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Robust PID Control Design via Mixed Particle Swarm Optimization Algorithm and Gap Metric

Xinghong Qiao[1,2], Fei Luo[1], Yuge Xu[1]

[1]*College of Automation Science and Engineering, South China University of Technology, Guangzhou 510641, China*
[2] *Guangzhou Institute of Energy Testing, Guangzhou 511447, China*
qiaoxinghong@126.com

Abstract

This paper proposes a novel tuning approach for robust proportional-integral-derivative (PID) controller based on H_∞ loop shaping synthesis in combination with gap metric and Particle Swarm Optimization (PSO) algorithm. Different from the traditional research, the controller is designed through the search region constrained by H_∞ loop shaping synthesis and gap metric theorem. PSO algorithm is used for tuning the robust PID controller parameters based on the underlying constrained optimization problems without resolving complex arithmetical calculations. The control technique is applied for the robust controller design so as to get a low order structured controller and achieve robust performance and the ability of restraining disturbance. The simulation shows that the proposed method can character the set of all values of the controller parameters that guarantees the robust stability with any supposed accuracy and achieves favorable control performance for uncertain systems.

Keywords*: gap metric, robust controller, PSO algorithm, model uncertainty, loop shaping*

1. Introduction

A feedback control system consists of two basic parts: the controlled object plant and controller. In the analysis and synthesis of the feedback uncertain systems, many control techniques have been developed. The classical approach of the robust controllers for nonlinear uncertain systems designed by using the H_2, H_∞, or μ formulations can produce extremely fragile controllers [1]. The robust controller is designed for structured uncertain systems by singular value synthesis [2], but this kind of control design is computationally intractable [3]. The H_∞ loop shaping design procedure (LSDP) is an effective method to design robust controller [4] and is being used in a wide range of applications in industry. The gap metric was introduced into control field to study of robustness properties of uncertainty systems by Zames and El-Sakkary [5, 6]. The graph metric and similar metric for normalized coprime-factorized models was introduced by Vidyasagar [7]. The v-gap metric was proposed by Vinnicombe [8]. Georgiou and Smith proved that the problem of robustness optimization in the gap metric was equivalent to robustness optimization for perturbations in normalized coprime factors [9]. Qiu and Davison proposed pointwise gap metrics for uncertain systems to study stability robustness [10]. Although these metrics are different numerical, they are the same topology. The gap metric has been widely used to design the robust controller in the presence of uncertainty plant [11-14].

However, the resulting controllers designed by the above approaches are computationally intractable and the orders of H_∞ loop shaping controllers are high. The stochastic optimization for tuning of PID controller method is a worthwhile direction to

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overcome the difficulties. The paper is concerned with the development of robust PID-controllers based on H∞ loop shaping synthesis in combination with gap metric. In our approach, the H∞ loop shaping and v-gap metric which is maximal gap metric between the nominal system model and any other models are used to define a family of robustly stable controllers. However, the problem arises the computation of optimal solutions. This constrained optimization problems can be solved by using Particle Swarm Optimization (PSO). PSO algorithm has demonstrated the ability to deal with non-convex optimization problems [16, 17].

The main goal of this paper is to further develop robust control synthesis approaches for uncertain systems. The controller designed based on conventional mathematical and analytical methods is usually computationally intractable. In order to simplify mathematical calculations, the v-gap theorem and H∞ specifications in combination with PSO algorithm are used to solve the underlying constrained optimization problems. The rest of this paper is structured as follows. In Section 2, the proposed PID controller in state-space structure and the conventional H∞ loop shaping are illustrated. The gap measurement and robust stability analysis are constructed in Section 3. In Section 4, PSO algorithm is briefly introduced and used to solve the optimization problem constrained by the v-gap theorem and H∞ specifications. The simulation study is carried out by Matlab/Simulink to verify the comparatively strong robust and the ability of restraining disturbance in Section 5. Finally, the concluding remarks are drawn in Section 6.

2. Problem Setting and Derivation of Constraint

2.1. PID Controller Structure

The proportional-integral-derivative (PID) controller is defined as follows [18]:

$$K_{PID}(s) = K_P + K_I\frac{1}{s} + K_D\frac{s}{1+\tau s} = \left[\begin{array}{cc|c} 0 & 0 & K_I \\ 0 & -\frac{1}{\tau}I & -\frac{1}{\tau^2}K_D \\ \hline I & I & K_P + \frac{1}{\tau}K_D \end{array}\right] = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \tag{1}$$

where K_P , K_I , K_D are the proportional, integral and derivative matrix gains, respectively, and τ is the time constant. PID controller can be used a simple first-order low-pass filter to ensure the properness and physical realizability of PID controller. The matrices A_K , B_K , and C_K must satisfy the structures described in (1); the matrix D_K must satisfy the well-posedness condition.

2.2. H_{inf} Loop Shaping

The robust control design method based a combination of loop shaping and robust stabilization was proposed in McFarlane and Glover [19, 20]. In this framework, the open-loop scaled plant G(s) is shaped with the pre-compensator $W_1(s)$ and post-compensator $W_2(s)$ as shown in Figure 1. The $W_1(s)$ and $W_2(s)$ are chosen so that the weighted plant is sufficiently at all frequencies, typically a large gain at low frequencies for good robust stability and a small gain at high frequencies for disturbance attenuation.

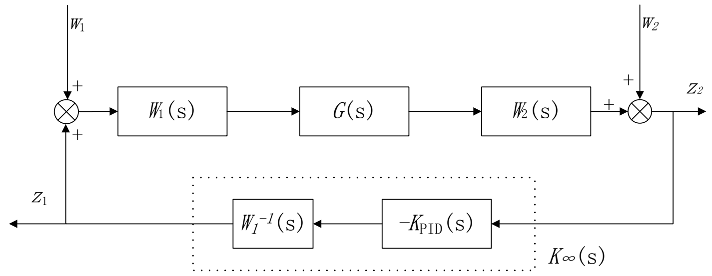

Figure 1. Standard Block Diagram of H_{inf} Loop Shaping Control Method

The transfer functions from disturbances w_1 and w_2 to the outputs z_1 and z_2 can be written as follows:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T_{zw}\left(K_\infty\right)\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{2}$$

where $T_{zw} = \begin{bmatrix} I \\ -W_1^{-1}K_{PID} \end{bmatrix}\left(I + W_2 G K_{PID}\right)\left[W_2 G W_1 \quad I\right]$. The controlled system with the proposed PID controller should achieve perfect dynamic performance and strong robustness. The design problem can be expressed for solving a minimization problem as follows:

$$\min_{stab \;\; K_{PID}} \left\|T_{zw}\right\|_\infty = \min_{stab \;\; K_{PID}} \left\| \begin{bmatrix} I \\ -W_1^{-1}K_{PID} \end{bmatrix}\left(I + W_2 G K_{PID}\right)\left[W_2 G W_1 \quad I\right] \right\|_\infty \tag{3}$$

The above numerical constrained optimization is equivalent to maximise stability margins for stabilising PID controllers. The non-convex minimization problems can not be solved easily. However, a solution can be search based on evolutionary algorithm if the optimisation problem is structured in the state-space matrix inequalities. The state-space realization for the transfer matrix from w_1 and w_2 to z_1 and z_2 can be obtained as [21].

$$T_{\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \to \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}} = \left[\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl} & D_{cl} \end{array}\right] = \left[\begin{array}{cccc|cc} A + BD_K C & BC & 0 & BC_K & BD & BD \\ 0 & 0 & 0 & 0 & B & 0 \\ \tilde{B}_1 D_K C & 0 & \tilde{A}_1 & \tilde{B}_1 C_K & 0 & BD \\ BC & 0 & 0 & A & 0 & B \\ \hline C & 0 & 0 & 0 & 0 & I \\ \tilde{D}_1 D_K C & 0 & \tilde{C}_1 & \tilde{D}_1 C_K & 0 & \tilde{D}_1 D_K \end{array}\right] \tag{4}$$

where $W_1 = \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right], W_1^{-1} = \left[\begin{array}{c|c} \tilde{A}_1 & \tilde{B}_1 \\ \hline \tilde{C}_1 & \tilde{D}_1 \end{array}\right], W_2 G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$.

3. Gap Measurement and Robust Stability Analysis

3.1. Gap Measurement and Geometry Meanings

The controller for uncertain systems should be satisfied not only adequate level of performances but also robust metrics. The ν-gap can offer a sensible measure of uncertain system. The spherical distance between c_1 and c_2, denoted by $\theta(c_1, c_2)$, is defined as the shortest length of an arc on Riemann sphere connecting $\varphi(c_1)$ and $\varphi(c_2)$, and the center of the Riemann sphere as shown in Figure 2.

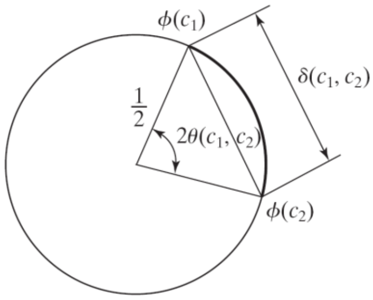

Figure 2. The Relationship Between $\delta(c_1,c_2)$ and $\theta(c_1,c_2)$

$$\delta(c_1,c_2)=\left\|\varphi(c_1)-\varphi(c_2)\right\|=\frac{|c_1-c_2|}{\sqrt{1+c_1^2}\sqrt{1+c_2^2}} \tag{5}$$

$$\theta(c_1,c_2)=\arcsin\delta(c_1,c_2)=\arcsin\frac{|c_1-c_2|}{\sqrt{1+c_1^2}\sqrt{1+c_2^2}} \tag{6}$$

The transfer functions $G_i(s)$, $i=1,2$, of any single-input–single-output (SISO) systems can be given as the following:

$$G_i(s)=\frac{b_i(s)}{a_i(s)} \tag{7}$$

where $a_i(s), b_i(s), i=1,2$, are polynomials. The chordal distance between two systems $G_1(s)$ and $G_2(s)$ is presented as

$$\delta(G_1,G_2)=\frac{|a_2b_1-a_1b_2|}{\sqrt{|a_1|^2+|b_1|^2}\sqrt{|a_2|^2+|b_2|^2}} \tag{8}$$

The normalized right coprime factorizations of multiple-input-multiple-output (MIMO) systems $G_1(s)$ and $G_2(s)$ are (D_1, N_1) and (D_2, N_2), respectively. Then the gap between $G_1(s)$ and $G_2(s)$ can be calculated by the following formula [23]:

$$\delta(G_1,G_2)=\max\left\{\delta(G_1,G_2),\delta(G_2,G_1)\right\} \tag{9}$$

where $\delta(G_1,G_2)=\inf\limits_{Q_1\in M(H_\infty)}\left\|\begin{bmatrix}D_1\\N_1\end{bmatrix}-\begin{bmatrix}D_2\\N_2\end{bmatrix}Q_1\right\|$, $\delta(G_2,G_1)=\inf\limits_{Q_1\in M(H_\infty)}\left\|\begin{bmatrix}D_2\\N_2\end{bmatrix}-\begin{bmatrix}D_1\\N_1\end{bmatrix}Q_2\right\|$,

$M(H_\infty)$ denotes the set of all matrices with elements in H_∞.

All models denoted by $\mathcal{B}_\delta(G, r)$ with the graph uncertainty centered at nominal plant G with radius r are defined by

$$\mathcal{B}_\delta(G, r) = \{G_1 : \delta(G_1,G) \leq r\} \tag{10}$$

where $\delta(.\,,.)$ is the spherical distance, G is nominal system, r is uncertain radius. Similarly, θ(. , .) is the spherical distance. A set of uncertainty models $\mathcal{B}_\theta(G, r)$ is defined by

$$\mathcal{B}_\theta(G, r) = \{G_1 : \theta(G_1,G) \leq \arcsin r\} \tag{11}$$

3.2. Robust Stability Based on Gap Measurement

Consider the standard feedback configuration depicted in Figure 3. The standard feedback is stable if and only if the following equation is correct.

$$T_{ew}=\begin{bmatrix}(1-CP)^{-1} & C(1-PC)^{-1}\\ P(1-CP)^{-1} & (1-PC)^{-1}\end{bmatrix}\in RH_\infty \tag{12}$$

where T_{we} is the transfer function from $w=\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ to $z=\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$.

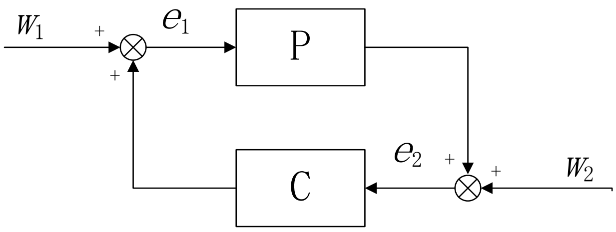

Figure 3. Standard Feedback Configuration

If the feedback system is stable for load disturbances, it is necessary to minimize the effects of T_{we}, which means that the cost minimization function in (13) is solved.

$$\rho \overset{\Delta}{=} \left\| \begin{bmatrix} I \\ P \end{bmatrix} (I-CP)^{-1} \begin{bmatrix} I & -C \end{bmatrix} \right\|_{\infty} \tag{13}$$

The objective is to find a stabilizing controller ensuring the best possible disturbance rejection. The corresponding optimal H_∞ controller is the solution for the following optimization problem:

$$\text{minimize} \quad \gamma \tag{14}$$

subject to $\|T_{ew}\|_\infty \le \gamma$

Alternately, it can be defined as

$$b_{P,C} \overset{\Delta}{=} \frac{1}{\rho} = \left\| \begin{bmatrix} I \\ P \end{bmatrix} (I-CP)^{-1} \begin{bmatrix} I & -C \end{bmatrix} \right\|_{\infty}^{-1} \tag{15}$$

$b_{P,C}$ is called the robust stability radius, which was introduced by Tryphon T. Georgiou and Malcolm C. Smith in terms of the controller and the plant [9]. It is regarded as the generalized stability margin.

Lemma1[21] The uncertain gap measure $\delta(P_1,P_2)$ and the stability margin $b_{P,C}$ can be related by the following inequality:

$$\arcsin\left(b_{P_2,C}\right) \ge \arcsin\left(b_{P_1,C}\right) - \arcsin\delta\left(P_1,P_2\right)$$

Theorem1 The controller C stabilizes any given system $\tilde{P}$ if and only if $\delta\left(P,\tilde{P}\right) < 1/\gamma_{\min}$,where $\gamma_{\min}$ is the minimal H_∞ norm of the closed-loop transfer matrix T_{ew}, P is nominal system.

Proof：According to the Lemma1, the following inequality can be get:

$$b_{P_2,C} \ge b_{P_1,C} - \delta\left(P_1,P_2\right) \tag{16}$$

Since the closed-loop system $[P_2,C]$ is stable if $b_{P_2,C} > 0$, the inequality $b_{P_1,C} > \delta\left(P_1,P_2\right)$ exists. On the other, the controller C can stabilize P_1 and also stabilize P_2 .The P_1 and P_2 are any given systems.

4. Control Designer Based on PSO Algorithm

4.1. Introduction to PSO [24]

Particle swarm optimization(PSO) algorithm, which stems from the simulation of birds flock's looking for food, is a swarm intelligence-based evolutionary computing. Consider the following optimization problem:

$$\mathop{Min}_{x\in\chi} \quad f(x) \tag{17}$$

The typical PSO algorithm includes the following steps, which is repeated either a certain number of times or until a particular stopping condition.

1. Initialization. PSO is initialized with a population of random particles by assigning random position and velocities inside the problem space. It combines local search and global search activities, possessing high search efficiency.

2. Swarm evolution. Each particle has fitness values which is evaluated by the fitness function optimized, and has velocity which direct the flying of the particles.

3. After finding the two best values, the particle updates best fitness and positions with following equation

$$\hat{x}_{best,i}(t+1) = \hat{x}_{best,i}(t) \text{ , if } f\left[x_i(t+1)\right] > \hat{x}_{best,i}(t) \tag{18}$$

$$\hat{x}_{best,i}(t+1) = x_i(t+1) \text{ ,if } f\left[x_i(t+1)\right] \le \hat{x}_{best,i}(t) \tag{19}$$

$$g_{best}(t) = \min\left\{\hat{x}_{best,i}(t)\right\} \tag{20}$$

where $i \in [1,2,\cdots,n]$ and $n > 1$.

4. The velocities and positions of all particles are updated. Fitness evaluation is performed by calculating objective function. The best fitness and positions are updated, which are responsible for the optimization ability of PSO algorithm. The positions and velocities of all the particles are updated through the following equations:

$$v_i(t+1) = \omega v_i(t) + c_1 r_1\left[\hat{x}_i(t) - x_i(t)\right] + c_2 r_2\left[g(t) - x_i(t)\right] \tag{21}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{22}$$

where i, $v_i(t)$ and x_i represent the index, velocity and position of the particle, respectively.

4.2. Largest *ν*-gap Metric Based on PSO Algorithm

PID controller based on PSO algorithm structure diagram is shown in Figure 4. The controller is mainly for the following three parts to carry on the design.

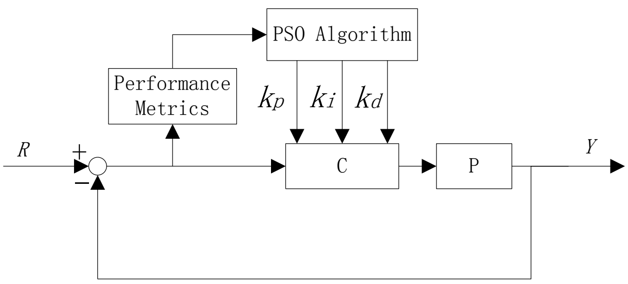

Figure 4. Structure Diagram of PID Control System Based on PSO

(1) The largest gap metric δ_{wc} between the nominal system and any given system $\tilde{P}$ is calculated by mixing particle swarm optimization algorithm and Matlab command “gapmetric”.

(2) According to the experience of the project, the designers explore and design the search space as $D = \left\{x \in R^3 : \underline{x}_i \le x_i \le \bar{x}_i \quad i = 1,2,3\right\}$.

(3) Fixed-order H$_\infty$ synthesis with PSO the problem can be solved as follows. At iteration *k*, after moving the swarm according to (20-24), do for each particle:

a. Build PID controller by $\left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$,

b. Build $\left[\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl} & D_{cl} \end{array}\right]$ according to (1)

c. Evaluate $\lambda_{Acl} = \mathrm{Re}\left(\lambda_i\left(A_{ck}\right)\right)$

If $\lambda_{Acl} \geq 0$, $f\left(x_p^k\right) = \inf$,

else $f\left(x_p^k\right) = \gamma$

d. Solve $\gamma_{\min}$ by PSO algorithm

e. if $\delta_{wc} \geq 1/\gamma_{\min}$ the controller C don't stabilize any given system $\tilde{P}$

else the controller stabilize any given system $\tilde{P}$

5. A Numerical Example

Consider the uncertain system shown as Figure 5 [25].

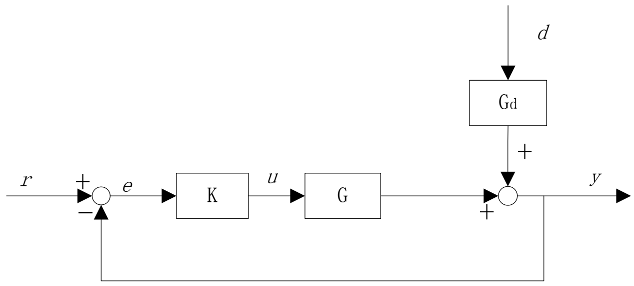

Figure 5. Block Diagram of a Feedback Control System

$$G(s) = \frac{k}{(as+1)(bs+1)^2} \qquad G_d(s) = \frac{100}{10s+1} \tag{23}$$

Assume that the coefficients k, a and b of the denominator lie within the following bounds, where $180 \leq k \leq 220$, $9 \leq a \leq 11$, $0.045 \leq b \leq 0.055$ .The loop shaping weighting functions W_1 and W_2 for the nominal systems are chosen as follows:

$$W_1 = \frac{0.75s + 4.001}{s + 0.001} \qquad W_2 = 1 \tag{24}$$

PID controller is given as follows:

$$K(s) = x_1 + \frac{x_2}{s} + \frac{x_3 s}{0.01s+1} \tag{25}$$

where $x := (x_1, x_2, x_3) = (k_p, k_i, k_d)$ and the search space is set as D := {x∈R^3 : (-3,-3,-3,)^T < x < (8,6,5)^T }. The above algorithm is applied for the system (23) to find the parameters: $k_p = 0.6$, $k_i = 1.3682$, $k_d = 0.016$, $b_{P,C} = 3.097$ $\gamma_{\min} = 0.295$ .The controller is obtained as following:

$$K_1(s) = 0.6 + \frac{1.3682}{s} + \frac{0.016s}{0.01s+1} \tag{26}$$

The corresponding robustness margin is $b_{P,C} = 1/3.097 = 0.323 > \delta_v\left(P, \tilde{P}\right) = 0.295$ .Therefore the controller is guaranteed to be stable against parametric uncertainties. The step responses of nominal system and various parameters system are shown in Figure 6 and Figure 7, respectively. As shown in Figure 7 shows, any given system can be controlled sufficiently well with the synthesis method.

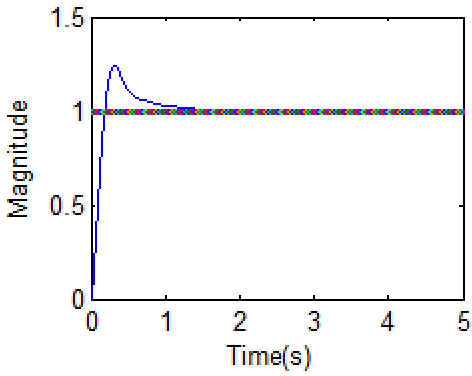

Figure 6. Step Response of Nominal System

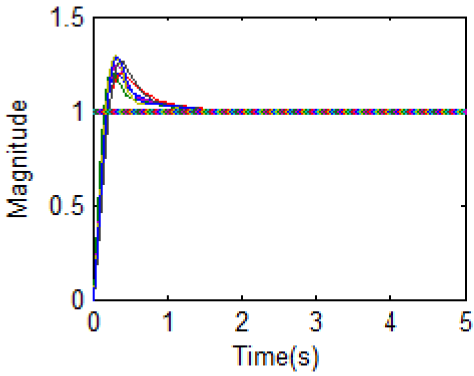

Figure 7. Step Response of Uncertain System

The full order controller is designed by the standard H_∞ loop shaping design method [25, 26] as following:

$$K_2(s) = \frac{123.8s^4 + 6080s^3 + 9.644\times10^4 s^2 + 5.212\times10^5 s + 8.584\times10^5}{s^5 + 127.8s^4 + 5853s^3 + 1.344\times10^5 s^2 + 5.668\times10^5 s + 566.7} \tag{27}$$

The unit step and loop shaping in the disturbance response for the uncontrolled case, full order controller and synthesis PID algorithm are shown in Figure 8 and Figure 9, respectively. The solid line, the dash-dot line and dash line present the uncontrolled case, full order controller and synthesis PID algorithm. Table 1 shows the results in detail. The gain margin (GM) is improved from 3.481 (for the full order case) to 19.700 (for synthesis PID algorithm), and the phase margin (PM) is reduced slightly from 51.492^O to 50.920^O. The gain crossover frequency is reduced 10.257 to 9.271(rad/s) .The results show that the synthesis PID algorithm and full order controller are robust to the uncertain-but-bounded system and the influent disturbances. But the synthesis PID algorithm controller is lower order and computationally tractable. Furthermore according to Table 1, the time-domain synthesis PID algorithm is quite competitive in comparison with the full order case.

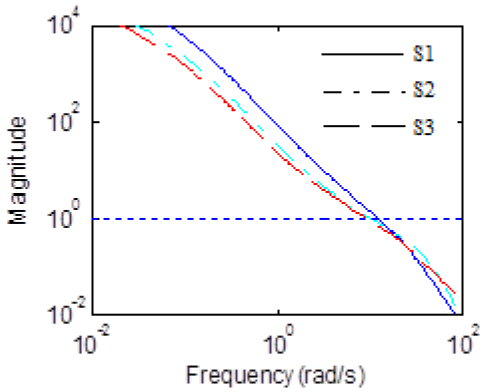

Figure 8. Loop Shapes

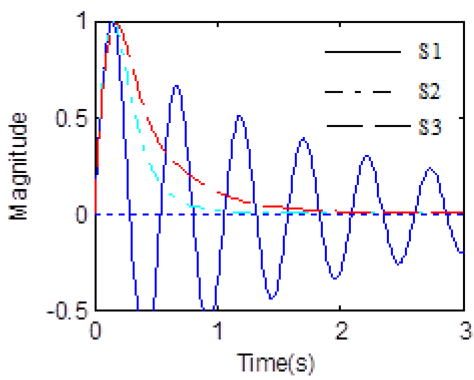

Figure 9. The Unit Step in the Disturbance Response

Table 1. Comparison of the Performance of Different Controllers

performance / method	γ_{min}	gain margin	phase margin	crossover frequency
the uncontrolled case	---	1.618	13.179^{O}	13.733 rad/s
the full order case	2.593	3.481	51.492^{O}	10.257 rad/s
synthesis PID algorithm	3.097	19.700	50.920^{O}	9.271 rad/s

6. Conclusions

In this paper, a synthesizing PID robust controller based on H_∞ loop shaping in combination with ν-gap metric and PSO algorithm is developed. Generally, the robust controller design problem using conventional mathematical and analytical methods is known to be computationally intractable and the order of the controller is high. PSO algorithm is used for tuning the robust PID controller parameters without resolving complex arithmetical calculations by minimizing a fitness function. The control algorithm is applied for the robust controller design so as to achieve robust performance. Simulate and experiment show that the control technology has comparatively strong robust and the ability of restraining disturbance. For instance, the proposed controller is more stable to the uncontrolled case and the order of the controller is lower to the full order case. PSO algorithm is quite generic and can be replaced by any other evolutionary algorithm. The control technique may provide a novel method for the robust PID design.

Acknowledgements

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Authors

Xinghong QIAO. He was born in 1979. He received his M.S. degree in University of Chinese Academy of Sciences in 2008. He is a Ph.D. candidate in South China University of Technology. His research interests include robust optimization with applications in process systems engineering.
Email: qiaoxinghong@126.com

Fei LUO, He received his M.S and Ph.D. degrees in 1986 and 1993 from University of Science & Technology Beijing, China. He is currently a professor with College of Automation Science and engineering, South China University of Technology, China. His research interests include His current research interests include nonlinear dynamical systems and control, nonlinear observer design, robust control, and applications.

Yuge XU. She received her M.S and Ph.D. degrees in 2003 and 2006 from South China University of Technology, China. She is currently a associate professor with College of Automation Science and engineering, South China University of Technology, China. Her research interests include robust control, controller design under risk specifications, robust and chance constrained optimization.

