

Point Stabilization for Wheeled Mobile Robots Using Model Predictive Control

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Abstract

A model predictive control method is proposed for the point stabilization of wheeled mobile robots (WMRs) subject to nonholonomic constraints. The problem is simplified by considering only the steering system and neglecting the vehicle dynamics. A linearized error model is then formulated by transforming the robot position into polar frame. The feedback control policy is obtained by minimizing a quadratic cost function which penalizes the predicted errors and control variables in each sampling time over a finite horizon. The proposed control law is proven to guarantee the exponential stability of the robot system by considering additive inequality constraints in the optimization process. The performance of the stabilization algorithm is verified through computer simulations showing that the proposed method has a good regulation performance and convergence.

Keywords: *model predictive control, polar frame, stabilization, wheeled mobile robots*

1. Introduction

Differentially steered wheeled mobile robots (WMRs) is a popular kind of WMR used in many areas due to its high mobility, high traction with pneumatic tires and a simple wheel configuration. The basic motion tasks of WMRs can be framed into two, namely: 1) follow a given trajectory, and 2) moving between two robot positions. The two main approaches to control of mobile robots on the other hand are trajectory tracking and point stabilization.

The aim of trajectory tracking is to have the robot follow a reference trajectory point [1-4] while stabilization can be regarded as the generation of control inputs to drive the robot from an initial point to a target point. Some early research handled the problem by developing a piecewise smooth controller to render the origin exponentially stable for any initial condition in the state space was developed [5], whereas another proposed a smooth feedback control law derived from the Lyapunov method [6]. From then onwards, research on stabilization problem has been extensive. A piecewise continuous law which guarantees the exponential stability of the closed-loop system was derived by using Lyapunov stability technique [7]. On the other hand, the point stabilization problem is solved via standard nonlinear control theory, *e.g.*, the state-space exact feedback linearization [8], where the controller is proved to be stable by considering three different conditions. An asymptotic control with driftless constraints was proposed based on empirical practice and the convergence is presented by using Lyapunov stability theory [9]. Furthermore, a method was developed for solving trajectory tracking as well as posture stabilization problems, based on the unifying framework of dynamic feedback linearization [10]. For a class of nonholonomic mechanical systems, time-varying feedback control laws were developed by considering chain form systems [11, 12] as well as driftless systems [13].

Model predictive control (MPC), also called receding horizon control (RHC), is in widespread use in the design of control for highly complex multivariable processes [14-16]. Such an MPC method represents a way of transforming an open-loop design methodology (i.e., optimal control) into a feedback one, as the input applied to the process depends on the most recent measurements at every time step. The contribution of this work is in using MPC method to deal with the point stabilization problem of differentially steered WMRs wherein the problem is simplified by neglecting the vehicle dynamics and considering the steering system only. The vehicle control inputs are computed by assuming that there is a “perfect velocity control” [17]. A stabilizing receding horizon regulator was previously developed based on a nonlinear control system since the linearized model of a nonholonomic system is not controllable [18]. The computation burden led to a problem even if the sub-optimal algorithm was utilized when the MPC was applied to this nonlinear robot model. System states on the other hand, were limited by additional constraints in order to guarantee the stability which in turn affected the performance of convergence.

An error model is first formed in this work by transforming the robot position into polar frame based on the target point posture. The control law is then formulated by introducing feedback factors so that the error model can be linearized at the equilibrium point without an uncontrollable problem. The main idea of MPC law is to optimize the cost function which penalizes the future point-tracking errors and control signals. This cause the feedback factors to be updated at every time point by the optimization process with the resulting control policy applied to the system during the following time interval. The exponential stability on the other hand, is guaranteed by considering corresponding variable constraints in the optimization rather than as restrictions to system states. Easy control computation and good convergence performance for point stabilization of WMRs are the advantages of MPC method. The proposed algorithm has the added advantage of achieving a more broad and clear stability analysis compared with time-varying feedbacks which guarantee exponentially stable systems [11-13].

This paper is organized into six parts wherein the kinematics of the mobile robot is introduced in Section 2 followed by the construction of the error model in Section 3. The MPC designed for the error model is in Section 4 while the computer simulation results are shown in Section 5. Concluding remarks are finally presented to round up the study.

2. Kinematics of the Wheeled Mobile Robot

The mobile robot shown in Figure 1 is a typical example of a nonholonomic mechanical system consisting of a vehicle with two driving wheels mounted on the same axis and two castors. The motion and orientation are achieved by independent actuators, *e.g.*, dc motors providing the necessary torques to the driving wheels. The position and orientation of the robot in a global Cartesian frame $\{O, X, Y\}$ are completely specified by $q = [x \ y \ \theta]^T$, which are the coordinates of P_c . Let P_c be the center of the axis of the driving wheels and also assumed as the mass center of the robot in our paper.

The nonholonomic kinematic constraint is described by

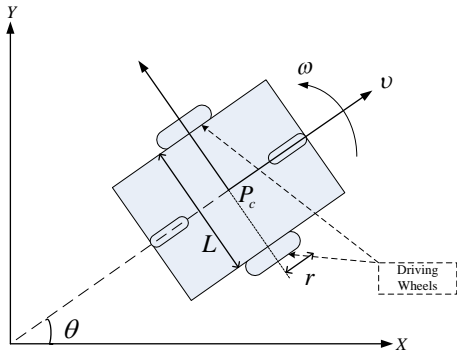


Figure 1. A Nonholonomic Wheeled Mobile Robot

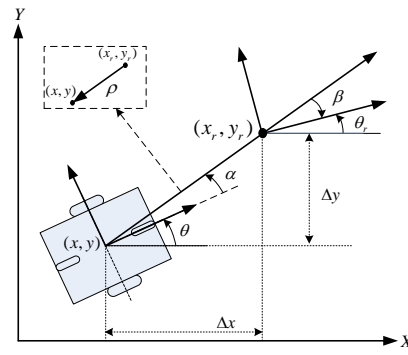


Figure 2. Coordinate Transformation of the Robot

$$A(q)\dot{q} = [\sin \theta \quad -\cos \theta \quad 0] \dot{q} = 0 \quad (1)$$

which corresponds to the ideal hypothesis of a “pure rolling and nonslipping” condition.

Let $S(q)$ be a full rank matrix formed by a set of smooth (i.e., continuously differentiable) and linearly independent vector fields in the null space of $A(q)$:

$$A(q)S(q) = 0. \quad (2)$$

It is easy to verify that $S(q)$ is given by

$$S(q) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix}. \quad (3)$$

The kinematic equations of motion in terms of its linear speed v and angular speed ω are

$$\dot{q} = S(q)u, \quad u = \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (4)$$

which is called the steering system of the vehicle.

In (4), the number of the control variables is less than the number of the state variables. Therefore, it is a standard nonholonomic system and the linearized model is not controllable [19].

3. The Definition of Error Model

The goal point should also be described by a state vector. For control (4) to move to the target, a new state vector can be expressed as follows (see Figure 2):

$$\begin{aligned} \rho &= \sqrt{\Delta x^2 + \Delta y^2} \\ \alpha &= -\theta + \text{atan2}(\Delta y, \Delta x) \\ \beta &= -\Delta\theta - \alpha \end{aligned} \quad (5)$$

where $\Delta x = x - x_r$, $\Delta y = y - y_r$ and $\Delta\theta = \theta - \theta_r$. Here, ρ is the distance from target to the robot and α is the angle from robot orientation to the opposite direction of the distance vector. Note that (5) can be regarded as an error model and the purpose is to design a controller that can converge three states to the origin.

The Cartesian frame $\{O, X, Y\}$ is rotated counterclockwise through the angle θ_r about the origin O in order to avoid the problem due to the target orientation. In the new frame $\{O, X', Y'\}$, (5) can be rewritten as (see Figure 3)

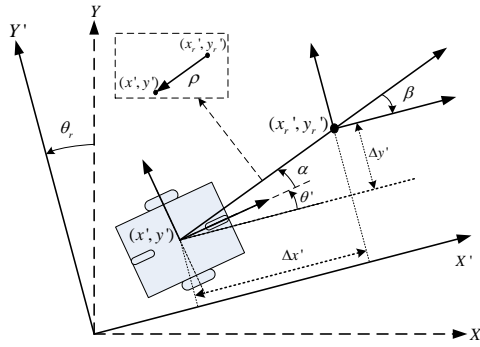


Figure 3. Coordinate Transformation After Rotation

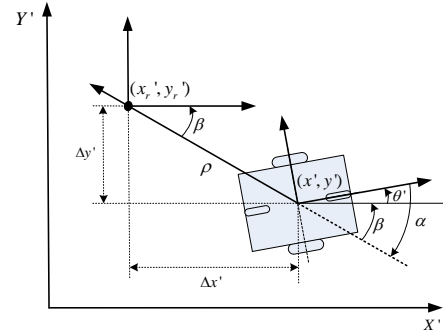


Figure 4. Coordinate Transformation for Backward Case

$$\begin{aligned}\rho &= \sqrt{\Delta x'^2 + \Delta y'^2} \\ \alpha &= -\theta' + \text{atan2}(\Delta y', \Delta x') \\ \beta &= -\theta' - \alpha\end{aligned}\quad (6)$$

where

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{matrix} x'-y' \\ x-y \end{matrix} \mathbf{R} \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x_r' \\ y_r' \end{bmatrix} = \begin{matrix} x'-y' \\ x-y \end{matrix} \mathbf{R} \begin{bmatrix} x_r \\ y_r \end{bmatrix}, \theta' = \theta - \theta_r \quad (7)$$

and

$$\begin{matrix} x' & y' \\ x & y \end{matrix} \mathbf{R} = \begin{bmatrix} \cos \theta_r & \sin \theta_r \\ -\sin \theta_r & \cos \theta_r \end{bmatrix} \quad (8)$$

is the corresponding rotation matrix.

The first order differential of the distance from the target to the starting point can then be defined as

$$\dot{\rho}(t) = -v \cos(\alpha(t)). \quad (9)$$

Meanwhile, the differentials of another two angular states can be obtained as

$$\dot{\beta}(t) = -v \sin(\alpha(t)) / \rho(t) \quad (10)$$

and

$$\alpha = -\beta - \theta', \dot{\alpha} = -\dot{\beta} - \dot{\theta}', \dot{\alpha} = v \sin(\alpha(t)) / \rho(t) - \omega(t). \quad (11)$$

The new kinematic equation of the motion in term of the state $[\rho \ \alpha \ \beta]^T$ and control input $[v \ \omega]^T$ is shown:

$$\begin{bmatrix} \dot{\rho}(t) \\ \dot{\alpha}(t) \\ \dot{\beta}(t) \end{bmatrix} = \begin{bmatrix} -\cos(\alpha(t)) & 0 \\ \sin(\alpha(t)) / \rho(t) & -1 \\ -\sin(\alpha(t)) / \rho(t) & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix}. \quad (12)$$

Case A: The angle state α belongs to the region defined as:

$$\mathbf{R}_1 = \left\{ \alpha \in \mathbb{R} \mid |\alpha| \leq \frac{\pi}{2} \right\}. \quad (13)$$

Note that the robot moves forward to the target in this case. By designing the feedback control law as

$$u(t) = \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} k_\rho \rho(t) \\ k_\alpha \alpha(t) + k_\beta \beta(t) \end{bmatrix}, \quad (14)$$

equation (12) can be rewritten as

$$\begin{bmatrix} \dot{\rho}(t) \\ \dot{\alpha}(t) \\ \dot{\beta}(t) \end{bmatrix} = \begin{bmatrix} -\cos(\alpha(t))k_{\rho}\rho(t) \\ \sin(\alpha(t))k_{\rho} - (k_{\alpha}\alpha(t) + k_{\beta}\beta(t)) \\ -\sin(\alpha(t))k_{\rho} \end{bmatrix}. \quad (15)$$

Subsequently, by linearizing the model (15) at the point $\rho = 0, \alpha = 0, \beta = 0$, the following linear model is obtained

$$\dot{x} = Ax, \quad A = \begin{bmatrix} -k_{\rho} & 0 & 0 \\ 0 & -(k_{\alpha} - k_{\rho}) & -k_{\beta} \\ 0 & -k_{\rho} & 0 \end{bmatrix} \quad (16)$$

where $x = [\rho(t) \quad \alpha(t) \quad \beta(t)]^T$.

Theorem 1:

If the factors satisfy the following constraints

$$k_{\rho} > 0, k_{\beta} < 0, k_{\alpha} - k_{\rho} > 0, \quad (17)$$

the origin $x_e = [0 \quad 0 \quad 0]^T$ is an exponentially stable equilibrium point for the system (16).

Proof:

If the eigenvalues λ of the matrix A satisfies $\text{Re}(\lambda) < 0$, the system is exponentially stable at x_e [20]. By using the Cramer's rule, it can be concluded that

$$\det(A - \lambda_i I) = 0 \quad (18)$$

which is known as the characteristic equation of A . Note that (18) can be written as

$$(\lambda + k_{\rho})(\lambda^2 + \lambda(k_{\alpha} - k_{\rho}) - k_{\rho}k_{\beta}) = 0. \quad (19)$$

By considering the condition of λ , the constraints of factors (17) can be obtained.

Case B: The angle state α belongs to the region as

$$R_2 = \left\{ \alpha \in \square \mid \left| \alpha \right| \geq \frac{\pi}{2} \right\}. \quad (20)$$

The robot should then move backward as in the case presented in Figure 4 with the corresponding state vector given as

$$\begin{aligned} \rho &= \sqrt{\Delta x'^2 + \Delta y'^2} \\ \alpha &= -\theta' + \text{atan2}(-\Delta y', -\Delta x') \\ \beta &= -\theta' - \alpha. \end{aligned} \quad (21)$$

Since the robot moves backward, the control law (14) is changed to

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} -k_{\rho}\rho(t) \\ k_{\alpha}\alpha(t) + k_{\beta}\beta(t) \end{bmatrix}. \quad (22)$$

Note that in this case, the linear model still has the same form as in (16). Second, the constraints of the feedback factors in Theorem 1 still hold as:

$$k_{\rho} > 0, k_{\beta} < 0, k_{\alpha} - k_{\rho} > 0. \quad (23)$$

By taking into account these two different cases of movement, the linear feedback control laws (14) and (22) with corresponding constraints (17) and (23) guarantee the exponential stability of the original error model (12). (14) and (22) has the advantage of simple control structure as well as broad stability analysis as compared to time-varying feedbacks that provide exponential convergence [11-13]. The methods used to obtain feedback factors subject to the stability constraints will be discussed in the following section.

4. The Model Predictive Controller

The continuous-time kinematic system (12) is proved to be exponentially stable with linear feedback control laws in the previous section. A constructive algorithm using MPC to obtain parameters $k_\rho, k_\alpha, k_\beta$ will now be presented. The main idea of MPC is to start with a model of the open-loop process that explains the relations among the system's variables. Constraint specifications on the system variables are then added such as input limitations and desired range so that the states and outputs should remain. The desired performance specifications that completes the control problem setup are expressed in the form of different weights on the tracking errors and actuator efforts. The rest of the MPC is then designed automatically. An optimal control problem based on the given model, constraints, and weights is constructed and translated into an equivalent optimization problem that depends on the initial state and reference signals. The optimization control problem is solved at each sampling time by taking the current (measured or estimated) state as its initial state. It is for this reason that the approach is said to be predictive, but the optimal control problem is in fact formulated over a time-interval that starts at the current time and extends up to a certain interval in the future. The result of the optimization is an optimal sequence of future control moves with only the first sample actually applied to the process and the remaining moves discarded. A new optimal control problem based on the new feedback state is solved over a shifted prediction horizon at the next time step which is the reason that this approach is also called "receding horizon" control.

The MPC concept is used to find the feedback factor values that minimize the quadratic cost function based on the predicted state and control input. By considering the computation convenience, linear models (16) is discretized by short time interval T as:

$$x(k+1)=Ax(k), x(k)=[\rho(k) \quad \alpha(k) \quad \beta(k)]^T \quad (24)$$

$$\text{s.t. } A = \begin{bmatrix} 1-k_\rho T & 0 & 0 \\ 0 & 1-T(k_\alpha - k_\rho) & -k_\beta T \\ 0 & -Tk_\rho & 1 \end{bmatrix}; \quad k_\rho > 0, k_\beta < 0, k_\alpha - k_\rho > 0 \quad (25)$$

Formulate the open-loop optimal control problem as:

$$J^*(x(k)) \square \min_{k_\rho, k_\alpha, k_\beta} \sum_{j=1}^N [x^T(k+j)Qx(k+j) + u^T(k+j-1)Ru(k+j-1)] \quad (26)$$

$$\text{s.t. } k_\rho > 0, k_\beta < 0, k_\alpha - k_\rho > 0 \quad (27)$$

where $x(k+j)$ denotes the state at time $k+j$, obtained by starting from the initial state $x(k)$ and applying input sequence $U=[u(k), u(k+1), \dots, (k+j-1)]^T$ to the model. N is the prediction horizon and $Q \in \mathbb{R}^{3 \times 3}$, $R \in \mathbb{R}^{2 \times 2}$ are weighting matrices

for state and control variables where $Q \geq 0$ and $R > 0$. The parameters $k_\rho, k_\alpha, k_\beta$ are regarded as optimal variables in (26).

In the moving time frame, the predictive error state within the horizon prediction N can be written as:

$$\begin{aligned} x(k+1) &= Ax(k) \\ x(k+2) &= A^2x(k) \\ &\vdots \\ x(k+N) &= A^Nx(k). \end{aligned} \quad (28)$$

It is now possible to rewrite the optimization problem in the usual quadratic form hence the following predictive state vectors is introduced:

$$X = Fx, F = [A \quad A^2 \quad \dots \quad A^N]^T \quad (29)$$

where $x = x(k)$ is the current state.

The control signal of the robot can be expressed as

$$\begin{aligned} u(k) &= \Phi(k)x, \\ \text{case A: } \Phi(k) &= \begin{bmatrix} k_\rho & 0 & 0 \\ 0 & k_\alpha & k_\beta \end{bmatrix}, \\ \text{case B: } \Phi(k) &= \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & k_\alpha & k_\beta \end{bmatrix}, \end{aligned} \quad (30)$$

and the predictive control vector written as

$$U(k) = [u(k), u(k+1), \dots, u(k+N-1)]^T = \Phi(k)F_{N-1}x \quad (31)$$

where

$$F_{N-1} = [I \quad A^2 \quad \dots \quad A^{N-1}]^T. \quad (32)$$

Rewrite (26)-(27) using vector notion (29) and (31) as:

$$\begin{aligned} J^*(x(k)) &\square \min_{k_\rho, k_\alpha, k_\beta} (X^T Q X + U^T R U) \\ &\square \min_{k_\rho, k_\alpha, k_\beta} \left\{ x^T \left[F_N^T Q F_N + F_{N-1}^T \Phi(k)^T R \Phi(k) F_{N-1} \right] x \right\} \end{aligned} \quad (33)$$

$$\text{s.t. } k_\rho > 0, k_\beta < 0, k_\alpha - k_\rho > 0 \quad (34)$$

where $Q = \text{diag}\{Q, \dots, Q\}$, $\bar{Q} \in \mathbb{R}^{(3 \cdot N) \times (3 \cdot N)}$ and $R = \text{diag}\{R, \dots, R\}$, $R \in \mathbb{R}^{(2 \cdot N) \times (2 \cdot N)}$. Denote that $K^*(k) = \{k_\rho^*, k_\alpha^*, k_\beta^*\}$ is the optimal solution to (33)–(34). Note that the control signal applied to the system (12) during a short time interval T is

$$\begin{aligned} u^*(t) &= \Phi^*(k)x(t), t \in [kT, (k+1)T], \\ \text{case A: } \Phi(k) &= \begin{bmatrix} k_\rho^* & 0 & 0 \\ 0 & k_\alpha^* & k_\beta^* \end{bmatrix}, \quad \text{case B: } \Phi(k) = \begin{bmatrix} -k_\rho^* & 0 & 0 \\ 0 & k_\alpha^* & k_\beta^* \end{bmatrix}. \end{aligned} \quad (35)$$

When a new state $x(k+1)$ is measured, the optimization procedure is repeated for another prediction horizon N :

$$J^*(x(k+1)) \square \min_{k_\rho, k_\alpha, k_\beta} \sum_{j=1}^N [x^T(k+1+j)Qx(k+1+j) + u^T(k+1+j-1)Ru(k+1+j-1)]. \quad (36)$$

Now the vector of feedback factors is $K^*(k+1)$ and the control

is $u^*(t) = \Phi^*(k+1)x(t), t \in [(k+1)T, (k+2)T]$.

Theorem 2: System (12) with an MPC derived state feedback controller is stable if and only if the optimization problem (26)-(27) is feasible for all $x(\cdot) \in \mathbb{R}^3$ at every time step.

Proof:

If the optimization problem is feasible at every time point, the corresponding parameters can be derived from the moving horizon strategy of MPC as:

$$K^*(k) = \{k_\rho^* \quad k_\alpha^* \quad k_\beta^*\}, \quad (37)$$

the feedback control law for (12) in the following time interval is

$$v(t) = \begin{cases} k_\rho^* \rho(t), & \text{case A} \\ -k_\rho^* \rho(t), & \text{case B} \end{cases} \quad (38)$$

$$\omega(t) = k_\alpha^* \alpha(t) + k_\beta^* \beta(t).$$

Note that the set of stable constraints in Theorem 1 are considered to be inequality constraints subject to the minimization problem (26). Consequently, the optimal solution (37) guarantees the exponential stability of the continuous system (12) and the convergence is not affected by the discretization.

5. Implementation of the Control

The bounded velocity constraints are considered in the control of a wheeled mobile robot. The robot's linear velocity v and angular velocity ω are restricted as

$$-0.5 \text{ m/s} \leq v \leq 0.5 \text{ m/s}, \quad -5 \text{ rad/s} \leq \omega \leq 5 \text{ rad/s}. \quad (39)$$

The parameters of the cost function are selected as:

$$N = 4, R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}. \quad (40)$$

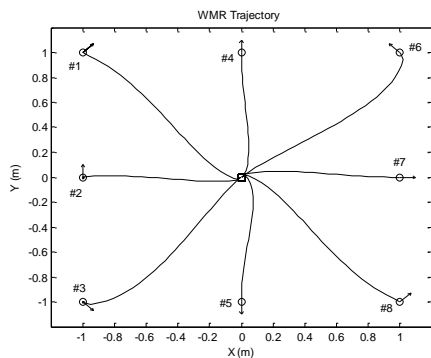


Figure 5. Trajectories from Several Starting Points

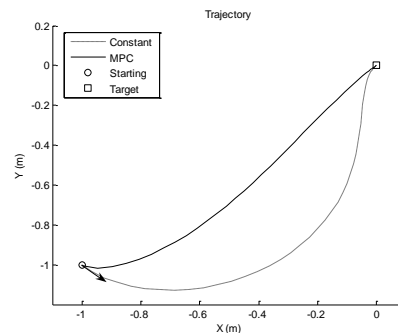


Figure 6. Trajectory Starting from Point #3

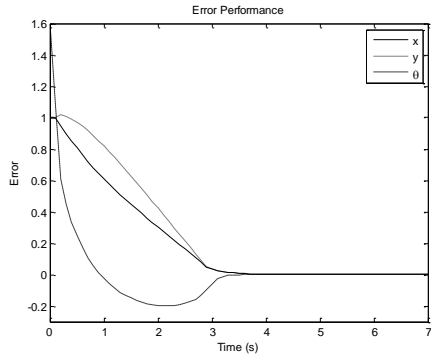


Figure 7. Position and Angle Errors

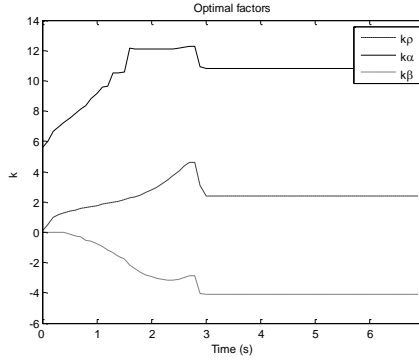


Figure 8. Linear and Angular Velocities

Simulation results for the control performance of the robot as it moves towards three consecutive targets is shown in Figures 12 and 13 indicating good stabilization and convergence. The results present an interesting opportunity for the application of the trajectory tracking of WMRs. Note that in Figure 12, the predictive error state of (28) was first considered using the fixed target point #1 and the system is then driven to a prescribed small neighbourhood of the desired configuration after which the target is switched to #2. The performance might be affected when the robot tracks trajectory in real time since only one reference point can be considered in the predicted optimization process.

6. Conclusion

The model predictive controller for the point stabilization of differentially steered wheeled mobile robots is presented in this paper. The linearized error model is formed in the polar frame and then the feedback control factors are derived from the optimization of the quadratic cost function which penalizes the predictive tracking error and control variables in each sampling time. The exponential stability of the system is guaranteed by considering the constraints during the minimization process. Simulation results demonstrate the good stabilization performance of the proposed algorithm. It was proven that the robot converges to the target more efficiently and accurately when compared to the constant feedback factors case and nonlinear MPC approach.

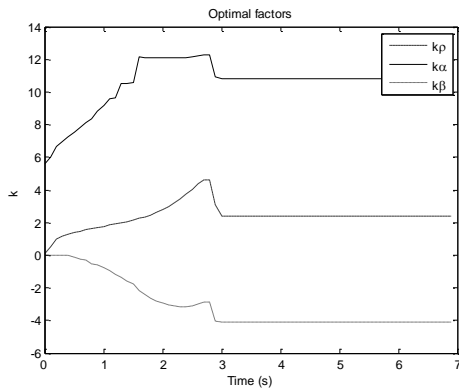


Figure 9. Optimal Feedback Factors

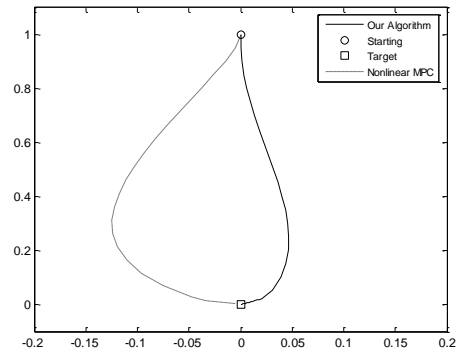


Figure 10. Robot Trajectory Starting from Point #4

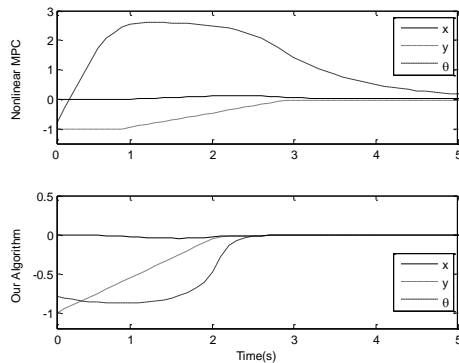


Figure 11. Position and Angle Errors

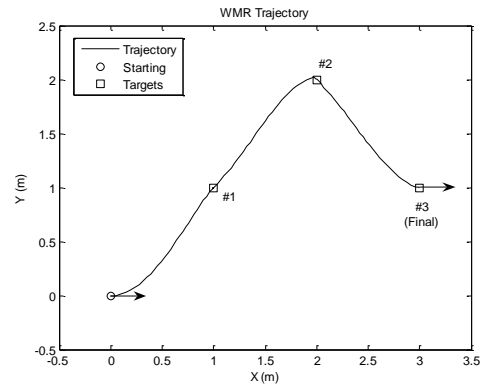


Figure 12. Trajectory from a Point to Three Consecutive Targets

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