_2+i\omega\hat{c}_{33}'/N_2\,(C_2e^{i\omega/N_2z_2}-D_2e^{-i\omega/N_2z_2})]e^{-i\omega t}\\D_{z2}(z_2,t)=-\eta_{33}'E_2e^{-i\omega t}\end{cases}$$
(9b)

3. Transfer Matrix Method For Dispersion Equation

The method of transfer matrix will be used to calculate the dispersion relation. A state vector is defined for convenience of statement of the interface condition and the periodic condition. As we know, the reflected and transmitted waves of an incident in-plane wave are combination of P- and SV-waves. Then, let $\mathbf{V}_j(z_j,t)=(u_j,w_j,\tau_{zxj},\sigma_{zj},\varphi_j,D_{zj})^{\mathrm{T}}(z_j,t)$ $(j=1,2)$.Therefore, for the perfect interface, the interface condition can be expressed as

$$\mathbf{V}_1(a_1,t)=\mathbf{T}_1'\mathbf{V}_1(0,t),\mathbf{V}_2(a_2,t)=\mathbf{T}_2'\mathbf{V}_2(0,t),\mathbf{V}_1(a_1,t)=\mathbf{V}_2(0,t)$$
(10)

where subscript "1" denotes component A and subscript "2" denotes component B. From Eq. 10, it can be found that

$$\mathbf{V}_2(a_2,t)=\mathbf{T}_2'\mathbf{T}_1'\mathbf{V}_1(0,t)$$
(11)

For in-plane waves, $\mathbf{T}_1'$and $\mathbf{T}_2'$ derived from Eq. 10 are both 6×6 matrixes. Hence, let

$\mathbf{T_1'} = (\mathrm{A_{ij}})_{6\times6}$, $\mathbf{T_2'} = (\mathrm{B_{ij}})_{6\times6}$,
(12)

in which

$A_{11} = cos\alpha_1$, $A_{13} = \frac{M_1}{\omega c_{44}} sin\alpha_1$, $A_{22} = cos\beta_1$, $A_{24} = \frac{N_1}{\omega c_{11}} sin\beta_1$, $A_{31} = -\frac{\omega c_{44}}{M_1} sin\alpha_1$, $A_{33} = cos\alpha_1$, $A_{42} = -\frac{\omega c_{11}}{N_1} sin\beta_1$, $A_{44} = cos\beta_1$,$A_{55} = A_{66} = 1$, $\mathrm{A}_{56} = -a_1/\eta_{11}$,other elements equal to 0, $\alpha_1 = \omega a_1/M_1$, $\beta_1 = \omega a_1/N_1$;

and

$B_{11} = cos\alpha_2$, $B_{13} = \frac{M_2}{\omega c_{44}'} sin\alpha_2$, $B_{22} = cos\beta_2$, $B_{24} = \frac{N_2}{\omega \hat{c}_{33}'} sin\beta_2$, $B_{26} = \frac{pN_2}{\omega \hat{c}_{33}'} sin\beta_2$,$B_{31} = -\frac{\omega c_{44}'}{M_2} sin\alpha_2$, $B_{33} = cos\alpha_2$, $B_{42} = -\frac{\omega \hat{c}_{33}'}{N_2} sin\beta_2$, $B_{44} = cos\beta_2$, $B_{46} = p[cos\beta_2 - 1]$, $B_{54} = \frac{pN_2}{\omega \hat{c}_{33}'} sin\beta_2$, $B_{55} = B_{66} = 1$, $\mathrm{B}_{56} = \frac{p^2N_2}{\omega \hat{c}_{33}'} sin\beta_2$,other elements equal to 0, $\alpha_2 = \omega a_2/M_2$, $\beta_2 = \omega a_2/N_2$.

According to Bloch theorem, the state vectors in a periodic structure satisfy

$\mathbf{V}_2(a_2, t) = e^{ika}\mathbf{V}_1(0, t)$
(13)

where $a = a_1 + a_2$ is the thickness of the primitive cell. Substituting Eq. 13 into Eq. 11, the following eigenvalue equation is finally obtained

$f(k, \omega, \sigma_z^0, \tau_{\mathrm{zx}}^0) = \left|\mathbf{T}(\omega, \sigma_z^0, \tau_{\mathrm{zx}}^0) - \mathbf{I}e^{ika}\right| = 0$
(14)

Eq. 14 is the dispersion equation with four variables (wavenumber k, frequency ω, initial normal stress σ_z^0 and initial shear stress τ_{zx}^0). The effect of the stresses σ_z^0 and τ_{zx}^0 on the dispersion relation will be observed respectively.

4. Numerical Examples with Initial Stresses $\boldsymbol{\sigma_z^0}$ And $\boldsymbol{\tau_{zx}^0}$

In the numerical example, the material combination of polythene and PZT-4 is considered. The lattice constant $\mathrm{a}(= \mathrm{a}_1 + \mathrm{a}_2)$is 20mm, and the filling ratio $\mathrm{f}(= \mathrm{a}_1/\mathrm{a}_2)$ is 1:3. The material constants of polythene and PZT-4 are listed in Table 1.

Table 1. Material Constants Of Polythene And PZT-4[18,19]

	ρ	c11	c13	c33	c44	e15	e31	e33	η11	η33
Polythene	1.12	0.78	0.466	0.78	0.157	0	0	0	0.3984	0.3984
PZT-4	7.50	13.9	7.43	11.5	2.56	12.7	-5.2	15.1	64.64	56.22

ρ($\times10^3$kg/m3); c_{ij}($\times10^{10}$N/m2); e_{ij}(×C/m2); η_{ij}($\times10^{-10}$F/m)

The dispersive curves of the in-plane wave propagating normal to the laminated structure without initial stresses are evaluated in Fig. 2. In all the figures $\mathrm{k_{nd}}$ and ω_{nd} denotes that non-dimensional wavenumber and frequency.

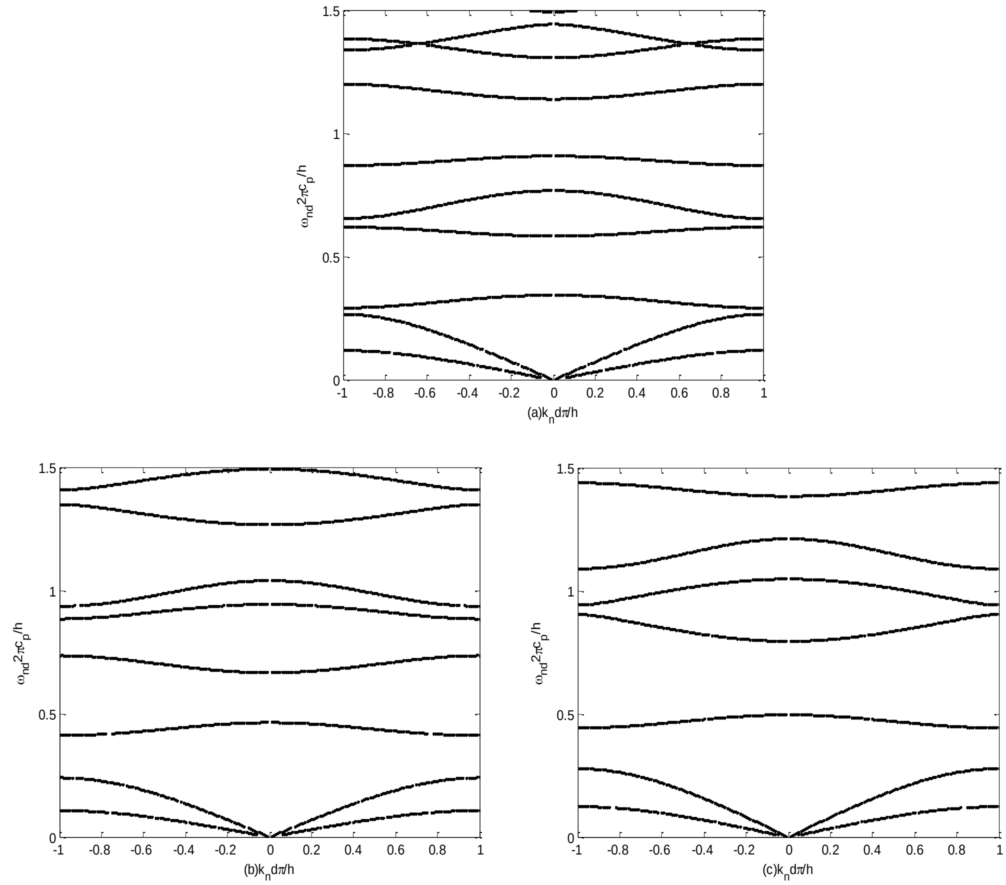

Figure 2. In-Plane Waves Propagating In Pcs Without Initial Stress: (A) F=3:1; (B) F=1:1; (C) F=1:3

Figure 2 shows the band structures shift due to the filling ratio f for normal incident in-plane wave. It's obviously that the dispersive curves turn to the higher frequency range with the increase of filling ratio. The width and center frequency of the first band gap increase monotonously with the filling ratio, while the width of the second band gap decreases as the filling ratio further increases. The comparison shows that filing ratio can be used to tune the band structures of phononic crystals. We use the filling ratio 1:3 to further quantify the initial stress effect on the band structures, because the band gaps are much easier for observing.

It can easily be found that when the initial stress and the material stiffness constants have almost the same order of magnitude the effect of initial stress are more explicit. Therefore, we let $\sigma_z = -\sigma_z^0/c_{44}$ and $\tau_{zx} = -\tau_{zx}^0/c_{44}$ ($c_{44} = 0.157 \times 10^{10} \mathrm{N/m^2}$) and then observe the change of the band gaps. The dispersive curves of the in-plane wave propagating normal to the laminated structure with initial stress are evaluated. The influences of the initial normal compressive stress on the dispersive curves of in-plane waves are shown in Fig. 3.

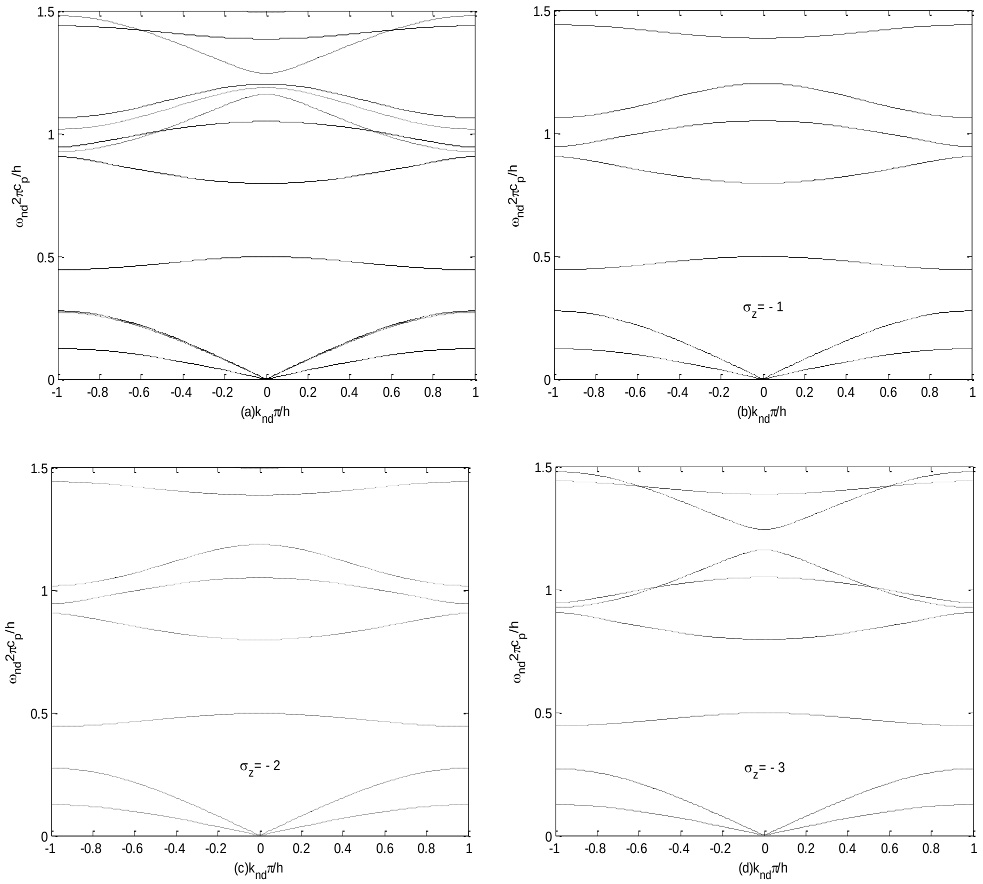

Figure 3. The Effect of Initial Normal Stress σ_z on Band Gaps Of In-Plane Waves (A) Comprehensive Figure; (B)$\sigma_z = -1$; (C)$\sigma_z = -2$; (D)$\sigma_z = -3$

Table 2. Dimensionless Width and Mid-Frequency of the First Three Band Gaps

ω_{nd}	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
Lower-1st	0.2768	0.2748	0.2712
Upper-1st	0.4445	0.4445	0.4445
Width-1st	0.1677	0.1697	0.1733
Midfre-1st	0.3607	0.3597	0.3579
Lower-2nd	0.4975	0.4975	0.4975
Upper-2nd	0.7962	0.7962	0.7962
Width-2nd	0.2987	0.2987	0.2987
Midfre-2nd	0.6469	0.6469	0.6469
Lower-3rd	0.9045	0.9045	0.9045
Upper-3rd	0.9435	0.9435	0.9265
Width-3rd	0.0390	0.0390	0.0220
Midfre-3rd	0.9240	0.9240	0.9155

In order to investigate the effect of the initial stress, in Fig. 3(a) comparison of the results with three different sizes of initial stress σ_z is shown and in Table 2 the numerical results of the upper and lower band edges, width and central frequency of the first three band gaps are given. In Fig. 3(a) it can be found that, when the size of the initial normal stress changing, the higher frequency band gaps are more sensitive than the lower ones. The second dispersive curve subjects to small perturbation and the sixth one is greatly affected, while others are not changed. Hence, the variation of the width and central frequency is mainly caused by the two curves. The width of the first band gap becomes larger and that of the second band gap keeps unchanged. With the increase of the initial normal stress σ_z^0 , the first band gap shows a sharp change in that the width(0.1677<0.1697<0.1733) is gradually increasing but the central frequency(0.3607<0.3597<0.3579) is moving towards the lower frequency. Obviously, there is a rule about the effect of the initial normal stress on the first band gap.

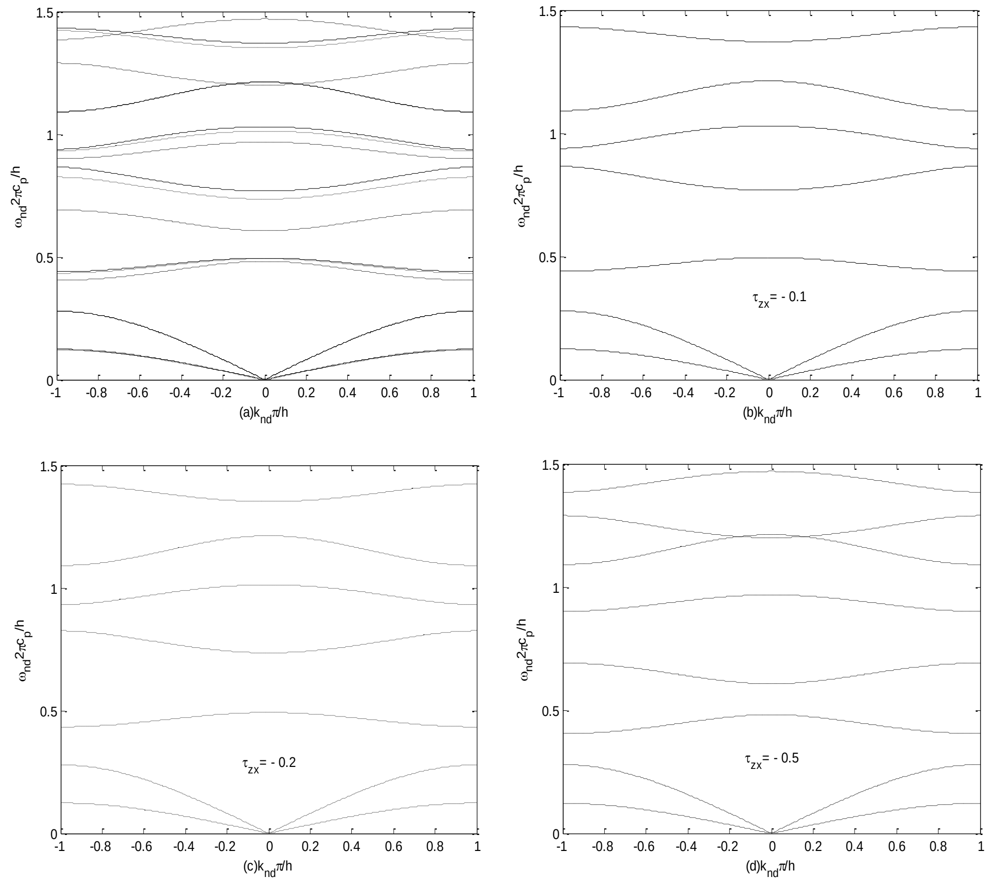

Figure 4. The Effect of Initial Shear Stress τ_z on Band Gaps of In-Plane Waves (A) Comprehensive Figures; (B)$\tau_{zx} = -0.1$; (C)$\tau_{zx} = -0.2$; (D)$\tau_{zx} = -0.5$

Table 3. Dimensionless Width and Mid-Frequency of the First Three Band Gaps

ω_{nd}	Fig. 4(b)	Fig. 4(c)	Fig. 4(d)
Lower-1st	0.2788	0.2788	0.2788
Upper-1st	0.4395	0.4335	0.4045
Width-1st	0.1607	0.1547	0.1257
Midfre-1st	0.3592	0.3562	0.3417
Lower-2nd	0.4955	0.4925	0.4815
Upper-2nd	0.7682	0.7352	0.6062
Width-2nd	0.2727	0. 2427	0.1247
Midfre-2nd	0.6319	0.6139	0.5439
Lower-3rd	0.8655	0.8245	0.6912
Upper-3rd	0.9385	0.9315	0.9005

Width-3rd	0.0730	0. 1070	0.2093
Midfre-3rd	0.9020	0.8780	0.7959

Similarly, in Fig. 4 a comparison of the results with three different sizes of initial shear stress τ_{zx} is shown and in Table 3 the numerical results of the upper and lower band edges, width and central frequency of the first three band gaps are given. In fig. 4(a) it can be found that, with the change of the shear initial stress τ_{zx}, the higher frequency band gaps are more sensitive than the lower ones, too. The second dispersive curve and the sixth one are not affected, while others are changed. The widths of the first and the second band gaps become smaller and meanwhile the mid-frequencies turn to the lower zone. With the increase of the initial normal stress σ_z, the second and third band gaps show a sharp change.

4. Analytical Solutions of Examples

These results are due to that P-wave and SV-wave are decoupled when in-plane waves vertically propagating. Eq. 14 can be rewritten as

$$\left|\mathbf{T}(\omega,\sigma_z^0,\tau_{zx}^0)-\mathbf{I}e^{ika}\right|$$
$$=[(K_{22}-e^{ikh})(K_{44}-e^{ikh})-K_{24}K_{42}][(K_{11}-e^{ikh})(K_{33}-e^{ikh})-K_{13}K_{31}]$$
$$=0$$
(15)

where $\mathbf{K}=\mathbf{T_2'}\mathbf{T_1'}=(K_{ij})_{6\times6}$, $K_{11}=A_{11}B_{11}+A_{13}B_{31}$, $K_{13}=A_{11}B_{13}+A_{13}B_{33}$, $K_{22}=A_{22}B_{22}+A_{24}B_{42}$, $K_{24}=A_{22}B_{24}+A_{24}B_{44}$, $K_{26}=A_{22}B_{26}+A_{24}B_{46}$, $K_{31}=A_{31}B_{11}+A_{33}B_{31}$, $K_{33}=A_{31}B_{13}+A_{33}B_{33}$, $K_{42}=A_{42}B_{22}+A_{44}B_{42}$, $K_{44}=A_{42}B_{24}+A_{44}B_{44}$, $K_{46}=A_{42}B_{26}+A_{44}B_{46}$, $K_{52}=B_{52}$, $K_{56}=A_{56}+B_{56}$, $K_{66}=1$, other elements equal to 0.

Then, Eq. 15 can be recast as

$$cos(kh)=cos\beta_1cos\beta_2-\frac{1}{2}\left(\frac{\hat{c}_{33}'c_{P1}}{c_{11}c_{P2}}+\frac{c_{11}c_{P2}}{\hat{c}_{33}'c_{P1}}\right)sin\beta_1sin\beta_2$$
(16a)

$$cos(kh)=cos\alpha_1cos\alpha_2-\frac{1}{2}\left(\frac{c_{44}'c_{SV1}}{c_{44}c_{SV2}}+\frac{c_{44}c_{SV2}}{c_{44}'c_{SV1}}\right)sin\alpha_1sin\alpha_2$$
(16b)

It can be observed that, Eq. 16a won't be affected when initial normal stress σ_z^0 changed, while Eq. 16b won't be affected when initial shear stress τ_{zx}^0 changed. That's what Fig. 2 and Fig. 3 show.

5. Conclusion

The influences of initial stresses on dispersive curves in 1D elastic/piezoelectric periodic structure are investigated in this paper. The normal and shear initial stresses ($\sigma_z^0$and τ_{zx}^0) are considered respectively. In-plane waves coupled with P- and SV- waves propagating normally are involved. It can be found that: The influences can be observed obviously just when the initial stresses have the same dimension with the stiffness coefficient. There are greater effects on dispersive curves in higher frequency range than that in lower frequency range. And the initial normal stresses have more evident influences than initial shear stresses. When initial normal stresses or shear stresses get changed parts of dispersive curves are affected while others are not changed. It can be proved that P-wave and SV-wave are decoupled when propagating normally through PCs. With initial stresses increased dispersive curves shift toward the lower frequency. The effects of initial normal and shear stresses on the width of the first band gap are opposite, that is, the width gets increased with increased normal stresses while decreased with increased shear stresses.

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