Parametric Uncertainties Prone Adaptive Control Method for Omni Directional Vehicle

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Abstract

The trajectory tracking problem of a wheeled omnidirectional robot is addressed in this paper. The proposed controller takes into account the dynamics of the wheeled omnidirectional robot when adaptive linear control with computed torque is utilized for tracking its trajectory. The computed torque method was employed to follow a reference trajectory but there were certain degrees of uncertainties in parameters such as estimation of mass and inertia which caused the performance to deviate from the ideal case. The adaptive linear control method is used to overcome the degradation of the computed torque when there is drift in vehicle parameters. Lyapunov stability criteria was utilized to prove that the proposed adaptive linear control with computed torque method is stable. The proposed strategy was evaluated through numerical simulation which showed the better performance in tracking a reference signal of the adaptive linear control with computed torque method as compared to computed torque method only.

Keywords: adaptive control, computed torque, omni-vehicle, wheeled mobile robots

1. Introduction

Wheeled omnidirectional vehicles are popular in the robotics community due to its simultaneous rotation and translation capability. There have been various approaches in the construction of this type of vehicles such as that of Endo [1] which was constructed using a spherical ball that was rolled to provide the motion. An omni directional vehicle for rough terrain that utilized a widely used type of wheel called Swedish or Omni wheels was also proposed [2]. This type of vehicles can be constructed with a different number of wheels with a minimum of three wheels required for omni-directional motion [3] wherein some of these vehicles were designed as an omnidirectional mobile platform (OMP) with three wheels having manipulators on-board [4-6].

Trajectory generation and tracking are important aspects of an autonomous vehicle with several computationally efficient trajectory generation algorithms applied to wheeled omnidirectional vehicles. Some examples are when Purwin proposed the generation and tracking of near optimal minimum trajectories using the dynamic model of the vehicle [7] while trajectories were generated using the kinematic model by Indiveri [8]. The dynamic model of an omnidirectional mobile robot with computed torque strategy was used in order to solve the path-tracking problem by Vasquez [9]. Watanabe utilized analysis and control for an omnidirectional mobile manipulator by first deriving the dynamic model followed by employing computed torque and resolved acceleration control methods [4]. Velasco-Villa addressed trajectory-tracking control problem of an omnidirectional mobile robot by means of a full state information time varying feedback based which exploited the passivity properties of the exact tracking error dynamics [10].
The backstepping tracking controller based on Lyapunov method was proposed by Dinh in tracking a desired path for an OMP [5]. Penaloza-Mejia addressed the problem by designing a passive nonlinear controller that is combined with a tracking controller in a negative feedback connection structure [11]. Viet utilized a differential sliding mode controller (DSMC) based on dynamic modeling of the OMP with force external disturbances to obtain control inputs driving the OMP [6].

Several tracking controllers for non-linear systems can be utilized such as Feedback Linearization, Sliding Mode Control, or $H_{-\infty}$. Feedback Linearization also known as computed torque (CT) is usually adopted for mechanical systems with the CT method previously applied on wheeled omnidirectional vehicles [12]. Robust adaptive method in conjunction with sliding mode control was also previously proposed but one of its drawback is the chattering in the input [13]. The decrease in performance when using CT due to the presence of uncertainties in the parameters of the model led to the use of the adaptive linear control method in this work to overcome the degradation of the computed torque. Lyapunov stability criteria was utilized to prove that the proposed adaptive linear control with computed torque method is stable. The same way that several of the previous works utilized Lyapunov to prove the stability of their systems [5, 6, 11]. Numerical simulations confirmed the better performance in tracking a reference signal of the adaptive linear control with computed torque method as compared to computed torque method only.

The paper is arranged as follows: The structure and the dynamics of the vehicle are presented in Section 2 followed by a discussion on computed torque and adaptive methods in Section 3. The simulation results are given in Section 4 followed by the concluding remarks.

2. Omni Vehicle Structure and Dynamics

Omni-wheels (Figure 1a) consisting of discs at its perimeter is a main component of an Omni-vehicle which has the advantage of simultaneously rolling and sliding laterally. The omni-vehicle utilized for the study consists of three (3) omni-wheels with the axis of rotation at an angular interval of 120 degrees (Figure 2). The positive angles $a_1$, $a_2$, and $a_3$ are the angles inscribed by the axis on the X-axis of body reference frame $B$ of the vehicle while $l$ denotes the distance of the wheels from the centre of gravity. Details of the completely constructed Lego Mindstorm NXT Robot omni-vehicle is shown in Figure 1b (www.HiTechnique.com).

![Omni-wheel and Lego Mindstorm NXT](http://www.superdroidrobots.com/)

Figure 1. Holonomic Vehicle (http://www.superdroidrobots.com/)

![Arrangement of Omni-Wheels on the Vehicle](http://www.superdroidrobots.com/)

Figure 2. Arrangement of Omni-Wheels on the Vehicle
2.1. Omni Vehicle Dynamics

The motion of a robot can be described in terms of its configuration variables. A ground vehicle with motion confined in the XY plane (Figure 3) is given as
\[
q = \begin{bmatrix} x, y, \theta \end{bmatrix}^T \in \mathbb{R}^3,
\]
where \((x, y)\) represent the (X, Y) positions while angle \(\theta\) is inscribed by X-axis of \(B\) on the X-axis of inertial frame. The ground vehicle has \((v_x, v_y)\) as the velocity vectors along X and Y axis of \(B\) while \(\psi\) is the turning rate of the vehicle.

The following nonlinear dynamic model for the motion of the vehicle was derived through the Euler-Lagrange formalism used by Stonier [12]:

\[
M(q)\ddot{q} + N(q, \dot{q})\dot{q} = \tau,
\]

where

\[
M(q) = \begin{bmatrix} l_w D_2^{-1} R^T(q) + r_w D_1^{-1} R^T(q) \text{diag}(m_r, m_r, I_r) \end{bmatrix},
\]

\[
N(q, \dot{q}) = \begin{bmatrix} \frac{b}{r_w} D_2^{-1} R^T(q) + \frac{l_w}{r_w} D_2^{-1} S(\dot{\theta}) R^T(q) \end{bmatrix},
\]

\[
\tau_m = [\tau_1, \tau_2, \tau_3]^T \in \mathbb{R}^3
\]

are the input vectors to the system

\[
D_1 = \begin{bmatrix} -\sin \alpha_1 & -\sin \alpha_2 & -\sin \alpha_3 \\ \cos \alpha_1 & \cos \alpha_2 & \cos \alpha_3 \\ l & l & l \end{bmatrix}, \quad D_2 = \begin{bmatrix} -\sin \alpha_1 & -\sin \alpha_2 & -\sin \alpha_3 \\ \cos \alpha_1 & \cos \alpha_2 & \cos \alpha_3 \\ 1/l & 1/l & 1/l \end{bmatrix}
\]

while

\[
R(q) = \begin{bmatrix} \cos q_3 & -\sin q_3 & 0 \\ \sin q_3 & \cos q_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

is the rotational matrix from \(B\) to inertial frame. The following skew matrix \(S(\dot{\theta})\)

\[
S(\dot{\theta}) = \begin{bmatrix} 0 & -\dot{q}_3 & 0 \\ \dot{q}_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

with \(m_r\) as the mass of the robot, \(I_r\) as the moment of inertia of the robot, \(m_w\) as the mass of the wheel, \(I_w\) as the moment of inertia of the wheel, and \(b\) is the stiffness coefficient of the motor attached to the wheels.

The summary of the parameters of the robot used are listed in Table 1 with the motor dynamics and slipping of the wheels not considered in (1). The derivation of the model of the vehicle which includes the dynamics due to motors attached to the

![Figure 3. General Vehicle Moving on Ground Plane Where \(v_x, v_y, \psi\) are Velocity Vectors along X-Axis & Y-Axis of B, and Turning Rate of Vehicle.](image)
wheels will be given in the succeeding section.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of robot ($m_r$)</td>
<td>0.5 Kg</td>
</tr>
<tr>
<td>Mass of wheel ($m_w$)</td>
<td>0.038 Kg</td>
</tr>
<tr>
<td>Inertia of robot ($I_r$)</td>
<td>0.0034 Kg.m$^2$</td>
</tr>
<tr>
<td>Inertia of wheel ($I_w$)</td>
<td>0.0021 Kg.m$^2$</td>
</tr>
</tbody>
</table>

2.2. Motor Dynamics

The approximated parameters for the direct current (DC) motors (Lego Mindstorm NXT) connected to each of the wheels is given in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_b$</td>
<td>0.5036 V(rad/s)</td>
</tr>
<tr>
<td>$K_T$</td>
<td>0.32 N.m/A</td>
</tr>
<tr>
<td>$R_a$</td>
<td>5.11 ohm</td>
</tr>
<tr>
<td>$b$</td>
<td>0.0012 N/m</td>
</tr>
</tbody>
</table>

The dynamics of the armature voltage controlled DC motor is given as

$$V_a = R_a i_a + K_b \phi, \quad \tau_m = K_T i_a$$

where $V_a$ is the voltage applied across the armature, $i_a$ is the current flowing through it, $K_b$ is back emf constant, and $K_T$ is the torque constant.

Eliminating $i_a$ from (2) and noting that $\phi = \frac{v_w}{r_w}$ will give

$$\tau_m = \rho v_a - \beta v_m,$$

where

$$\rho = \frac{K_T}{R_a}, \quad \beta = \frac{K_T K_a}{R_a r_w}.$$

In vector form,

$$\tau_m = \rho v_a - \beta v_m.$$  \hspace{1cm} (3)

Back substituting (3) to (1) and then manipulating the variable will give the following dynamics of the vehicle and of the motor:

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} = \tau,$$  \hspace{1cm} (4)

where

$$M(q) = \frac{1}{\rho} \left( \frac{I_w}{r_w} D_z^{-1} R^T(q) + r_w D_z^{-1} R^T(q) \text{diag}(m_r, m_w, I_r) \right),$$

$$N(q, \dot{q}) = \left( \frac{\beta}{\rho} + \frac{b}{r_w \rho} \right) D_z^{-1} R^T(q) + \frac{I_w}{\rho r_w} D_z^{-1} S(q) R^T(q),$$

$$\tau_m = v_a.$$
3. Control

The CT method, which is a variant of feedback linearization, was employed to follow a reference trajectory. There were certain degrees of uncertainties in parameters such as estimation of mass and inertia which caused the performance to deviate from the ideal case. An adaptive method similar to the one used for the trajectory tracking of a manipulator [14] was additionally applied to address this problem. A brief introduction of CT and adaptive method are presented in the following sub-sections.

3.1. Computed Torque (CT)

A robot whose dynamics are similar to the form in (4) with \( n \) state variables was considered for the study. For the CT method, an input of the following form is applied to the system

\[
\tau = \dot{M}(q)(\ddot{q} - K_c \dot{e} - K_p \dot{e}) + \dot{N}(q, \dot{q}) \dot{q}
\]

(5)

where \( \dot{M} \in \mathbb{R}^{nn} \) and \( \dot{N} \in \mathbb{R}^{nn} \) are the estimates of inertial (\( M \)) and carioliis (\( N \)) matrices, \( \ddot{q} \) is the second derivative of desired trajectory vector, \( e = q - q_d \) is the error vector, \( \dot{e} = \dot{q} - \dot{q}_d \) is the derivative of error vector, \( K_c = k_c I \) is the derivative gain matrix, and \( K_p = k_p I \) is the proportional gain matrix. A closed loop system is produced if \( \dot{M} = M \) and \( \dot{e} + K_c \dot{e} + K_p \dot{e} = 0 \).

The closed loop system is guaranteed to be stable for positive values of \( K_p \) and \( K_c \) gain matrices.

3.2. Adaptive Computed Torque

The dynamics of the robot should first be parameterized in order to apply the adaptive method and this is only possible if the system is linearly dependent on its parameter(s). The dynamics of the system can be represented as

\[
W(q, \dot{q}, \ddot{q})^T p = M(q) \ddot{q} + N(q, \dot{q}) \dot{q},
\]

(6)

where \( p \in \mathbb{R}^r \) is the parameter vector and \( W(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{rn} \) is the regressor matrix.

Applying approximated CT input (5) to the system will allow the dynamics to be represented as:

\[
\dot{M}(q)(\ddot{q} - K_c \dot{e} - K_p \dot{e}) = -M(q) \ddot{q} - \dot{N}(q, \dot{q}) \dot{q} = -W(q, \dot{q}, \ddot{q}) \dot{p}
\]

or

\[
\ddot{p} + K_c \dot{e} + K_p \dot{e} = -M(q)^{-1}W(q, \dot{q}, \ddot{q}) \dot{p}
\]

where \( \dot{M}(q) = M(q) - \dot{M}(q), \dot{N}(q, \dot{q}) = \dot{N}(q, \dot{q}), \) and \( \dot{p} = p - \dot{p} \). With \( \dot{p} \) representing estimate of \( p \).

The equation can be represented in the following state space form after further modification:

\[
\dot{x} = A x - B \dot{M}(q)^{-1}W(q, \dot{q}, \ddot{q}) \dot{p},
\]

(7)

where
The following Lyapunov-like candidate function is chosen

\[ V(x, \hat{p}) = x^T P x + \hat{p}^T \Gamma \hat{p}, \]  

where \( P \in \mathbb{R}^{2n \times 2n} \) is a positive definite symmetric matrix, and \( \Gamma \) is a positive definite diagonal matrix.

\[ \dot{V}(x, \hat{p}) = x^T P x + x^T P \dot{x} + \hat{p}^T \Gamma \hat{p} + \hat{p}^T \Gamma \dot{p} \]

(9)

Equations (7) and (9) plus the fact that \( \hat{p}^T \Gamma \hat{p} = (\hat{p}^T \Gamma \hat{p})^T \), will give

\[ \dot{V}(x, \hat{p}) = (Ax - B \dot{M}(q)^{-1} W(q, \ddot{q}, \dot{q}))^T P x + x^T P (Ax - B \dot{M}(q)^{-1} W(q, \ddot{q}, \dot{q})) + 2 \hat{p}^T \Gamma \dot{p} \]

or

\[ \dot{V}(x, \hat{p}) = x^T A^T P x + x^T P A x - 2 \hat{p}^T \left( B \dot{M}(q)^{-1} W(q, \ddot{q}, \dot{q}) \right)^T + 2 \hat{p}^T \Gamma \dot{p} \]

which can be simplified as

\[ \dot{V}(x, \hat{p}) = -x^T Q x - 2 \hat{p}^T \left( \dot{M}(q)^{-1} W(q, \ddot{q}, \dot{q}) \hat{p} \right) \]

(10)

where \( A^T P + PA = -Q \) and \( Q \in \mathbb{R}^{2n \times 2n} \) is a positive definite symmetric matrix.

Applying the following update rule

\[ \dot{\hat{p}} = -\Gamma^{-1} W(q, \ddot{q}, \dot{q}) \hat{p} \]

(11)

to (8) will give \( \dot{V}(x, \hat{p}) = -x^T Q x < 0 \). Hence the closed loop system of (7) is stable in a Lyapunov sense.

### 3.3. Regressor Matrix for the Vehicle

The adaptive update rule given in (11) require regressor matrix \( W(q, \ddot{q}, \dot{q}) \) which should be calculated for every clock cycle of the controller. The dynamics of the system should be represented in parametric form as in (6) consisting of parametric vector \( p \) in order to calculate the regressor matrix. The parameter vector for the vehicle is selected as

\[ p = \begin{bmatrix} \rho_1 & \rho_2 & \rho_3 & \rho_4 \end{bmatrix}^T \]

(12)

The regressor matrix can then be calculated from (6) as

\[ W(q, \ddot{q}, \dot{q}) = [W_1 W_2 W_3 W_4], \]

(13)

where

\[ W_1 = D_2^{-1} R^T(q) \ddot{q} + D_2^{-1} S(q) R^T(q) \dot{q}, \quad W_2 = D_1^{-1} R^T(q) [\ddot{q}_1 \ddot{q}_2 \ddot{q}_3 T, \quad W_3 = D_4^{-1} R^T(q)[0 0 \ddot{q}_3 T, \quad W_4 = D_2^{-1} R^T(q) \dot{q}. \]
3.4. Estimation of $\dot{q}$

Regressor matrix $W(q, \dot{q}, \ddot{q})$ requires the calculation of $\dot{q}$. Since $\dot{q}$ cannot be measured directly, it is estimated from $\ddot{q}$ by taking the time derivative and then applying discretization using impulse invariance transformation. The $Z$ transform for the differentiator using impulse invariance transformation is given by

$$Z\left(\frac{dy}{dt}\right) = \frac{2(1-z^{-1})}{T(1+z^{-1})},$$

(14)

where $T$ is the sampling period, and $z$ is the complex variable.

4. Simulation Results

Eight-shaped reference trajectories were generated for the simulations using the following parametric function

$$x_d(t) = r_d \cos(2\pi F t), \quad y_d(t) = r_d \sin(\pi F t), \quad \theta_d(t) = 0$$

(15)

where $x_d(t)$ is the reference state along X-axis, $y_d(t)$ is the reference state along Y-axis, $\theta_d(t)$ is the reference orientation of the vehicle, $r_d = 2m$ is the radius of the desired trajectory, and $F = 0.2 \text{ Hz}$ is the frequency.

Errors when tracking the reference states of (15) are shown in Figure 4. These are defined as $e_x = x - x_d, e_y = y - y_d, \text{and} e_{\theta} = \theta - \theta_d$ with their corresponding derivatives.

Figure 4. CT Only (Blue) and Adaptive CT (Green): (A) Position & (B) Velocity Errors Along X-Axis, (C) Position & (D) Velocity Errors Along Y-Axis, (E) Orientation Angle Error, (F) Angular Velocity Error
It can be seen from the glitches in the figure that CT is not able to track the reference states properly due to parameter uncertainties as compared to adaptive CT. This becomes much more evident in the plot of \( \dot{\hat{e}_x} \) in Figure 5 which shows an oscillating tracking error. The actual orientation angle tracking for adaptive CT is shown in Figure 6 while the trajectories for both CT only and Adaptive CT methods are shown in Figure 7 for comparative purposes. The simulation results showed that Adaptive CT method is superior than CT method only due to better estimation of parameters (closer to the actual values).

![Figure 5. Expanded View of Position Error Along X-Axis \( \dot{\hat{e}_x} \)](image)

![Figure 6. Estimation Of \( \phi_2 \) Using Adaptive CT Method](image)

![Figure 7. Vehicle Trajectories for CT Only (Blue) and Adaptive CT (Red)](image)

### 5. Conclusion

A strategy in addressing trajectory tracking problem of a wheeled omnidirectional robot through adaptive nonlinear control in conjunction with Computed Torque (CT) is presented in this paper. The dynamics of the wheeled omnidirectional robot is taken into account when adaptive linear control with computed torque is utilized for tracking the trajectory. The computed torque method was used to follow a reference trajectory but certain degrees of uncertainties in some of the parameters such as estimation of mass and inertia caused the performance to deviate from the ideal case. The adaptive linear control method was utilized to overcome the degradation of the computed torque due to drift in vehicle parameters. The proposed adaptive linear control with computed
torque method was proven stable through Lyapunov stability criteria. Numerical simulations showed that the proposed adaptive linear control with computed torque method performed much better when tracking the wheeled omnidirectional robot as compared to computed torque method only.

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