

Dynamic Fault-Tolerant Control on Faults with Prior Knowledge for Discrete Systems with Multiple Transfer Delays

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Abstract

This paper researches fault diagnosis and dynamic fault-tolerant control methods for a class of discrete system with measurement noises and multiple transfer/measurement delays. Different from the existing results which regard the transfer delays at the two ends of systems as an input delay, this paper simultaneously considers the transfer delays in input and output ends of systems. Different from the existing fault-tolerant control methods which don't consider and use the prior knowledge for faults, this paper considers the fault tolerant control approach by using the prior knowledge for faults. Furthermore, a dynamic fault-tolerant controller is derived which is composed of two compensators. Then, in order to solve the physically unrealizable problem of fault-tolerant controller, a reconstruction method is proposed to diagnose fault and reconstruct system information by structuring a servo fault estimator. Simulation results demonstrate the effectiveness of the proposed approaches.

Keywords: Fault-Tolerant Control; Fault Diagnosis; Delay-Free Transform; Delay Systems

1. Introduction

The modern control systems and equipment are becoming more and more complex, and the higher and higher requirements for safety and reliability are proposed for a lot of practical systems. Especially in some special fields, for example aerospace, nuclear industry, chemical industry and so on, the economic loss and disaster could be caused by faults. Therefore improving the system reliability and ensuring the safety of faulty systems have become an important research subject for many experts and scholars[1]. As the effective measures to ensure the reliability and safety, the fault diagnosis and fault-tolerant control have drawn much attention. As we all known, there are lots of good research results on fault diagnosis[2-4], but the research on fault-tolerant control is relatively less[5-7]. Generally, the existing methods in fault-tolerant control design can be classified into two main methods: the passive method and the active method[8]. Compared to the active fault tolerant, the passive fault tolerant has very high conservatism, which can't handle the unexpected faults. So, the active fault-tolerant control has received more and more attentions. For example, [9] proposed a reliable H_∞ control for uncertain nonlinear discrete systems when multiple intermittent faults occurred in sensors and/or actuators. In [10], a post fault tolerant control approach was proposed for sensor fault of

an induction motor and validated by an experiment of induction motor. For a stuck-actuator fault, [11] presented an adaptive fault-tolerant control approach to accommodate the incipient faults. Another adaptive fault-tolerant control scheme was proposed in [12] based on leader-follower consensus control when actuator faults occurred. [13] presented a self-restore fault tolerant control approach based on the diagnosed faults. For the faults of nonlinear discrete systems, [14] proposed a fault-tolerant control strategy and applied it to the twin-rotor system. For a class of switched systems, [15] proposed a robust fault-tolerant control approach when there were structural uncertainties, and applied it to haptic display systems. In [16], an active fault tolerant control was proposed for nonlinear systems when the actuator and sensor faults simultaneously occurred. [17] presented a fault-tolerant strategy which was composed of a fault estimator and a robust fault-tolerant controller, and applied it to a practical quadruple-tank process. [18] proposed a fault-tolerant control method which was based on algebraic derivative estimation. [19] proposed a flexible fault-tolerant topology scheme for the faults of switched reluctance motor. An optimal fault-tolerant control for a kind of faults was proposed in [20]. [21-22] gave a survey on fault diagnosis and fault-tolerant control.

The above scholars and the other scholars have given some good results on fault-tolerant control. Whereas, the existing method seldom consider the priori knowledge of faults^[13, 21]. [13] and [20] considered the priori knowledge of faults to realize the self-restore control and optimal fault-tolerant control, but they all didn't consider the multiple transfer delays in both input end and output end of system. [23] considered the fault diagnosis for systems with delayed measurements and inputs, but it didn't consider the fault-tolerant control. On the basis of the above, this paper researches the fault-tolerant control approach by means of the prior knowledge of faults when the measure/transfer disturbances exist.

The objective of this paper is to research the fault-tolerant control approach for discrete systems with measurement/transfer noise and transfer delays in the input and output ends of systems. A kind of dynamic fault-tolerant control is proposed by taking use of the known prior knowledge of faults. Furthermore, a design scheme is proposed to solve the physically unrealizable problem of dynamic fault-tolerant control.

The outline of this paper is as follows. The problem is formulated in Section 2. In Section 3, the fault-tolerant controller is designed and the physically unrealizable problem is given. Section 4 gives a delay-free transformation method to overcome the difficulty of fault diagnosis and fault-tolerant control due to the existence of delays. In Section 5, the dynamic fault-tolerant controller is proposed that consists of two compensators and a servo fault estimator, and the physically unrealizable problem is solved. Section 6 gives a numerical example. Finally, some conclusions are drawn in Section 7.

2. System Model and Problem Statement

Consider a class of linear discrete system with faults, multiple transfer delays and measurement/transfer noise as follows

$$\begin{aligned}
 x(k+1) &= Ax(k) + Bu(k-d) + D_1 f(k), \quad k=0,1,2,\dots, \\
 x(0) &= x_0, \\
 u(k) &= 0, \quad k=0,1,\dots,d-1, \\
 y_0(k) &= Cx(k) + D_2 f(k), \quad k=0,1,2,\dots, \\
 y(k) &= \begin{cases} y_0(k-h) + Nn(k), & k=h, h+1, h+2, \dots, \\ 0, & k=0,1,\dots,h-1. \end{cases}
 \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^p$ and $y_0(k) \in \mathbb{R}^q$ are the state vector, the control input vector and the output vector of the generalized controlled plant, respectively. $y_0(k) \in \mathbb{R}^q$

and $y(k) \in \mathbb{R}^q$ are the actual delay-free output vector and the transferred measurement output vector with output transfer delay, respectively. d and h are the known transfer delays in input end and output end, respectively. $h \geq 1$ and $d \geq 1$ are positive integers. $\varphi(k) \in \mathbb{R}^m$ is the indirectly measurable fault vector. A , B , C , D_1 , D_2 and N are real constant matrices of appropriate dimensions. $n(k)$ is the measurement/transfer noise. Without loss of generality, assume that $n(k)$ is white noise.

In general, we have certain prior knowledge for faults. Here, we discuss a kind of faults with prior knowledge, *i.e.*, the dynamic characteristics of the faults are known. We can derive the fault vector $f(k)$ as the following exosystem in the discrete form [23].

$$\begin{aligned} \psi(k+1) &= G\psi(k), \quad k = k_0, k_0+1, k_0+2, \dots, \\ \psi(k_0) = \psi_{k_0} &= \begin{cases} [\psi_a^T(\alpha) \quad \mathbf{0}]^T, & \alpha = k_0 < \beta, \\ [\mathbf{0} \quad \psi_s^T(\beta)]^T, & k_0 = \beta < \alpha, \end{cases} \\ \psi(k) &= \mathbf{0}, \quad k = 0, 1, \dots, k_0-1, \\ f(k) &= F\psi(k), \quad k = 0, 1, 2, \dots. \end{aligned} \quad (2)$$

where

$$\begin{aligned} \psi(k) &= \begin{bmatrix} \psi_a(k) \\ \psi_s(k) \end{bmatrix}, \quad f(k) = \begin{bmatrix} f_a(k) \\ f_s(k) \end{bmatrix}, \\ G &= \begin{bmatrix} G_a & 0 \\ 0 & G_s \end{bmatrix}, \quad F = \begin{bmatrix} F_a & 0 \\ 0 & F_s \end{bmatrix}. \end{aligned}$$

here, $\psi(k) \in \mathbb{R}^r$ is the state vector of exosystem (2). $\psi_a(k) \in \mathbb{R}^{r_1}$, $f_a(k) \in \mathbb{R}^{m_1}$, t_a and ψ_{a0} represent the actuator fault state vector, actuator fault vector, initial time instant and initial state vector of actuator fault, respectively. $\psi_s(k) \in \mathbb{R}^{r_2}$, $f_s(k) \in \mathbb{R}^{m_2}$, t_s and ψ_{s0} represent the sensor fault state vector, sensor fault vector, initial time instant and the initial state vector of sensor fault, respectively. $r = r_1 + r_2$, $m = m_1 + m_2 \leq r$. t_a , ψ_{a0} , t_s and ψ_{s0} are unknown. G_a , G_s , F_a , and F_s are known constant matrices of appropriate dimensions. α is the first sampling instant after occurring the actuator fault, $(\alpha-1)T < t_a \leq \alpha T$. β is the first sampling instant after occurring the sensor fault, $(\beta-1)T < t_s \leq \beta T$.

Remark 1: Practically, we often have some priori knowledge for faults, for example, the broken rotor bar fault for squirrel cage motor^[24], the electrical short-circuit fault [25], the resonance fault^[26] and so on, whose dynamic characteristics are all known.

3. Design of Dynamic Fault-Tolerant Control

Consider the faults described by (2), a dynamic fault-tolerant controller is proposed when faults occur, which is comprised of a servo fault compensator and a stabilizing compensator. The fault-tolerant control system is shown in Figure1, where the fault acts as a switch. When there are no any faults, the linkage switches are all at the position (1), then the dynamic fault-tolerant controller is not connected with the system, so the fault-tolerant controller takes no effect. When faults occur, the three linkage switches are simultaneously switched from position (1) to position (2), then the dynamic fault-tolerant controller takes effect.

When faults occur, in order to compensate the effect of faults on the system's outputs, one can design the following servo fault compensator according to the dynamic

characteristics of faults

$$\begin{aligned} \psi_w(k+1) &= G_w \psi_w(k) + B_w e(k), \\ e(k) &= \begin{cases} u_0(k) - y(k) = u_0(k) - y_0(k-h), \hat{f}(k) = 0 \\ u_0(k) - \hat{y}_0(k), \hat{f}(k) \neq 0 \end{cases} \\ \psi_w(0) &= \psi_{w0}, \\ \bar{u}(k) &= \begin{cases} 0, \hat{f}(k) = 0, \\ C_w \psi_w(k), \hat{f}(k) \neq 0. \end{cases} \end{aligned} \quad (3)$$

where $\psi_w(k) \in \mathbb{R}^r$ ($t \leq r$) and $\bar{u}(k) \in \mathbb{R}^l$ are the state vector and output vector of the servo fault compensator, respectively. $\psi_w(k)$ consists of non-asymptotically stable state vector of $\psi(k)$. $u_0(k)$ and $e(k)$ are the desirable input signal vector and the tracking error vector of the controlled system, respectively. $\hat{y}_0(k)$ is the feedback transfer delay-free output, i.e., the estimated $y_0(k)$ without the output transfer delay. B_w is real constant matrices of appropriate dimensions, and C_w is the control gain of the servo fault compensator.

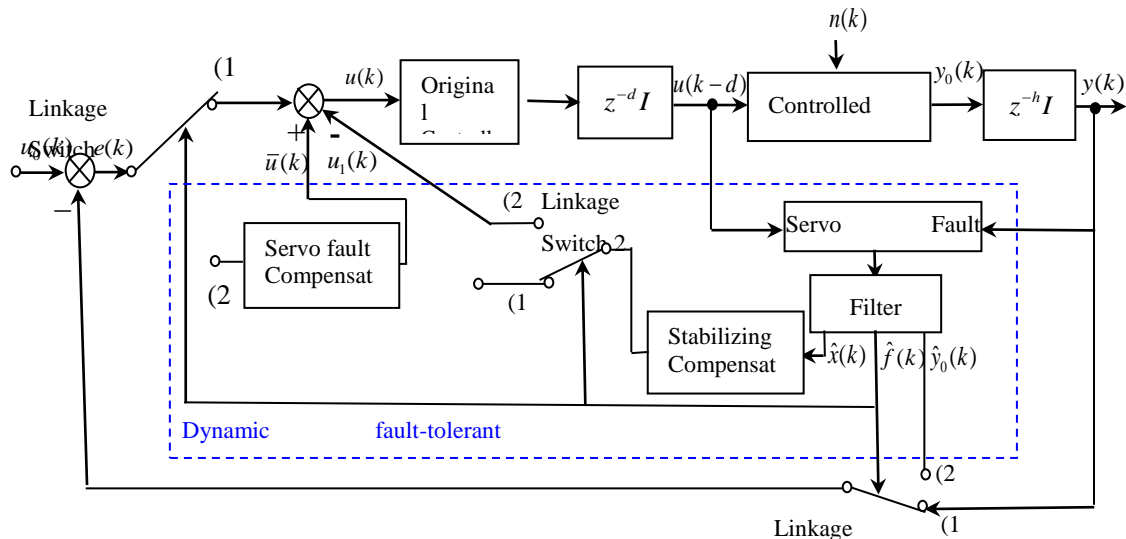


Figure 1. Block Diagram of Control System

When faults occur, the designed servo fault compensator puts into effect which is put in the forward channel before the fault action point to compensate the effect of faults.

Remark 2: The stability of the system will be influenced when there exist transfer delays. The existence of output transfer delays usually results in the divergence of system if we use the actual delayed output $y(k)$ as the output feedback. In order to solve the problem, we propose to restructure the delay-free output $\hat{y}_0(k)$ instead of the delayed output $y(k)$ as the input signal of the servo fault compensator. $\hat{y}_0(k)$ is restructured in Section 5.

When the servo fault estimator takes into effect, the following stabilizing compensator simultaneously takes into effect which is constructed in order to realize the asymptotic stability of the control system

$$u_1(k) = C_m x(k). \quad (4)$$

where, $u_1(k)$ and C_m are the output vector and control gain of the stabilizing

compensator, respectively.

Based on (3) and (4), one will obtain the output $u(k)$ of the fault-tolerant controller

$$u(k) = u_0(k) - \hat{y}_0(k) + \bar{u}(k) - u_1(k) = u_0(k) - \hat{y}_0(k) + C_w \psi_w(k) - C_m x(k). \quad (5)$$

Substituting (1) and (3) into (5), one obtains the dynamic fault-tolerant control law

$$\begin{aligned} \varphi(k+1) &= A_3 \varphi(k) + B_2 \varphi(k-d) + B_3 (u_0(k) - \hat{y}_0(k)) + D_{01} f(k), \\ u(k) &= \begin{cases} u_0(k) - y(k), & f(k) = 0, \\ u_0(k) - \hat{y}_0(k) + K \varphi(k), & f(k) \neq 0. \end{cases} \end{aligned} \quad (6)$$

where

$$\begin{aligned} \varphi(k) &= \begin{bmatrix} x(k) \\ \psi_w(k) \end{bmatrix}, \quad A_3 = \begin{bmatrix} A & 0 \\ 0 & G_w \end{bmatrix}, \quad B_2 = \begin{bmatrix} -BC_m & BC_w \\ 0 & 0 \end{bmatrix}, \\ B_3 &= \begin{bmatrix} 0 \\ B_w \end{bmatrix}, \quad D_{01} = \begin{bmatrix} D_1 \\ 0 \end{bmatrix}, \quad K = [-C_m \quad C_w]. \end{aligned}$$

From (6), one can see that the dynamic fault-tolerant control law is physically unrealizable. The reasons lie in: (i) $u(k)$ comprises the system state vector $x(k)$, which maybe isn't practical physical variables and can't be directly measured or is practical physical variable but the measurement cost is expensive; (ii) $\hat{y}_0(k)$ is the k th instant delay-free output, but we can't directly obtain it due to the existing of the output measurement/transfer delay; (iii) Fault-tolerant control law is switched according to $f(k)$, but $f(k)$ is unknown and can't be directly measured. As we all know, it is difficult to solve the physically unrealizable problem due to the existence of transfer delays. So we firstly introduce the following delay-free transformation approach.

4. Functional-Based Delay-Free Transformation

Let $w(k) = [x(k)^T \quad \psi(k)^T]^T$. Integrating (1) with (2), we have the following state space expression

$$\begin{aligned} w(k+1) &= A_1 w(k) + B_1 u(k-d), \quad k = 0, 1, 2, \dots, \\ w(0) &= [x_0^T, \psi(0)^T]^T, \\ u(k) &= 0, \quad k = -d, -d+1, \dots, -1, \\ y_0(k) &= C_1 w(k), \\ y(k) &= \begin{cases} y_0(k-h) + Nn(k), & k = h, h+1, h+2, \dots, \\ 0, & k = 0, 1, \dots, h-1. \end{cases} \end{aligned} \quad (7)$$

where

$$A_1 = \begin{bmatrix} A & D_1 F \\ 0 & G \end{bmatrix}, \quad B_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C_1 = [C \quad D_2 F].$$

From the system (7), one can obtain

$$\begin{aligned} w(k) &= A_1^k w(0) + \sum_{i=0}^{k-1} A_1^{k-i-1} E u(i) - \sum_{i=k-d}^{k-1} A_1^{k-i-1} E u(i), \\ y(k) &= H \left[A_1^k w(0) + \sum_{i=0}^{k-h-1} A_1^{k-i-1} E u(i) - \sum_{i=k-d-h}^{k-h-1} A_1^{k-i-1} E u(i) \right] + Nn(k), \quad k = h, h+1, h+2, \dots. \end{aligned} \quad (8)$$

where

$$E = A_1^{-d} B_1, \quad H = C_1 A_1^{-h}.$$

In order to transform the system with transfer delays into one in delay-free form, one introduces the following functional-based delay-free transformations

$$z(k) = w(k) + \sum_{i=k-d}^{k-1} A_1^{k-i-1} Eu(i), \quad (9)$$

$$\eta(k) = y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} Eu(i) - Nn(k), \quad k = h, h+1, h+2, \dots.$$

Based on (8) and (9), one rewrites (7) as

$$\begin{aligned} z(k+1) &= A_1 z(k) + Eu(k), \quad k = 0, 1, 2, \dots, \\ z(0) &= w(0), \\ u(k) &= 0, \quad k = -d, -d+1, \dots, -1, \\ \eta(k) &= Hz(k), \quad k = 0, 1, 2, \dots, \\ y(k) &= \begin{cases} \eta(k) - H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} Eu(i) + Nn(k), & k = h, h+1, h+2, \dots, \\ 0, & k = 0, 1, \dots, h-1. \end{cases} \end{aligned} \quad (10)$$

5. Physically Realizable Fault-tolerant Control

In order to solve the physically unrealizable problem of fault-tolerant control, we propose to reconstruct output information by structuring a servo fault estimator which can estimate the state vector, delay-free output vector, and the fault vector. Firstly, we construct a nonsingular matrix

$$Q = \begin{bmatrix} H_{\perp}^T & H^T \end{bmatrix}^T \in \mathbb{R}^{(n+r) \times (n+r)},$$

where $H_{\perp} \in \mathbb{R}^{(n+r-q) \times (n+r)}$ is the orthogonal complement of H . Let $P = Q^{-1} = \begin{bmatrix} P_1 & P_2 \end{bmatrix}$, $P_1 = \begin{bmatrix} P_{11}^T & P_{21}^T \end{bmatrix}^T \in \mathbb{R}^{(n+r) \times (n+r-q)}$, $P_2 = \begin{bmatrix} P_{21}^T & P_{22}^T \end{bmatrix}^T \in \mathbb{R}^{(n+r) \times q}$, $P_{21} \in \mathbb{R}^{r \times (n+r-q)}$, $P_{22} \in \mathbb{R}^{r \times q}$. About the design of the dynamic fault-tolerant controller, the following theorem is given.

Theorem 1: Consider system (1) with transfer delays and faults described by (2). Assume that the pair (C_1, A_1) is completely observable, then there exists a feedback gain matrix $L \in \mathbb{R}^{(n+r-q) \times q}$, such that the following system is a physically realizable dynamic fault-tolerant controller with servo fault estimator:

$$\begin{aligned} \hat{\mu}(k+1) &= (H_{\perp}^T - LH) \left[A_1 P_1 \hat{\mu}(k) + A_1 (P_1 L + P_2) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} Eu(i) - Nn(k) \right) + Eu(k) \right], \\ \hat{x}(k) &= P_{11} \hat{\mu}(k) + (P_{11} L + P_{12}) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} Eu(i) - Nn(k) \right) - [I \ \mathbf{0}] \left(\sum_{i=k-d}^{k-1} A_1^{k-i-1} Eu(i) \right), \\ \hat{\psi}(k) &= P_{21} \hat{\mu}(k) + (P_{21} L + P_{22}) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} Eu(i) - Nn(k) \right) - [\mathbf{0} \ I] \left(\sum_{i=k-d}^{k-1} A_1^{k-i-1} Eu(i) \right), \\ \hat{f}(k) &= F \hat{\psi}(k), \\ \hat{y}_0(k) &= C \hat{x}(k) + D_2 \hat{f}(k), \\ u(k) &= \begin{cases} u_0(k) - y(k), \hat{f}(k) = 0, \\ u_0(k) - \hat{y}_0(k) + C_w \psi_w(k) - C_m \hat{x}(k), \hat{f}(k) \neq 0. \end{cases} \end{aligned} \quad (11)$$

where $\hat{\mu}(k)$, $\hat{x}(k)$, $\hat{\psi}(k)$, $\hat{y}_0(k)$, and $\hat{f}(k)$ are the state vector of the dynamic fault-tolerant controller, estimated state vector of controlled plant, diagnosed fault state vector, estimated delay-free output vector, and estimated fault vector, respectively. L is the feedback gain matrix of fault-tolerant controller. I and $\mathbf{0}$ denote the identity matrix and zero matrix, respectively.

Proof: Let

$$\bar{z}(k) = Qz(k) = \begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix} \quad (12)$$

where $\bar{z}_1(k) \in \mathbb{R}^{(n+r-q) \times 1}$, $\bar{z}_2(k) \in \mathbb{R}^{q \times 1}$. one has

$$z(k) = P_1 \bar{z}_1(k) + P_2 \bar{z}_2(k). \quad (13)$$

Then system (10) is rewritten as

$$\begin{aligned} \bar{z}_1(k+1) &= H_{\perp}^T A_1 P_1 \bar{z}_1(k) + H_{\perp}^T A_1 P_2 \bar{z}_2(k) + H_{\perp}^T E u(k), \\ \bar{z}_2(k+1) &= H A_1 P_1 \bar{z}_1(k) + H A_1 P_2 \bar{z}_2(k) + H E u(k), \\ \eta(k) &= \bar{z}_2(k), \end{aligned} \quad (14)$$

$$y(k) = \begin{cases} \eta(k) - H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) + N n(k), & k = h, h+1, h+2, \dots, \\ 0, & k = 0, 1, \dots, h-1. \end{cases}$$

Introducing a variable transformation

$$\mu(k) = \bar{z}_1(k) - L \eta(k). \quad (15)$$

where L is the feedback gain matrix. Then the system (14) becomes

$$\mu(k+1) = (H_{\perp}^T - LH) \left[A_1 P_1 \mu(k) + A_1 (P_1 L + P_2) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) - N n(k) \right) + E u(k) \right], \quad (16)$$

$$\bar{z}_1(k) = \mu(k) + L \eta(k).$$

Substituting (16) into (13), one will obtain

$$\mu(k+1) = (H_{\perp}^T - LH) \left[A_1 P_1 \mu(k) + A_1 (P_1 L + P_2) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) - N n(k) \right) + E u(k) \right] \quad (17)$$

$$z(k) = P_1 \mu(k) + (P_1 L + P_2) \eta(k).$$

So we can construct a reduced-order fault estimator as follows

$$\hat{\mu}(k+1) = (H_{\perp}^T - LH) \left[A_1 P_1 \hat{\mu}(k) + A_1 (P_1 L + P_2) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) - N n(k) \right) + E u(k) \right], \quad (18)$$

$$\hat{z}(k) = P_1 \hat{\mu}(k) + (P_1 L + P_2) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) - N n(k) \right).$$

where $\hat{\mu}(k)$ and $\hat{z}(k)$ are the state vector and the output vector of the estimator (18), respectively.

Separate the states of $\hat{z}(k)$, then one rewrites (18) as

$$\hat{\mu}(k+1) = (H_{\perp}^T - LH) \left[A_1 P_1 \hat{\mu}(k) + A_1 (P_1 L + P_2) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) - N n(k) \right) + E u(k) \right],$$

$$\hat{x}(k) = P_{11} \hat{\mu}(k) + (P_{11} L + P_{12}) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) - N n(k) \right) - [I \ \mathbf{0}] \left(\sum_{i=k-d}^{k-1} A_1^{k-i-1} E u(i) \right), \quad (19)$$

$$\hat{\psi}(k) = P_{21} \hat{\mu}(k) + (P_{21} L + P_{22}) \left(y(k) + H \sum_{i=k-d-h}^{k-1} A_1^{k-i-1} E u(i) - N n(k) \right) - [\mathbf{0} \ I] \left(\sum_{i=k-d}^{k-1} A_1^{k-i-1} E u(i) \right),$$

$$\hat{f}(k) = F \hat{\psi}(k).$$

Replacing $x(k)$ and $f(k)$ of $y_0(k)$ in (1) with the estimated $\hat{x}(k)$ and $\hat{f}(k)$ in (19), respectively, then we obtain the reconstructed delay-free output $\hat{y}_0(k)$ of k th instant of (11). Replacing $x(k)$ of (5) with the estimated $\hat{x}(k)$ of (19), then one can

obtain the dynamic fault-tolerant control law $u(k)$ of (11).

Let the estimation error $\tilde{\mu}(k) = \mu(k) - \hat{\mu}(k)$, one obtains the estimation error equation

$$\tilde{\mu}(k+1) = (H_{\perp}^T A_1 P_1 - L H A_1 P_1) \tilde{\mu}(k). \quad (20)$$

Since (H, A_1) is completely observable and A_1 is nonsingular, and noting that P_1 is the full column rank and $P_1 H_{\perp}^T = I$, one obtains

$$\text{rank} \left[(H A_1 P_1)^T \quad (H A_1 P_1 H_{\perp}^T A_1 P_1)^T \quad (H A_1 P_1 (H_{\perp}^T A_1 P_1)^2)^T \quad \cdots \quad (H A_1 P_1 (H_{\perp}^T A_1 P_1)^{n+r-q-1})^T \right]^T = n+r-q \quad (21)$$

Then the pair $(H A_1 P_1, H_{\perp}^T A_1 P_1)$ is completely observable. We can choose the estimator gain matrix L such that all the eigenvalues of the matrix $(H_{\perp}^T - L H) A_1 P_1$ are assigned to the expected positions of Z plane. So one can obtain

$$\lim_{k \rightarrow \infty} \hat{\mu}(k) = \mu(k). \quad (22)$$

Based on (17), (18) and (22), one can obtain

$$\lim_{k \rightarrow \infty} \hat{z}(k) = z(k). \quad (23)$$

Based on (23) and (9), one will obtain

$$\begin{aligned} \lim_{k \rightarrow \infty} (\hat{z}(k) - z(k)) &= 0, \\ \lim_{k \rightarrow \infty} (\hat{w}(k) - w(k)) &= 0. \end{aligned} \quad (24)$$

i.e.,

$$\begin{aligned} \lim_{k \rightarrow \infty} (\hat{x}(k) - x(k)) &= 0, \\ \lim_{k \rightarrow \infty} (\hat{\psi}(k) - \psi(k)) &= 0, \\ \lim_{k \rightarrow \infty} (\hat{w}(k) - w(k)) &= 0. \end{aligned} \quad (25)$$

From formula (2) and (25), one can obtain

$$\begin{aligned} \lim_{k \rightarrow \infty} (\hat{f}_a(k) - f_a(k)) &= 0 \\ \lim_{k \rightarrow \infty} (\hat{f}_s(k) - f_s(k)) &= 0 \end{aligned} \quad (26)$$

where $\hat{f}_a(k)$ and $\hat{f}_s(k)$ are the diagnosed actuator fault vector and the diagnosed sensor fault vector, respectively.

Remark 3: According to introducing the functional-based delay-free transformation and constructing the servo fault estimator, we reconstruct the delay-free output $\hat{y}_0(k)$ of k th instant and the variables which can't be directly measured according to using the delayed inputs and outputs. So the physically unrealizable problem of dynamic fault-tolerant control is solved.

6. A Simulation Example

Consider the system with transfer delay and noise described by (1), where

$$\begin{aligned} A &= \begin{bmatrix} 0.9909 & 0.0861 \\ -0.1722 & 0.7326 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1042 \\ 0.0771 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0.2084 & 0 \\ 0.1541 & 0 \end{bmatrix}, \quad x(0) = [0.3 \quad 0.2]^T, \\ C &= [1 \quad 0], \quad D_2 = [0 \quad 1], \quad N = 1, \\ d &= 80, \quad h = 10, \quad T = 0.1. \end{aligned} \quad (27)$$

Consider the fault $f(k)$ described by (2), where

$$G = \begin{bmatrix} 0.9950 & 0.0998 & 0 & 0 \\ -0.0998 & 0.9950 & 0 & 0 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \end{bmatrix},$$

$$\psi_a(\alpha) = [1 \ 0]^T, \quad \psi_s(\beta) = [0 \ 1]^T, \quad (28)$$

$$\alpha = 200, \quad \beta = 50.$$

The poles of the servo fault estimator are assigned to 0.9 , $0.8 \pm j0.03$, $0.9 \pm j0.04$. According to Ackermann formula, one can obtain the gain matrix as the following

$$L = [-2.3005 \ 2.3721 \ 9.1912 \ 15.4071 \ 154.1641]^T.$$

The following servo fault compensator is designed which is described by (3) according to (28), where

$$G_w = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 3.99 & -5.98 & 3.99 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \quad (29)$$

The poles of the servo fault compensator are assigned to $0.8 \pm j0.01$, $0.7 \pm j0.01$. According to the pole assignment method, one can obtain the feedback gain matrix of the fault-tolerant controller as the following

$$K = [C_m \ \vdots \ -C_w] = [5.3143 \ 0.7749 \ \vdots \ 1.0919 \ -3.3649 \ 3.4633 \ -1.1905]. \quad (30)$$

The reference input signal of system (27) is $u_0 = 5$, and $n(k)$ is the Gaussian white noise. The system's measurement output with disturbance is shown in Figure 2. The system's filtered measurement output is shown in Figure 3 when system (27) occurs the faults described by (28). The diagnosed actuator fault from the servo fault estimator and the actual actuator fault are shown in Figure 4, and the diagnosed sensor fault from the servo fault estimator and the actual sensor fault are shown in Figure 5. The state $x_1(k)$, $x_2(k)$ and measurement output are shown in Figure 6, Figure 7 and Figure 8, respectively.

Figure 2 and Figure 3 illustrate that there is a output transfer delay of 1 second in the output $y(k)$, and a sinusoidal form of fault and a ramp form of fault are superimposed to the system output at $t = 21s$ and $t = 6s$, respectively. So we can infer that the actual faults occur at $t = 20s$ and $t = 5s$, respectively. Figure 4 and Figure 5 illustrate that the servo fault estimator correctly estimates the sensor fault and the actuator fault. Figure 6 and Figure 7 illustrate that the servo fault estimator correctly estimates the states of system. Figure 8 illustrates that the output of the system is completely isolated from faults as $k \rightarrow \infty$ after introducing the dynamic fault-tolerant control.

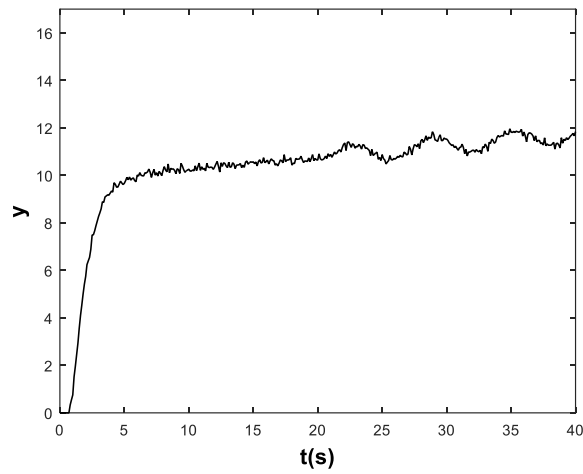


Figure 2. Measurement Output $y(k)$

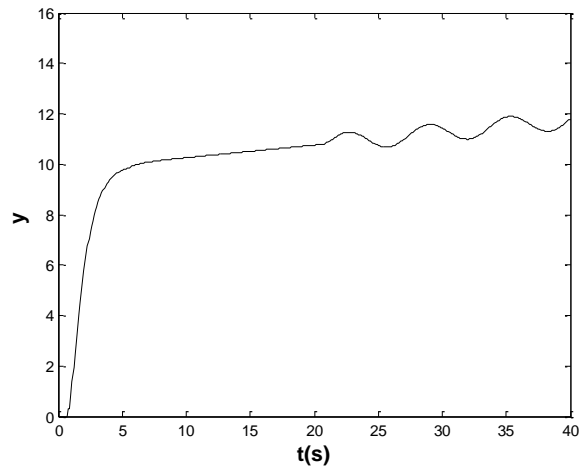


Figure 3. Filtered Measurement Output $y(k)$

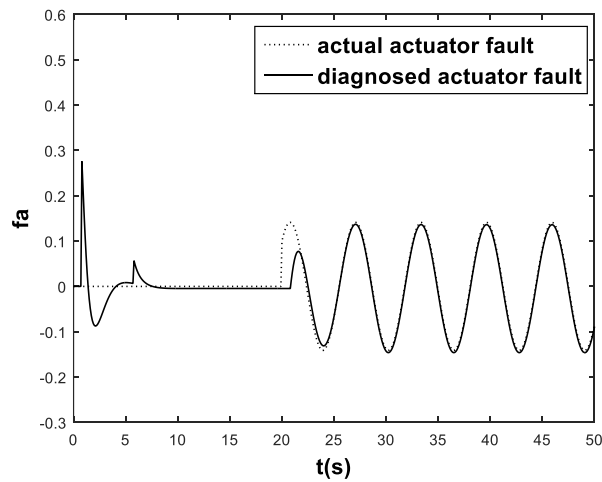


Figure 4. Diagnosed Actuator Fault $\hat{f}_a(k)$

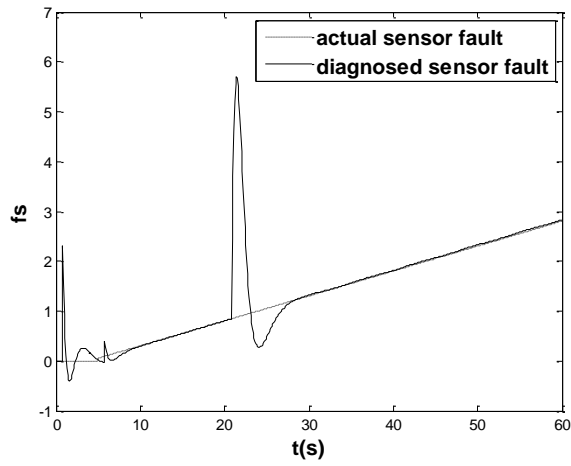


Figure 5. Diagnosed Sensor Fault $\hat{f}_s(k)$

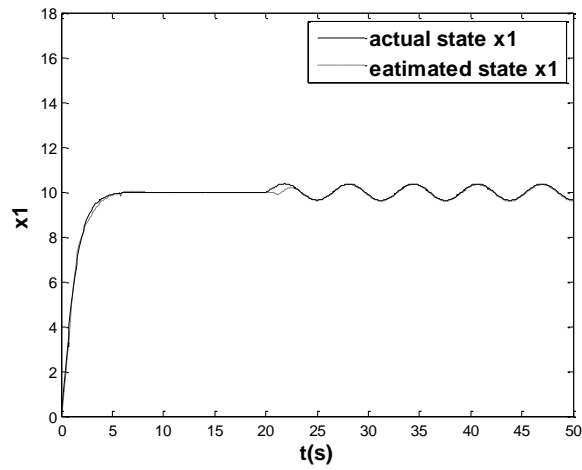


Figure 6. Trajectories of State $x_1(k)$

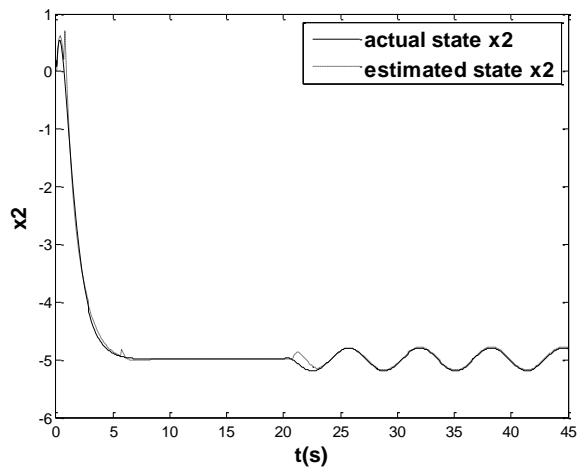


Figure 7. Trajectories of State $x_2(k)$

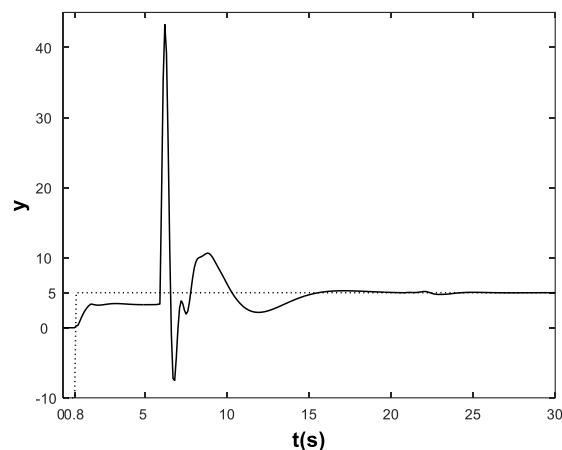


Figure 8. Measurement Output $y(k)$ By Fault-Tolerance

7. Conclusions

This paper has researched dynamic fault-tolerant control and fault diagnosis problem for systems with input and output transfer delays and measurement/transfer noises when a kind of faults with certain prior knowledge occur. A dynamic fault-tolerant controller is designed which includes two compensators by taking use of the known dynamic characteristics of faults. In order to solve the physically unrealizable problem of fault-tolerant control, a servo fault estimator is designed to reconstruct indirectly measurable system information and diagnose faults. Simulation results have demonstrated the feasibility and validity of the proposed scheme. The presented fault-tolerant control design approach and the idea of taking use of the priori knowledge can be extensively used in some other occasions.

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