International Journal of Control and Automation
Vol. 9, No. 5 (2016), pp. 21-32
http://dx.doi.org/10.14257/ijca.2016.9.5.03

Co-design of H_∞ Filter and Scheduling Strategy for NCS with Stochastic Nonlinearities and Multiplicative Noises via Delay-Partitioning Method

Yong Zhang*, Ning Wang, Jingjing Hu

College of Information and Control Engineering, China University of Petroleum
paul_zhangyong@163.com, wnqfnu@163.com, hu_589@163.com

Abstract

The co-design problem of H_∞ filter and scheduling strategy for NCS with stochastic nonlinearities and multiplicative noises is addressed in this paper. With consideration of communication constraints, the NCS are first modeled as a set of discrete-switched subsystems by applying scheduling strategy. Then a novel Lyapunov-Krasovskii functional and delay-partitioning technique are used to design an available filter such that the filtering error system is asymptotically mean-square stable and achieves a prescribed H_∞ performance level. A numerical example is given to illustrate the effectiveness of the proposed method.

***Keywords**: networked control systems; H_∞ filter; scheduling strategy; delay-partitioning*

1. Introduction

Networked control systems (NCS) are the novel frame of control system theories born with control, network and computer application technologies. Multilayer feedforward networks are used to design the nonlinear flight control systems. PLC networked control systems are used in automobile vehicle bridge welding machine production line’s control system. PLC networked control systems not only reduced the production cost in automobile vehicle bridge welding production line application design, but raised the vehicle bridge production system’s productivity. With the increasing application of networks in complex dynamic environment such as advanced aircraft, automobile, and manufacturing processes, the networked control systems (NCS) are currently receiving considerable attention [1-3]. Compared with the traditional control systems, the advantages of NCS include decreased wiring and cost, ease of installation and maintenance, and increased system flexibility. At the mean time the challenges are developed, such as transmission delays [4], packed dropouts [5] and communication constraints [6].

Filtering problem has been a significant research topic in the filed of control systems. The well-known Kalman filtering is the most representative one among various filtering approaches under the assumption that all noise signals have a Gaussian distribution. However, when noise signals are unknown, to handle this problem, H_∞ filtering has been well developed whose main idea is to design an estimator for a given system to estimate a combination of unknown states [7]. In the past years, various approaches, which include the linear matrix inequality (LMI) approach [8-9] and Riccati equation approach [10] have been used to deal with filtering problem. In [11], H_∞ output-feedback control of discrete-time systems with multiplicative noises has been investigated. H_∞ filtering problem for NCS with multiplicative noises need to be further investigated.

ISSN: 2005-4297 IJCA

On the other hand, it is inevitable that there exist time-delay and communication constraints in NCS. In the last decades, a great number of results have been obtained for NCS with various types of delays [12]. Recently, Delay-partitioning technique has been widely used to address stability analysis problems, which can effectively decrease the possible conservatism of stability criteria [13]. While because of the influence of limited network bandwidths, network resources need to be carried on the reasonable scheduling strategy. For example, stability of NCS subject to communication constraints has been considered in [14], which given a protocol, such as the Round-Robin (RR) and Try-Once-Discard (TOD) protocol [15], provides criteria for computing the Maximum Allowable Transmission Interval (MATI) and Maximum Allowable Delay (MAD). In [16], NCS with communication constraints are modeled as a class of optimization problems and then design the appropriate scheduling strategy to maximum satisfy performances of the system.

In this paper, the co-design problem of H_∞ filter and scheduling strategy for NCS with stochastic nonlinearities and multiplicative noises is investigated. First of all, H_∞ filtering problem of NCS under the influence of communication constraints are modeled as a set of filtering error discrete-switched systems with uncertainty parameters via scheduling strategy. Then considering time-delay, in order to decrease the conservatism of NCS, delay-partitioning technique and a novel Lyapunov-Krasovskii functional are applied to develop an asymptotically mean-square stable condition of the filtering error discrete-switched system with a prescribed H_∞ performance level. Furthermore by solving LMI, the reliable H_∞ filter gains are obtained. Finally, a simulation example is utilized to illustrate the effectiveness of the proposed method.

2. Problem Formulation

The structure of NCS with stochastic nonlinearities and multiplicative noises to be considered is shown in Figure 1.

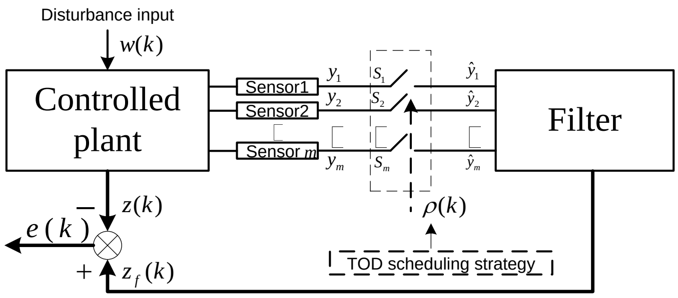


Figure 1. H_∞ Filtering Error System Model of NCS with Stochastic Nonlinearities and Multiplicative Noises

Consider the following controlled plant with stochastic nonlinearities and multiplicative noises:

$$\begin{cases} x(k+1) = (A+\Delta A)x(k) + (A_{d0} + \Delta A_{d0})x(k-d(k)) + A_{d1}\sum_{m=1}^{\infty}\mu_m x(k-m) + Bw(k) + Ef(k) \\ y(k) = Cx(k) + Dw(k) \\ z(k) = Lx(k) \\ x(k) = \phi(k),\ -\infty < k \le 0 \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state vector; $w(k) \in R^q$ is the disturbance input which belongs to $l_2[0,\infty)$; and $y(k) \in R^p$ is the plant output; $z(k) \in R^l$ is the signal to be estimated;

$\phi(k)$ is the initial state of system; $d(k)$ denotes the time-varying delay with lower and upper bounds $d_m \le d_k \le d_M$; the lower bound of delay d_m can be always described by $d_m = \tau m$ where τ and m are integers. $A, A_{d0}, A_{d1}, B, C, D, E$ and L are known real matrices with appropriate dimensions.

ΔA and ΔA_{d0} are unknown matrices representing parameter uncertainties that are assumed to satisfy the following admissible condition:

$$\begin{bmatrix}\Delta A & \Delta A_{d0}\end{bmatrix} = MF\begin{bmatrix}N & N_{d0}\end{bmatrix},\ FF^T \le I \tag{1}$$

where M, N and N_{d0} are known constant matrices with appropriate dimensions.

The function $f(k)$ describes the well-known stochastic nonlinearities that consist of $x(k), x(k-d(k))$ and $\sum_{m=1}^{\infty}\mu_m x(k-m)$.

The constants $\mu_m \ge 0,\ (m = 1, 2, ...)$ satisfy the following convergence conditions:

$$\bar{\mu} := \sum_{m=1}^{\infty}\mu_m \le \sum_{m=1}^{\infty} m\mu_m \le \infty \tag{2}$$

Consider the filter of the following form:

$$\begin{cases} x_f(k+1) = A_f x_f(k) + B_f \hat{y}(k) \\ z_f(k) = C_f x_f(k) \end{cases} \tag{3}$$

where $x_f(k) \in R^n$ is the filter's state, $\hat{y}(k)$ is the filter's input, $z_f(k)$ is the estimated output; A_f, B_f and C_f are parameters to be determined.

Due to the influence of the limited bandwidth, sensor nodes need to be scheduled at every sampling period, for example $\rho_1(k) = 1$ represents S_1 closed. $\Pi(k)$ is defined by

$$\Pi(k) = diag\{\rho_1(k), \rho_2(k), \cdots, \rho_m(k)\} \tag{4}$$

At the kth sampling period, scheduling function $\Pi(k)$ determines which sensor nodes can communicate with the filter through the shared network. Then $\Pi(k)y(k)$ are valid updated dates which received by filter, other signals those are not transmitted remain the last value by used zero-order-holders, so the filter input is described as follows:

$$\hat{y}(k) = \Pi(k)y(k) + (I - \Pi(k))\hat{y}(k-1) \tag{5}$$

Because the NCS have b sensor nodes, and only d_s sensor nodes can communicate with filter through real-time network at every transmission instant. The scheme that which nodes are chosen to access the network is defined as a mode, and then there are $L = b!/(b-d_s)!$ modes in the system.

The overall NCS have L modes, that is, L subsystems. The lth mode corresponds to $\Pi_l(k), (l = 1, 2, \cdots, L)$. In this paper, TOD protocol is used to determine switching law among all those subsystems.

TOD protocol, sometimes also called the Maximum Error First (MEF) refers that nodes with the maximum weighted error have higher transmission priority; others that are not transmitted will be replaced by the next update values. However, in this paper, the problem is that not a single node but multi-nodes under each mode can be transmitted at every transmission instant. Define filter error as follows:

$$e(k) = z_f(k) - z(k) \tag{6}$$

Switching law function $\delta(k)$ is obtained according to the TOD dynamic scheduling protocol:

$$\delta(k) = \arg\max\{\|\Pi_1(k)e(k)\|, \|\Pi_2(k)e(k)\|, \cdots, \|\Pi_N(k)e(k)\|\} \tag{7}$$

where $\delta(k) \in U \overset{def}{=} \{1, 2, \cdots, L\}$, that is, the subsystem with the maximum error is chosen to communicate data.

Under the influence of TOD scheduling protocol, the dynamic filter is chosen as the following form:

$$\begin{cases} x_f(k+1) = A_{f\delta(k)}x_f(k) + B_{f\delta(k)}\hat{y}(k) \\ z_f(k) = C_{f\delta(k)}x_f(k) \\ \hat{y}(k) = \Pi_{\delta(k)}(k)y(k) + (I - \Pi_{\delta(k)}(k))\hat{y}(k-1) \end{cases} \tag{8}$$

By defining $\xi(k) = \begin{bmatrix} x^T(k) & \hat{y}^T(k-1) & x_f^T(k) \end{bmatrix}^T$, combining with **Error! Reference source not found.** and (8), we obtain the following filtering error discrete-switched system:

$$\hat{S}_{\delta(k)}: \begin{cases} \xi(k+1) = \tilde{A}\xi(k) + \tilde{A}_{d0}\xi(k-d(k)) + \tilde{A}_{d1}\sum_{m=1}^{+\infty}\mu_m\xi(k-m) + \tilde{B}w(k) + \tilde{E}f(k) \\ e(k) = \Psi\xi(k) \end{cases} \tag{9}$$

where

$$\tilde{A} = \begin{bmatrix} A+\Delta A & 0 & 0 \\ \Pi_{\delta(k)}(k)C & (I-\Pi_{\delta(k)}(k)) & 0 \\ B_{f\delta(k)}\Pi_{\delta(k)}(k)C & B_{f\delta(k)}(I-\Pi_{\delta(k)}(k)) & A_{f\delta(k)} \end{bmatrix} =$$

$$\begin{bmatrix} A & 0 & 0 \\ \Pi_{\delta(k)}(k)C & (I-\Pi_{\delta(k)}(k)) & 0 \\ B_{f\delta(k)}\Pi_{\delta(k)}(k)C & B_{f\delta(k)}(I-\Pi_{\delta(k)}(k)) & A_{f\delta(k)} \end{bmatrix} + \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} N & 0 & 0 \end{bmatrix} = \bar{A} + \bar{M}F\bar{N}$$

$$\tilde{A}_{d0} = \begin{bmatrix} A_{d0}+\Delta A_{d0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_{d0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} N_{d0} & 0 & 0 \end{bmatrix} = \bar{A}_{d0} + \bar{M}F\bar{N}_{d0}$$

$$\tilde{A}_{d1} = \begin{bmatrix} A_{d1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \tilde{B} = \begin{bmatrix} B \\ \Pi_{\delta(k)}(k)D \\ B_{f\delta(k)}\Pi_{\delta(k)}(k) \end{bmatrix}, \ \tilde{E} = \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix}, \ \Psi = \begin{bmatrix} -L & 0 & C_{f\delta(k)} \end{bmatrix} \tag{10}$$

In this paper, for TOD dynamic scheduling protocol $\delta(k) \in U$ and any given performance index $\gamma > 0$, the purpose is to design a filter like as (8) for system (9) so that the following conditions are established:

Ⅰ. System (9) is asymptotically mean-square stable when $w(k) = 0$.

Ⅱ. Under zero initial condition for non-zero disturbance signal $w(k) \in l_2[0, \infty)$, system (9) satisfies the following inequality

$$\sum_{k=0}^{\infty} \mathrm{E}\{\|e(k)\|^2\} \le \sum_{k=0}^{\infty} \mathrm{E}\{\gamma^2 \|w(k)\|^2\} \tag{11}$$

where γ is called H_∞ filtering performance level.

3. Main Results

Lemma 1[17] Let $G \in R^{n\times n}$ be a positive semi-definite matrix, $x_i \in R^n$ and constants $a_i > 0 (i = 1, 2, \cdots)$. If the series concerned is convergent, then

$$\left(\sum_{i=1}^{+\infty} a_i x_i\right)^T G \sum_{i=1}^{+\infty} a_i x_i \le \sum_{i=1}^{+\infty} a_i x_i^T G a_i x_i \tag{12}$$

Lemma 2[Schur complement] Given a matrix $\bar{S} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} \\ \bar{S}_{12}^T & \bar{S}_{22} \end{bmatrix}$, where $\bar{S}_{11} \in R^{r\times r}$. The following three conditions are equivalent:

① $\bar{S} < 0$

② $\bar{S}_{11} < 0,\ \bar{S}_{22} - \bar{S}_{12}^T \bar{S}_{11}^{-1} \bar{S}_{12} < 0$

③ $\bar{S}_{22} < 0,\ \bar{S}_{11} - \bar{S}_{12} \bar{S}_{22}^{-1} \bar{S}_{12}^T < 0$

Lemma 3[18] Let $J = J^T$, $\tilde{M}$ and $\tilde{N}$ be real matrices of appropriate dimensions with $\tilde{F}$ satisfying $\tilde{F}\tilde{F}^T \le I$, then $J + \tilde{M}\tilde{F}\tilde{N} + \tilde{N}^T \tilde{F}^T \tilde{M}^T < 0$ if and only if there exists a positive scalar ε such that

$$J + \varepsilon^{-1}\tilde{M}\tilde{M}^T + \varepsilon \tilde{N}^T \tilde{N} < 0 \tag{13}$$

Theorem 1 For the filtering error system (9) with TOD dynamic scheduling protocol $\delta(k) \in M$ and a H_∞ performance level $\gamma > 0$. If there exist matrices $P > 0, R > 0,$ $Q_1 > 0, Q_2 > 0, Q_3 > 0, S_1 > 0, S_2 > 0$ and a scalar $\bar{\mu} > 0$ such that the following linear matrix inequality holds,

$$\begin{bmatrix}
\Omega_1 & * & * & * & * & * & * & * & * & * & * \\
W_{p1}^T & -P^{-1} & * & * & * & * & * & * & * & * & * \\
W_{p2}^T & 0 & \Omega_2^{-1} & * & * & * & * & * & * & * & * \\
W_{p3}^T & 0 & 0 & P^{-1} & * & * & * & * & * & * & * \\
W_{p4}^T & 0 & 0 & 0 & -P_4^{-1} & * & * & * & * & * & * \\
W_{p5}^T & 0 & 0 & 0 & 0 & P_5^{-1} & * & * & * & * & * \\
W_{Q1}^T & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_1^{-1} & * & * & * & * \\
W_{Q2}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_2^{-1} & * & * & * \\
W_{Q3}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_3^{-1} & * & * \\
W_{R1}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R^{-1} & * \\
W_{R2}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R^{-1}
\end{bmatrix} < 0 \tag{14}$$

where

$$\Omega_1 = \Xi^T[\tau S_1 + (d_M - \tau m)S_2]\Xi, \quad \Omega_2 = -[P + \tau S_1 + (d_M - \tau m)S_2],$$

$$\bar{Q}_1 = \begin{bmatrix} Q_1 & 0 \\ 0 & -Q_1 \end{bmatrix}, \bar{Q}_2 = \begin{bmatrix} Q_2 & 0 \\ 0 & -Q_2 \end{bmatrix}, \bar{Q}_3 = \begin{bmatrix} Q_3 & 0 \\ 0 & -Q_3 \end{bmatrix}, P_4 = \Psi^T(k)\Psi(k), P_5 = \gamma^2 I,$$

$$W_{p1} = \begin{bmatrix} \tilde{A} & 0_{2n,2mn} & \tilde{A}_{d0} & 0_{2n} & \tilde{A}_{d1} & 0_n & \tilde{B} \end{bmatrix}, W_{p2} = \begin{bmatrix} 0_{2n,2mn+10n} & \tilde{E} & 0_{2n,q} \end{bmatrix},$$

$$W_{p3} = \begin{bmatrix} I_{2n} & 0_{2n,2mn+7n+q} \end{bmatrix}, W_{p4} = \begin{bmatrix} I_{2n} & 0_{2n,2mn+7n+q} \end{bmatrix}, W_{p5} = \begin{bmatrix} 0_{2n,2mn+7n+q} & I_{2n} \end{bmatrix}$$

$$W_{R1} = \begin{bmatrix} I_{2n} & 0_{2mn+7n+q} \end{bmatrix}, W_{R2} = \begin{bmatrix} 0_{2n,2mn+6n} & \lambda_2 I_{2n} & 0_{n+q} \end{bmatrix},$$

$$W_{Q1} = \begin{bmatrix} I_{2mn} & 0_{2mn,9n+q} & \\ 0_{2mn,2n} & I_{2mn} & 0_{2n,7n+q} \end{bmatrix}, W_{Q1} = \begin{bmatrix} I_{2n} & 0_{2n,2mn+7n+q} & \\ 0_{2n,2mn+4n} & I_{2n} & 0_{2n,3n+q} \end{bmatrix},$$

$$W_{Q3} = \begin{bmatrix} \lambda_1 I_{2n} & 0_{2n,2mn+7n+q} & \\ 0_{2n,2mn+2n} & I_{2n} & 0_{2n,5n+q} \end{bmatrix}, \Xi = W_{p1} - W_{p3},$$

$$\lambda_1 = \sqrt{d_M - \tau m + 1}, \lambda_2 = \sqrt{\frac{1}{\bar{\mu}}}$$

then the filtering error system (9) is asymptotically mean-square stable with H_∞ filtering performance level γ.

Proof: Define the following Lyapunov-Krasovskii functional candidate as follows:

$$V(k)=\sum_{i=1}^{5}V_i(k) \tag{15}$$

where

$$\begin{aligned}
V_1(k)&=\xi^T(k)P\xi(k),\\
V_2(k)&=\sum_{i=k-\tau}^{k-1}\Gamma_i^T Q_1\Gamma_i+\sum_{i=k-d_M}^{k-1}\xi^T(i)Q_2\xi(i),\\
V_3(k)&=\sum_{j=-d_M+1}^{-\tau m+1}\sum_{i=k-1+j}^{k-1}\xi^T(i)Q_3\xi(i),\\
V_4(k)&=\sum_{j=-\tau}^{-1}\sum_{i=k+j}^{k-1}\delta_i^T S_1\delta_i+\sum_{j=-d_M}^{-\tau m-1}\sum_{i=k+j}^{k-1}\delta_i^T S_2\delta_i,\\
V_5(k)&=\sum_{m=1}^{+\infty}\sum_{i=k-m}^{k-1}\xi^T(i)R\xi(i)
\end{aligned}$$

with $\delta_i=\xi(i+1)-\xi(i),\quad \Gamma_i=\begin{bmatrix}\xi(i)\\ \xi(i-\tau)\\ \vdots \\ \xi(i-(m-1)\tau)\end{bmatrix}.$

Under the zero-initial condition, the difference of $V(k)$ can be obtained as follows:

$$\mathrm{E}\{\Delta V(k)\}=\mathrm{E}\{\Delta V_1(k)\}+\mathrm{E}\{\Delta V_2(k)\}+\mathrm{E}\{\Delta V_3(k)\}+\mathrm{E}\{\Delta V_4(k)\}+\mathrm{E}\{\Delta V_5(k)\} \tag{16}$$

where

$$\begin{aligned}
\mathrm{E}\{\Delta V_1(k)\}&=\mathrm{E}\{\xi^T(k+1)P\xi(k+1)-\xi^T(k)P\xi(k)\}\\
&=\mathrm{E}\{[\tilde{A}\xi(k)+\tilde{A}_{d0}\xi(k-d(k))+\tilde{A}_{d1}\sum_{m=1}^{+\infty}\mu_m\xi(k-m)+\tilde{B}w(k)+\tilde{E}f(k)]^T P\\
&\quad\cdot[\tilde{A}\xi(k)+\tilde{A}_{d0}\xi(k-d(k))+\tilde{A}_{d1}\sum_{m=1}^{+\infty}\mu_m\xi(k-m)+\tilde{B}w(k)+\tilde{E}f(k)]\\
&\quad-\xi^T(k)P\xi(k)\}\\
&=\mathrm{E}\{\alpha_k^T(W_{p1}^T PW_{p1}+W_{p2}^T PW_{p2}-W_{p3}^T PW_{p3})\alpha_k\}\\
\mathrm{E}\{\Delta V_2(k)\}&=\mathrm{E}\{\Gamma^T(k)Q_1\Gamma(k)-\Gamma^T(k-\tau)Q_1\Gamma(k-\tau)+\xi^T(k)Q_2\xi(k)-\\
&\quad\xi^T(k-d_M)Q_2\xi(k-d_M)\}\\
&=\mathrm{E}\{\alpha_k^T(W_{Q1}^T\bar{Q}_1W_{Q1}+W_{Q2}^T\bar{Q}_2W_{Q2})\alpha_k\}\\
\mathrm{E}\{\Delta V_3(k)\}&=\mathrm{E}\{(d_M-\tau m+1)\xi^T(k)Q_3\xi(k)-\sum_{i=k-d_M}^{k-\tau m}\xi^T(i)Q_3\xi(i)\}\\
&\le\mathrm{E}\{(d_M-\tau m+1)\xi^T(k)Q_3\xi(k)-\xi^T(k-d(k))Q_3\xi(k-d(k))\}\\
&=\mathrm{E}\{\alpha_k^T(W_{Q3}^T\bar{Q}_3W_{Q3})\alpha_k\}
\end{aligned}$$

$$\begin{aligned}\mathrm{E}\{\Delta V_4(k)\} &= \mathrm{E}\{\delta_k^T(\tau S_1+(d_M-\tau m)S_2)\delta_k - \sum_{i=k-\tau}^{k-1}\delta_i^T S_1\delta_i - \sum_{i=k-d_k}^{k-\tau m-1}\delta_i^T S_2\delta_i - \sum_{i=k-d_M}^{k-d_k-1}\delta_i^T S_2\delta_i\} \\ &= \mathrm{E}\{\alpha_k^T(\tau(\Xi^T S_1\Xi + (d_M-\tau m)\Xi^T S_2\Xi + \tau\bar{\varsigma}(1-\bar{\varsigma})W_{p2}^T S_1 W_{p2} + \\ &\quad (d_M-\tau m)W_{p2}^T S_2 W_{p2})\alpha_k - \sum_{i=k-\tau}^{k-1}\delta_i^T S_1\delta_i - \sum_{i=k-d_k}^{k-\tau m-1}\delta_i^T S_2\delta_i - \sum_{i=k-d_M}^{k-d_k-1}\delta_i^T S_2\delta_i\}\end{aligned}$$

$$\mathrm{E}\{\Delta V_5(k)\} = \mathrm{E}\{\frac{1}{\bar{\mu}}[\bar{\mu}\xi^T(k)R\xi(k) - \sum_{m=1}^{+\infty}\mu_m\xi^T(k-m)R\xi(k-m)]\}$$

According to Lemma1, one can obtain

$$\left(\sum_{m=1}^{+\infty}\mu_m\xi(k-m)\right)^T R\sum_{m=1}^{+\infty}\mu_m\xi(k-m) \le \sum_{m=1}^{+\infty}\mu_m\xi^T(k-m)R\mu_m\xi(k-m) \quad (17)$$

therefore

$$\mathrm{E}\{\Delta V_5(k)\} \le \mathrm{E}\{\alpha_k^T(W_{R1}^T R W_{R1} - W_{R2}^T R W_{R2})\alpha_k\} \quad (18)$$

with $\alpha_k = \begin{bmatrix}\Gamma^T(k) & \xi^T(k-m) & \xi^T(k-d(k)) & \xi^T(k-d_M) & \sum_{m=1}^{+\infty}\mu_m\xi^T(k-m) & f^T(k) & w^T(k)\end{bmatrix}^T$

In order to analyze the H_∞ filtering performance, let

$J(e,w) = \sum_{k=0}^{\infty}\mathrm{E}\left\{e^T(k)e(k) - \gamma^2 w^T(k)w(k)\right\}$, then

$$\begin{aligned}J(e,w) &= \sum_{k=0}^{\infty}\mathrm{E}\{e^T(k)e(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)\} + \mathrm{E}\{V(0)\} - \mathrm{E}\{V(\infty)\} \\ &\le \sum_{k=0}^{\infty}\mathrm{E}\{e^T(k)e(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)\}\end{aligned} \quad (19)$$

therefore

$$\begin{aligned}&\mathrm{E}\{e^T(k)e(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)\} \\ &= \mathrm{E}\{\xi^T(k)\Psi^T(k)\Psi(k)\xi(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)\} \\ &\le \mathrm{E}\{\alpha_k^T(W_{p4}^T P_4 W_{p4} - W_{p5}^T P_5 W_{p5})\alpha_k + \Delta V(k)\} \le \{\alpha_k^T(W_{p4}^T P_4 W_{p4} - W_{p5}^T P_5 W_{p5})\alpha_k \\ &+\alpha_k^T(W_{p1}^T P W_{p1} + W_{p2}^T P W_{p2} - W_{p3}^T P W_{p3})\alpha_k + \alpha_k^T(W_{Q1}^T \bar{Q}_1 W_{Q1} + W_{Q2}^T \bar{Q}_2 W_{Q2})\alpha_k \\ &+\alpha_k^T(W_{Q3}^T \bar{Q}_3 W_{Q3})\alpha_k + \alpha_k^T(\tau E^T S_1 E + (d_M-\tau m)E^T S_2 E + \tau W_{p2}^T S_1 W_{p2} \\ &+(d_M-\tau m)W_{p2}^T S_2 W_{p2})\alpha_k + \alpha_k^T(W_{R1}^T R W_{R1} + W_{R2}^T R W_{R2})\alpha_k\}\end{aligned} \quad (20)$$

By Schur complement and Theorem 1, it is easy to see that $\mathrm{E}\{e^T(k)e(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)\} < 0$, which implies that $J(e,w) < 0$. Therefore inequality (11) holds for non-zero disturbance signal $w(k) \in l_2[0,\infty)$, at the mean while, it is obvious that $\Delta V(k) < 0$ holds when disturbance signal $w(k) = 0$. So the filtering error system (9) is asymptotically mean-square stable. The proof is completed.

Remark 1 In Theorem 1, the Lyapunov-Krasovskii functional is constructed based on the method of delay-partitioning. The convex optimization algorithm [16] is used to solve LMI conditions. Note that the delay-partitioning technique can reduce conservatism [13], at the cost of increasing the computation burden; therefore, the partitioning number m should be properly chosen.

Theorem 2 For the filtering error system (9) with TOD dynamic scheduling protocol $\delta(k) \in M$ and a H_∞ performance index $\gamma > 0$. If there exist matrices $P > 0, R > 0,$ $Q_1 > 0, Q_2 > 0, Q_3 > 0, S_1 > 0, S_2 > 0,$ and scalars $\bar{\mu} > 0, \varepsilon > 0$ such that the following linear matrix inequality holds,

$$\begin{bmatrix} \bar{\Omega}_1 & * & * & * & * & * & * & * & * & * & * & * & * \\ \bar{W}_{p1}^T & -P^{-1} & * & * & * & * & * & * & * & * & * & * & * \\ W_{p2}^T & 0 & \Omega_2^{-1} & * & * & * & * & * & * & * & * & * & * \\ W_{p3}^T & 0 & 0 & P^{-1} & * & * & * & * & * & * & * & * & * \\ W_{p4}^T & 0 & 0 & 0 & -P_4^{-1} & * & * & * & * & * & * & * & * \\ W_{p5}^T & 0 & 0 & 0 & 0 & P_5^{-1} & * & * & * & * & * & * & * \\ W_{Q1}^T & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_1^{-1} & * & * & * & * & * & * \\ W_{Q2}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_2^{-1} & * & * & * & * & * \\ W_{Q3}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_3^{-1} & * & * & * & * \\ W_{R1}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R^{-1} & * & * & * \\ W_{R1}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R^{-1} & * & * \\ \bar{M} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ \varepsilon\breve{N} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{21}$$

where

$$\bar{\Omega}_1 = \bar{\Xi}^T[\tau S_1 + (d_M - \tau m)S_2]\bar{\Xi}, \quad \bar{\Xi} = \bar{W}_{p1} - W_{p3},$$

$$\bar{W}_{p1} = \begin{bmatrix} \bar{A} & 0_{2n,2mn} & \bar{A}_{d0} & 0_{2n} & \bar{A}_{d1} & 0_n & \tilde{B}_0 \end{bmatrix},$$

$$\breve{N} = \begin{bmatrix} \bar{N} & 0_{n_N} & \bar{N}_{d0} & 0_{n_N,5n+q} \end{bmatrix}$$

n_N is the dimension of row in matrix N, then the filtering error system (9) is asymptotically mean-square stable with H_∞ filtering performance level γ.

Proof: According to Schur complement, it can be seen that (21) is equivalent to

$$\tilde{\Omega} + \varepsilon^{-1}\breve{M}\breve{M}^T + \varepsilon\breve{N}^T\breve{N} < 0 \tag{22}$$

where

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_1 & * & * & * & * & * & * & * & * & * & * \\ \bar{W}_{p1}^T & -P^{-1} & * & * & * & * & * & * & * & * & * \\ W_{p2}^T & 0 & \Omega_2^{-1} & * & * & * & * & * & * & * & * \\ W_{p3}^T & 0 & 0 & P^{-1} & * & * & * & * & * & * & * \\ W_{p4}^T & 0 & 0 & 0 & -P_4^{-1} & * & * & * & * & * & * \\ W_{p5}^T & 0 & 0 & 0 & 0 & P_5^{-1} & * & * & * & * & * \\ W_{Q1}^T & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_1^{-1} & * & * & * & * \\ W_{Q2}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_2^{-1} & * & * & * \\ W_{Q3}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{Q}_3^{-1} & * & * \\ W_{R1}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R^{-1} & * \\ W_{R2}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R^{-1} \end{bmatrix} < 0,$$

$$\breve{M} = \begin{bmatrix} \bar{M} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\breve{N} = \begin{bmatrix} \breve{N} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By Lemma 3, it is easy to see that

$$\bar{\Omega} + \breve{M}F\breve{N} + \breve{N}^T F \breve{M}^T < 0 \tag{23}$$

By (10), (23) is equivalent to (14), therefore the filtering error system (9) is asymptotically mean-square stability with H_∞ filtering performance level γ. The proof is completed.

Remark 2 Based on Theorem 2, the filtering error system gains will be obtained by solving LMI. An LMI-based sufficient condition is derived for filter designs that ensure the mean-square asymptotic stability of the filtering error dynamics and reduce the effect of disturbance input. The delay-partitioning approach can obviously reduce the conservatism compared with Literature [8-10]. At the mean while, the larger of partitioning number m, the more complex computing the LMI inequality, therefore, how to select a proper partitioning number m is an interesting research issue for further investigation.

4. Numerical Example

In this section, a numerical example is given to illustrate the effectiveness of the proposed reliable H_∞ filter for delay NCS with nonlinearities and stochastic noises.

Consider system **Error! Reference source not found.** and (1) with parameters as follows:

$$A=\begin{bmatrix}0.3 & 0\\ 0 & 0.6\end{bmatrix}, A_{d0}=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}, A_{d1}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, B=\begin{bmatrix}0.5\\ 0.5\end{bmatrix},$$

$$E=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, C=\begin{bmatrix}0.1 & 0.2\\ 0.1 & 1\end{bmatrix}, D=\begin{bmatrix}0.5 & 0.5\end{bmatrix}$$

$$L=\begin{bmatrix}0.1 & -0.1\end{bmatrix}, M=\begin{bmatrix}1 & 1\end{bmatrix}^T, N=N_{d0}=\begin{bmatrix}0.001 & 0.001\end{bmatrix}$$

And the time-varying delay $d(k)=2+\dfrac{1+(-1)^k}{2}$, it is easy to see that

$$d_m=2<d(k)<3=d_M$$

The controlled plant outputs $y_1(k), y_2(k)$ are sent to remote filter by two different sensor nodes. Then filter estimates signal $z(k)$ according to the received measured signal $z_f(k)$. Due to the communication constraints, assume that only $d_s=1$ sensor node can communicate with filter through the shared network at every transmission instant, that is, the NCS have two modes. We choose $\Pi_1(k)=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, \Pi_2(k)=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$ and the dynamic scheduling strategy $\delta(k)$ is updated by (7).

The nonlinear function $f(k)$ is chosen as follows:

$$f(k)=\frac{1}{2}\begin{bmatrix}\dfrac{0.3x_1(k)+x_2(k)}{x_1^2(k)+x_2^2(k)+1}+0.1x_1(k)+0.3x_2(k) & 0.3x_1(k)+0.3x_2(k)\end{bmatrix}^T$$

And constants $\mu_m=3^{-(3+m)},\ (m=1,2,...)$, the following condition satisfying (2) holds.

$$\bar{\mu}=\sum_{m=1}^{\infty}\mu_m=\frac{1}{54}<\sum_{m=1}^{\infty}m\mu_m=\frac{1}{36}<\infty$$

The disturbance input $w(k)=\sin(2\pi*10/k)+n(k), n(k)=0.36*rand(1,k)$, and the disturbance input $w(k)$ is shown in Figure 2 .

The controlled plant outputs $y_1(k), y_2(k)$ are sent to remote filter by two different sensor nodes. Then filter estimates signal $z(k)$ according to the received measured signal $z_f(k)$. Due to the communication constraints, assume that only $d_s=1$ sensor node can communicate with filter through the shared network at every transmission instant, that is, the NCS have two modes. We choose $\Pi_1(k)=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, \Pi_2(k)=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$ and the dynamic scheduling strategy $\delta(k)$ is updated by (7).

Choose $m=1$, the H_∞ performance level is taken as $\gamma=0.5$, therefore, solving (21) via using LMI toolbox, the filter gains of two modes are gained as follows:

$$A_{f1}=\begin{bmatrix}0.2538 & 0.0623\\ -0.1340 & 0.5327\end{bmatrix}, B_{f1}=\begin{bmatrix}-0.4257\\ -1.7341\end{bmatrix}, C_{f1}=\begin{bmatrix}-0.0751 & 0.0883\\ 0.0694 & 0.0035\end{bmatrix},$$
$$A_{f2}=\begin{bmatrix}-0.3584 & 0.0218\\ 0.5418 & 0.0125\end{bmatrix}, B_{f2}=\begin{bmatrix}-1.2734\\ -0.4667\end{bmatrix}, C_{f2}=\begin{bmatrix}0.0665 & 0.2156\\ 0.7965 & 0.0096\end{bmatrix}$$

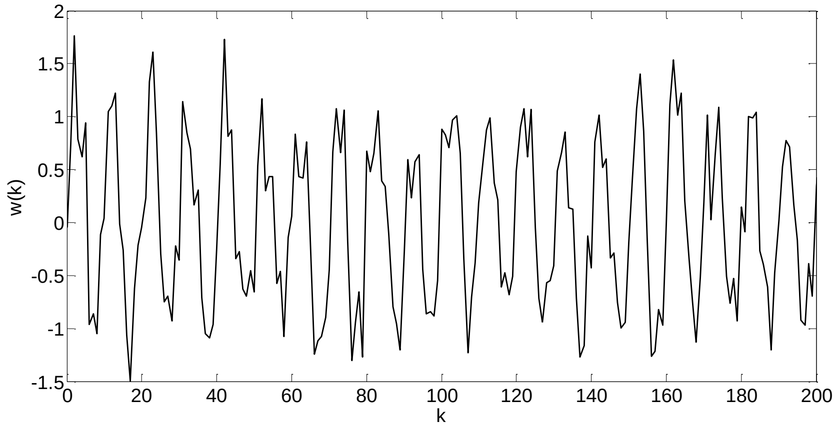


Figure 2. Disturbance Input $w(k)$

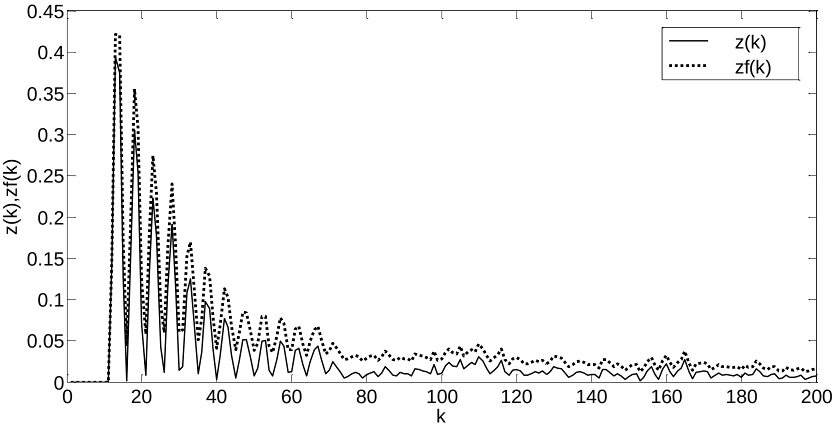


Figure 3. Output $z(k)$ **and Filter Output** $z_f(k)$

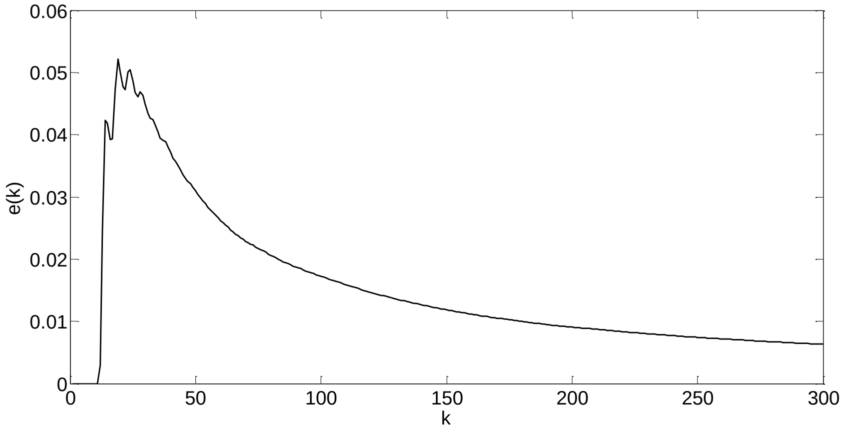


Figure 4. Estimated Error $e(k)$

And the calculated value $\gamma^* = \left[\mathrm{E}\{\|e(k)\|^2\} / \mathrm{E}\{\|w(k)\|^2\} \right]^{1/2} = 0.3958$. The simulation results are shown in Figure 3 and Figure 4, which verify that the filtering error system (9) is asymptotically mean-square stable and the expected system performance requirements, are well achieved.

5. Conclusion

In this paper, the co-design problem of H_∞ filter and scheduling strategy for a class of delay-NCS with stochastic nonlinearities and multiplicative noises has been investigated. A novel Lyapunov-Krasovskii functional and delay-partitioning technique have been applied to co-design H_∞ filter and scheduling strategy such that the filtering error system is asymptotically mean-square stable and achieves a prescribed H_∞ performance level. Then by solving a set of LMI inequalities, the available filter gains can be obtained. An illustrative example has been used to demonstrate the effectiveness of the proposed method.

It is obvious that the partitioning number m is relevant to solve LMI inequalities, how to choose m properly will be an important topic. Future work will involve the consideration of choosing m properly and large delay case (delays larger than the transmission interval). Moreover, we will apply the proposed method in industry, such as advanced aircraft, automobile, and manufacturing processes.

Acknowledgements

This work is partially supported by the National Natural Science Foundation of China (Grant 51407200) and the science and technology development plan project of Shandong province (Grant 2014GSF117035).

References

[1] F. Farokhi and K. Johansson, "Stochastic sensor scheduling for networked control systems", IEEE Transactions on Automatic Control, vol. 59, no. 5, **(2014)**, pp. 1147-1162.

[2] H. Wang, B. Zhou and C. C. Lim, "H_∞ fault-tolerant control of networked control systems with actuator failures", IET Control Theory & Applications, vol.8, no.12, **(2014)**, pp. 1127-1136.

[3] D. Antunes and J. P. Hespanha, "Stochastic networked control systems with dynamic protocols", Asian Journal of Control, vol. 17, no. 1, **(2015)**, pp. 99-110.

[4] D. Yue and E. Tian, "A delay system method for designing event-triggered controllers of networked control systems", IEEE Transactions on Automatic control, vol. 58, no. 2, **(2013)**, pp. 475-481.

[5] H. Li and Y. Shi, "Network-based predictive control for constrained nonlinear systems with two-channel packet dropouts", IEEE Transactions on Industrial Electronics, vol. 61, no. 3, **(2014)**, pp. 1574-1582.

[6] B. Xue, N. Li and S. Li, "Moving horizon scheduling for networked control systems with communication constraints", IEEE Transactions on Industrial Electronics, vol. 60, no. 8, **(2013)**, pp. 3318-3327.

[7] Y. Liu, Z. Wang and W. Wang, "Reliable H_∞ filtering for discrete time-delay systems with randomly occurred nonlinearities via delay-partitioning method", Signal Processing, vol. 91, no. 4, **(2011)**, pp. 713-727.

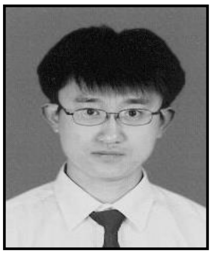
[8] R. A. Borges, R. C. L. F. Oliveira and C. T. Abdallah, "H_∞ filtering for discrete-time linear systems with bounded time-varying parameters", Signal Processing, vol. 90, no. 1, **(2010)**, pp. 282-291.

[9] E. Tian and D. Yue, "Reliable H_∞ filter design for T-S fuzzy model-based networked control systems with random sensor failure", International Journal of Robust and Nonlinear Control, vol. 23, no. 1, **(2013)**, pp. 15-32.

[10] E. Gershon, U. Shaked and I. Yaesh, "H_∞ control and filtering of discrete-time stochastic systems with multiplicative noise", Automatica, vol. 37, no. 3, **(2001)**, pp. 409-417.

[11] E. Gershon and U. Shaked, " H_∞ output-feedback control of discrete-time systems with state-multiplicative noise", Automatica, vol. 44, no. 2, **(2008)**, pp. 574-579.
[12] M. Liu, X. Liu and Y. Shi, "T-S fuzzy-model-based H_2 and H_∞ filtering for networked control systems with two-channel Markovian random delays", Digital Signal Processing, vol. 27, no. 1, **(2014)**, pp. 167-174.
[13] J. J. Hui, X. Y. Kong and H. X. Zhang, "Delay-partitioning approach for systems with interval time-varying delay and nonlinear perturbations", Journal of Computational and Applied Mathematics, vol. 281, no. 34, **(2015)**, pp. 74-81.
[14] W. P. M. H. Heemels, A. R. Teel and N. van de Wouw, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance", IEEE Transactions on Automatic Control, vol. 55, no. 8, **(2010)**, pp. 1781-1796.
[15] C. Zhou, M. Du and Q. Chen, "Co-design of dynamic scheduling and H-infinity control for networked control systems", Applied Mathematics and Computation, vol. 218, no. 21, **(2012)**, pp. 10767-10775.
[16] M. C. F. Donkers, W. P. M. H. Heemels and N. van de Wouw,"Stability analysis of networked control systems using a switched linear systems approach", IEEE Transaction on Automatic Control, vol. 56, no. 9, **(2011)**, pp. 2101–2115.
[17] Y. Liu, Z. Wang and J. Liang, "Synchronization and state estimation for discrete-time complex networks with distributed delays", IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol.38, no. 5, **(2008)**, pp. 1314-1325.
[18] B. R. Barmish, "Necessary and sufficient conditions for quadratic stabilizability of an uncertain system", Journal of Optimization Theory and Applications, vol. 46, no. 4, **(1985)**, pp. 399-408.

Authors



Yong Zhang, he was born in Shandong Province, China, in 1979. He received his B.S. degree and M.S degree in Qingdao University of science & technology in 2002 and 2005 respectively. In 2008, he received his PhD degree in Ocean University of China. Currently, he is a vice professor in College of Information and Control Engineering, China University of Petroleum. His research interests include networked control systems, delay systems and nonlinear systems.

Ning Wang, she was born in Shandong Province, China, in 1989. She received her B.S. degree in 2013. Now she is pursuing her M.S. degree from the China University of Petroleum. Her main research direction is networked control systems and switched systems.

Jingjing Hu, he was born in Shandong Province, China, in 1992. He received his B.S. degree in 2015. Now he is pursuing his M.S. degree in China University of Petroleum. His main research direction is switched systems.

