

MTPA Trajectory Tracking Control for Interior PMSM Based on Adaptive Parameter Identification

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Abstract

For an interior permanent magnet synchronous motor (IPMSM) control system, the actual maximum torque per ampere (MTPA) trajectory may deviate from the ideal one with parameter variances, and thus the system may not obtain the maximum torque output. In view of this problem, a novel parameter identification method based on the model reference adaptive system (MRAS) is proposed in this paper. In this method, the deviations of MTPA trajectories with parameter variances are analyzed and evaluated firstly, and then the q -axis inductance and the rotor flux linkage are identified on-line in a full rank MRAS model according to Popov Hyper Stability Theorem. Finally, the feasibility and validity of the designed method are proved through simulation results.

Keywords: interior permanent magnet synchronous machines, maximum torque per ampere control, model reference adaptive system, parameter identification

1. Introduction

A permanent magnet synchronous motor (PMSM) has many advantages such as its compact structure, high power density, low torque ripple and wide speed range. To make rational use of the reluctance torque, produced by an interior PMSM (IPMSM) with unequal d - q axis inductances, can obtain excellent torque-current and flux-weakening characteristics. Thus, the IPMSMs are widely used in many fields and especially the first choice in electrical vehicles (EV) [1].

For the IPMSM drive and control system in EV, its maximum torque output capacity is limited by the inverter's current rating in the constant-torque operation area under the base speed, where to adopt the maximum torque per ampere (MTPA) control method can maximize the torque output capability, or minimize its stator current and copper loss, and thus enhance the total power performances of EV [2, 3].

However, the electrical parameters of an IPMSM such as the d - q axis inductances and the rotor flux linkage may vary with armature reaction, magnetic saturation, temperature rise *etc.* under its actual operating speed and torque. And due to the parameter variances, the real MTPA trajectory may deviate from the ideal one, so the traditional design methods with invariable parameters can only obtain unchangeable and average characteristics in the overall speed-torque operation area [4, 5]. Thus, the parameter identification method is introduced to improve the MTPA trajectory tracking effect in the recent research work [6-8].

The practical and effective parameter identification methods now include recursive least square (RLS) method, model reference adaptive system (MRAS) method and extended Kalman filter (EKF) method. All the previous data are necessary to be reserved during recursive calculation in the RLS method, which results in an inevitable problem of data saturation [9, 10]. Although the EKF method is well used to identify the rotor position and speed in the sensorless control system, there are some practical difficulties to apply this method to the electrical parameter identification of an IPMSM. For

example, the four order identification model for the flux linkage may bring in a large amount of on-line calculation, and the identified errors may be much greater because of the smaller inputs or larger noises with lower speed or no-load [11].

In the MRAS method, the identified values of parameters are adjusted with the designed adaptive laws in order to make the output errors between the reference model and the adjustable model convergent to zero, and consequently the identified parameters in the adjustable model will asymptotically approach the real ones in the reference model. And the appropriate adaptive laws can be established through flexible design according to Popov Hyper Stability Theorem. However, in an IPMSM MTPA control system, there are 3 identified parameters related to the MTPA trajectory, while the rank of the stator voltage equation in the d-q reference frame is 2, so the deficient model of rank 2 can not ensure the identified accuracy and convergence of the 3 parameters. And the present research on the MRAS method mainly focuses on 1 or 2 parameters of surface-mounted PMSM [12-15]. In view of problems above, a novel and practical MRAS parameter identification method is investigated for the application of the IPMSM MTPA control system in this paper.

2. Mathematical Model of IPMSM MTPA Control System

The stator voltages of an IPMSM in the d-q synchronous rotating reference frame can be expressed as

$$\begin{cases} u_d = R_s i_d + L_d \frac{di_d}{dt} - L_q \omega_e i_q \\ u_q = R_s i_q + L_q \frac{di_q}{dt} + L_d \omega_e i_d + \omega_e \psi_f \end{cases} \quad (1)$$

where u_d and u_q are the d-q axis stator voltages, i_d and i_q are the d-q axis stator currents, L_d and L_q are the d-q axis inductances, R_s is the stator phase resistance, ψ_f is the rotor permanent magnet flux linkage, and ω_e is the rotor electrical angular speed. The corresponding electromagnetic torque is

$$T_e = 1.5p(\psi_f i_q + (L_d - L_q)i_d i_q) \quad (2)$$

Assuming that the stator current is i_s and the angle between i_s and q-axis is β , the d-q axis stator currents can be expressed as

$$\begin{cases} i_d = -i_s \sin \beta \\ i_q = i_s \cos \beta \end{cases} \quad (3)$$

In order to obtain the maximum ratio of the torque T_e to the current i_s , the angle between i_s and q-axis β must be configured as follows [4]

$$\sin \beta = \frac{\psi_f - \sqrt{\psi_f^2 + 8(L_d - L_q)^2 i_s^2}}{4(L_d - L_q)i_s} \quad (4)$$

It is evident that the desired values of the d-q axis currents i_d and i_q should be calculated on-line according to Equation (3) and Equation (4) so as to track the MTPA trajectory real-timely.

3. MTPA Trajectory with Parameter Variances

The IPMSM electrical parameters relative to the MTPA control, such as the d-axis inductance L_d , the q-axis inductance L_q and the rotor flux linkage ψ_f , may change with armature reaction, magnetic saturation, temperature rise etc in the actual operation of the motor.

Figure 1 shows the variation tendencies of the parameters L_d and L_q with the stator currents. In the linear magnetic area, the d-axis inductance L_d gets greater slowly with the increase of the d-axis current i_d , while the q-axis inductance L_q is nearly a constant. And in the saturated magnetic area, the d-axis inductance L_d is nearly a constant, while the q-axis inductance L_q gets smaller quickly with the increase of the q-axis current i_q . Furthermore, the d-axis inductance varies slowly and slightly (within 5-10% in general) in the whole operation area.

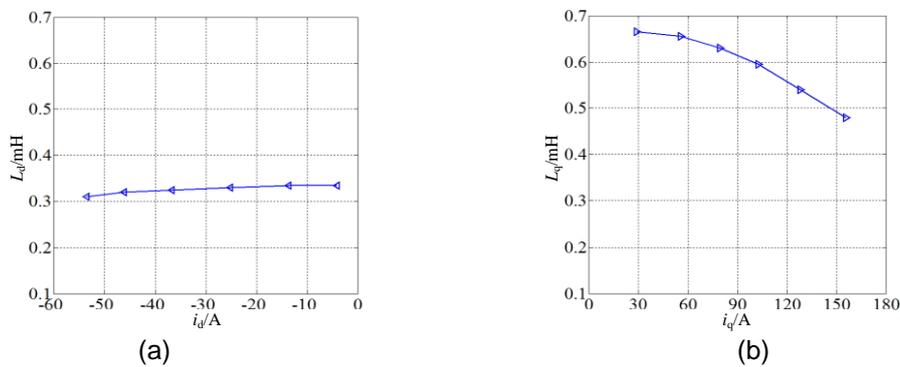


Figure 1. Variation Tendencies of Parameters. (A) L_d . (B) L_q

The actual MTPA trajectory may deviate from the ideal one due to parameter variances, as Figure 2 shows, where each parameter varies $\pm 10\%$ of its average value. Obviously seen from Figure 2, the variances of L_q and Ψ_f result in significant deviations of the MTPA trajectories and changes of the static MTPA operating points, while the variances of L_d lead to a negligible trajectory deviations and operating-point changes.

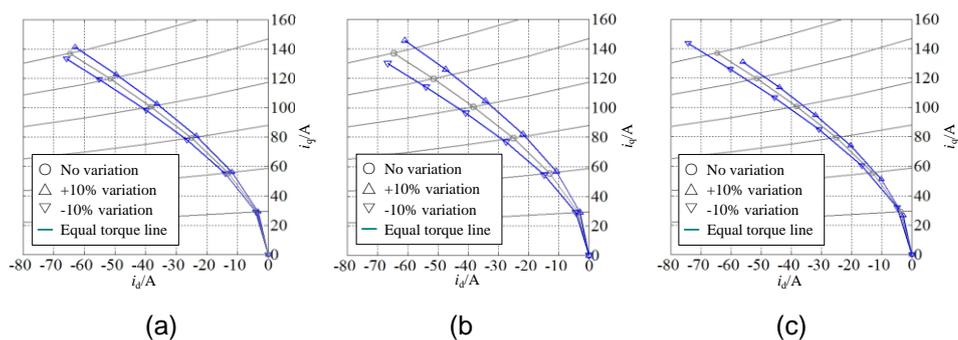


Figure 2. Deviations of MTPA Trajectories with Parameter Variances. (A) L_d . (B) L_q . (C) Ψ_f

To weigh between the MTPA control effect and the rank deficiency problem of the MRAS, a novel parameter identification method is proposed for the IPMSM MPTA control system, where the q-axis inductance L_q and the rotor flux linkage ψ_f are identified on-line with the d-axis inductance L_d considered to be a constant.

4. Construction of Model Reference Adaptive System

4.1. IPMSM Reference Model

The state space equation of the d-q axis current control system can be described from Equation (1) as

$$\begin{bmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \end{bmatrix} = \begin{bmatrix} -R_s a & \frac{a}{b} \omega_e \\ -\frac{b}{a} \omega_e & -R_s b \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega_e c \end{bmatrix} \quad (5)$$

where $a = \frac{1}{L_d}$, $b = \frac{1}{L_q}$, and $c = \frac{\psi_f}{L_q}$.

The reference model of an IPMSM system can be rewritten from Equation (5) as

$$p\mathbf{i} = \mathbf{A}\mathbf{i} + \mathbf{B}\mathbf{u} + \mathbf{E} \quad (6)$$

where $\mathbf{i} = [i_d \quad i_q]^T$, $\mathbf{u} = [u_d \quad u_q]^T$, $\mathbf{A} = \begin{bmatrix} -R_s a & \frac{a}{b} \omega_e \\ -\frac{b}{a} \omega_e & -R_s b \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, and $\mathbf{E} = \begin{bmatrix} 0 \\ -\omega_e c \end{bmatrix}$.

4.2. Adjustable Model

Considering the rotor speed ω_e is constant for parameter identification, the adjustable model can be constructed from the reference model Equation (6) as

$$p\hat{\mathbf{i}} = \hat{\mathbf{A}}\hat{\mathbf{i}} + \hat{\mathbf{B}}\mathbf{u} + \hat{\mathbf{E}} + \mathbf{G}(\mathbf{i} - \hat{\mathbf{i}}) \quad (7)$$

where $\hat{\mathbf{i}} = [\hat{i}_d \quad \hat{i}_q]^T$, $\hat{\mathbf{A}} = \begin{bmatrix} -R_s a & \frac{a}{\hat{b}} \omega_e \\ -\frac{\hat{b}}{a} \omega_e & -R_s \hat{b} \end{bmatrix}$, $\hat{\mathbf{B}} = \begin{bmatrix} a & 0 \\ 0 & \hat{b} \end{bmatrix}$, $\hat{\mathbf{E}} = \begin{bmatrix} 0 \\ -\omega_e \hat{c} \end{bmatrix}$, and $\mathbf{G} = \begin{bmatrix} -g_{11} & g_{21} \\ -g_{22} & -g_{12} \end{bmatrix}$

Every pole of the adjustable model Equation (7) should have a negative real component with an appropriate selection of the gain matrix \mathbf{G} , in order to ensure the observer asymptotic stability and the satisfactory convergence speed.

Subtracting the adjustable model Equation (7) from the reference model Equation (6) yields the error equation of MRAS

$$\dot{\mathbf{e}} = (\mathbf{A} + \mathbf{G})\mathbf{e} + \Delta\mathbf{A}\hat{\mathbf{i}} + \Delta\mathbf{B}\mathbf{u} + \Delta\mathbf{E} \quad (8)$$

where \mathbf{e} is the estimated error vector, $\mathbf{e} = \mathbf{i} - \hat{\mathbf{i}}$.

5. Design of Adaptive Parameter Identification Method

The MRAS method is designed based on Popov Hyper Stability Theorem in this paper. Firstly, the model reference adaptive system as in Equation (8) is equalized to a nonlinear time-varying feedback system, which consists of a linear constant forward path $\mathbf{G}(s)$ and a nonlinear feedback loop $\varphi(\mathbf{v})$. Then, an adaptive law of the parameter L_q or ψ_f is designed separately under the condition that the nonlinear feedback loop $\varphi(\mathbf{v})$ meets

Popov integral inequality. Finally, an appropriate gain matrix G is chosen to ensure the transfer function matrix of the linear forward path $G(s)$ to be a severe positive real matrix [16].

5.1. Equivalent Nonlinear Time-Varying Feedback System

Make $w = -(\Delta A \hat{i} + \Delta B u + \Delta E)$, and rewrite the error Equation (8) as

$$\dot{e} = (A + G)e - w \quad (9)$$

The equivalent nonlinear time-varying feedback system can be obtained from the above Equation (9), as shown in Figure 3, where $\varphi(v)$ represents the adaptive laws of the parameter matrices A , B and E .

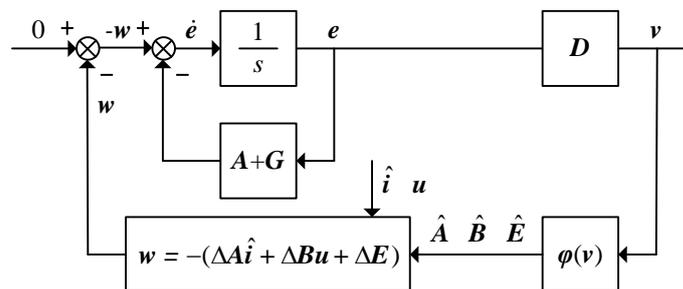


Figure 3. Structure Diagram of Equivalent Nonlinear Time-Varying Feedback System

5.2. Design of Adaptive Laws

The Popov integral inequality is

$$\eta(0, t_1) = \int_0^{t_1} e^T w dt \geq -\gamma_0^2 \quad (10)$$

where γ_0 is a finite positive constant independent of t for $t \geq 0$.

Take the q-axis inductance L_q and the rotor flux linkage ψ_f as the identified objects, and substitute w into the Popov integral inequality Equation (10),

$$\begin{aligned} \eta(0, t_1) = & \int_0^{t_1} (\hat{b} - b) \left[(u_q - R_s \hat{i}_q - \frac{1}{a} \omega_e \hat{i}_d)(i_q - \hat{i}_q) - \frac{a}{\hat{b}b} \omega_e \hat{i}_q (i_d - \hat{i}_d) \right] dt \\ & + \int_0^{t_1} -(\hat{c} - c) \omega_e (i_q - \hat{i}_q) dt \geq -\gamma_0^2 \end{aligned} \quad (11)$$

Equation (11) can be divided into two sub inequalities as

$$\begin{cases} \eta_1(0, t_1) = \int_0^{t_1} (\hat{b} - b) \left[(u_q - R_s \hat{i}_q - \frac{1}{a} \omega_e \hat{i}_d)(i_q - \hat{i}_q) - \frac{a}{\hat{b}b} \omega_e \hat{i}_q (i_d - \hat{i}_d) \right] dt \geq -\gamma_1^2 \\ \eta_2(0, t_1) = \int_0^{t_1} -(\hat{c} - c) \omega_e (i_q - \hat{i}_q) dt \geq -\gamma_2^2 \end{cases} \quad (12)$$

Generally, the PI adaptive law with respect to the parameter b can be expressed as

$$\hat{b} = \hat{b}(0) + \int_0^t G_1(\tau) d\tau + G_2(t) \quad (13)$$

where $\hat{b}(0)$ is an initial estimated value of the parameter b .

Substitute Equation (13) into Equation (12), $\eta_1(0, t_1)$ can be also divided into two sub inequalities as

$$\begin{cases} \eta_{11}(0, t_1) = \int_0^{t_1} [(u_q - R_s \hat{i}_q - \frac{1}{a} \omega_e \hat{i}_d)(i_q - \hat{i}_q) - \frac{a}{\hat{b}b} \omega_e \hat{i}_q (i_d - \hat{i}_d)] \int_0^t G_1(\tau) d\tau dt \geq -\gamma_{11}^2 \\ \eta_{12}(0, t_1) = \int_0^{t_1} [(u_q - R_s \hat{i}_q - \frac{1}{a} \omega_e \hat{i}_d)(i_q - \hat{i}_q) - \frac{a}{\hat{b}b} \omega_e \hat{i}_q (i_d - \hat{i}_d)] G_2(t) dt \geq -\gamma_{12}^2 \end{cases} \quad (14)$$

Thus, the PI adaptive law with respect to the parameter b can be obtained from Equation (13) and Equation (14)

$$\hat{b} = \hat{b}(0) + (\frac{k_{r1}}{s} + k_{r2})(u_q - R_s \hat{i}_q)(i_q - \hat{i}_q) \quad (15)$$

where k_{r1} and k_{r2} are both positive constants.

Similarly, the PI adaptive law with respect to the parameter c can be derived

$$\hat{c} = \hat{c}(0) - (\frac{k_{h1}}{s} + k_{h2})\omega_e (i_q - \hat{i}_q) \quad (16)$$

where $\hat{c}(0)$ is an initial estimated value of the parameter c , and k_{h1} and k_{h2} are both positive constants.

5.3. Design of Matrix Gain

As shown in the equivalent nonlinear time-varying feedback system Figure3, the state-space equation of the linear forward path $G(s)$ is

$$\begin{cases} \dot{e} = (A + G)e + B(-w) \\ v = Ce \end{cases} \quad (17)$$

where either B or C is a unit matrix I .

The corresponding transfer function matrix is

$$H(s) = [sI - (A + G)]^{-1} \quad (18)$$

According to Positive Real Lemma [16], for a controllable and observable system like Equation (17), if the positive definite and real symmetric matrices P and Q exist, and

$$\begin{cases} P(A + G) + (A + G)^T P = -Q \\ B^T P = C \end{cases} \quad (19)$$

then $H(s)$ is a severe positive real transfer function matrix.

It is evident that the unit matrix P is obviously a positive definite and real symmetric matrix. And for a positive definite and real symmetric matrix Q , substituting A and G into Equation (19) yields

$$\begin{cases} \frac{R_s}{L_d} + g_{11} = \frac{R_s}{L_q} + g_{12} > 0 \\ \frac{L_q}{L_d} \omega_e + g_{21} = \frac{L_d}{L_q} \omega_e + g_{22} \end{cases} \quad (20)$$

So $H(s)$ is a severe positive real transfer function matrix if the selected elements of the gain matrix G are confined within Equation (20).

Above all, the designed MRAS parameter identification system is shown in Figure4.

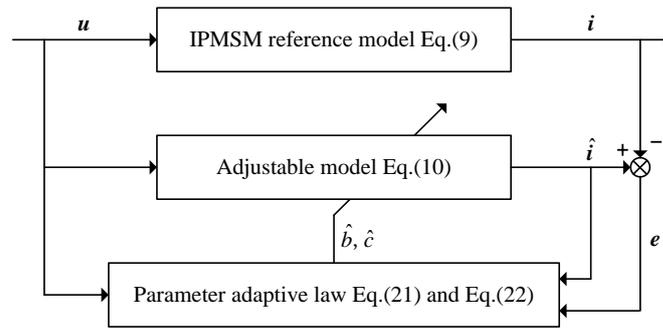


Figure 4. Structure Diagram of MRAS Parameter Identification System

6. Simulation Results and Analysis

A simulation model of the IPMSM drive and control system is built and examined in MATLAB, as illustrated in Figure 5.

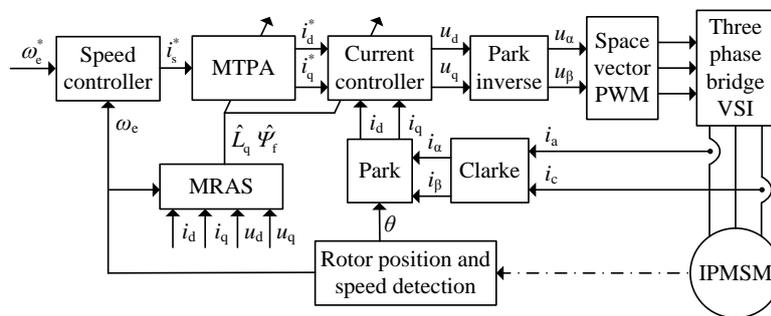


Figure 5. Structure Diagram of IPMSM MTPA Control System with Parameter Identification

The key parameters of the prototype IPMSM are shown as follows: the output power is 30kW, the rated speed is 4500r/min, the rated torque is 72N·m, the average values of L_d and L_q are 0.33mH and 0.63mH, and the average value of the flux linkage ψ_f is 0.068Wb. The PI parameters of the designed adaptive laws are set as: k_{f1} and k_{f2} for the inductance are 0.02 and 48.0 separately, and k_{h1} and k_{h2} for the flux linkage are 0.05 and 10.0 separately.

Figure 6, Figure 7 and Figure 8 show the tracking responses of the parameters, with 10% decrease of the average value of L_q , 10% increase of the average value of ψ_f and both of the above variances, respectively. The identified values respond quickly to the given ones, and the estimated errors of the d-q axis currents e_d and e_q have smaller overshoots and converge to zero rapidly. Thus, the designed parameter identification method has good tracking characteristics.

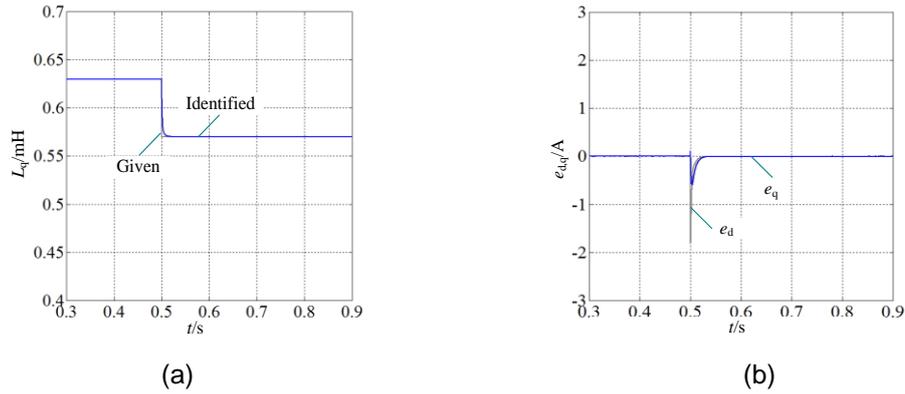


Figure 6. Tracking Responses of Identified Parameter L_q . (A) L_q . (B) Current Estimated Errors

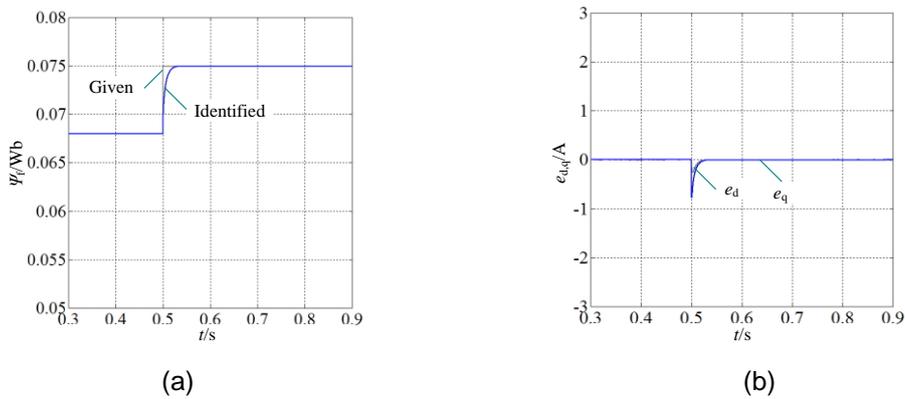


Figure 7. Tracking Responses of Identified Parameter ψ_f . (A) ψ_f . (B) Current Estimated Errors

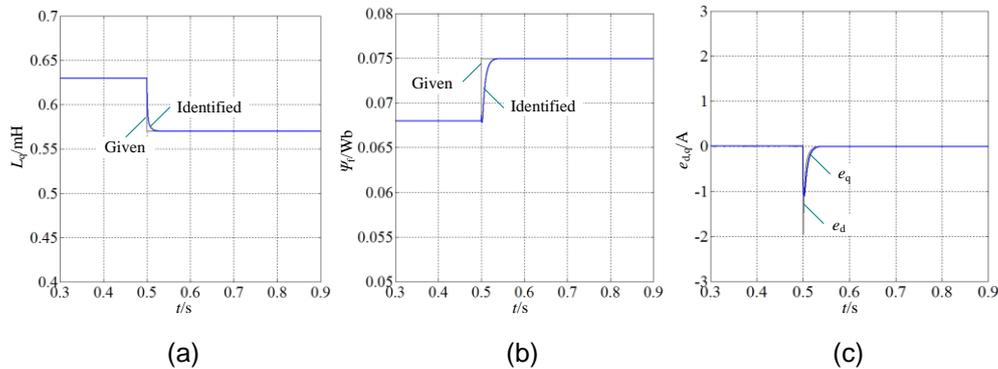


Figure 8. Tracking Responses of Identified Parameters L_q and ψ_f . (A) L_q . (B) ψ_f . (C) Current Estimated Errors

Figure 9, Figure 10 and Figure 11 show comparisons of the tracking responses of the d-q axis currents between the ideal MTPA trajectory, the one with invariable parameters, and the one with identified parameters. The parameter L_q decreases to 10% of its average value in Figure 9, the parameter ψ_f increases to 10% of its average value in Figure 10, and both of the above parameters changes in Figure 11. Under the conditions of parameter variances mentioned above, the designed method with invariable parameters can only obtain an operating point and a MTPA trajectory which deviates far away from the ideal one. And with the MRAS method, the d-q axis currents approach the ideal

operating point quickly and hence the actual MTPA trajectory is almost coincident to the ideal one. Thus, the designed parameter identification method enhances the MTPA tracking characteristics with parameter variances.

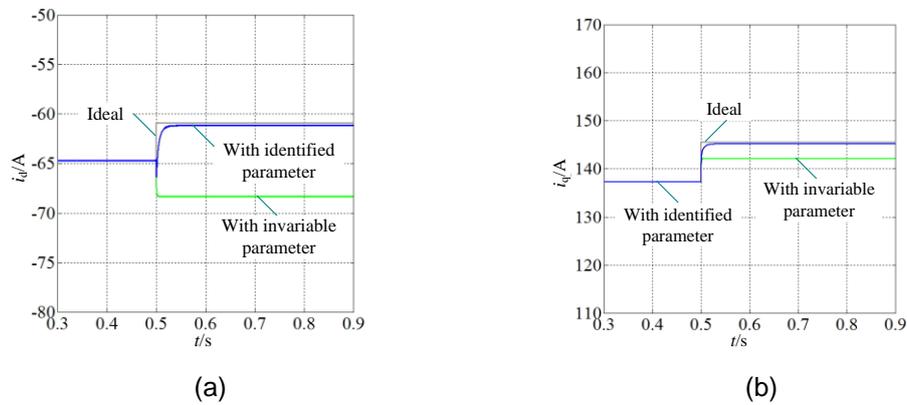


Figure 9. Comparisons of D-Q Axis Currents with L_q Variance. (A) I_d . (B) I_q

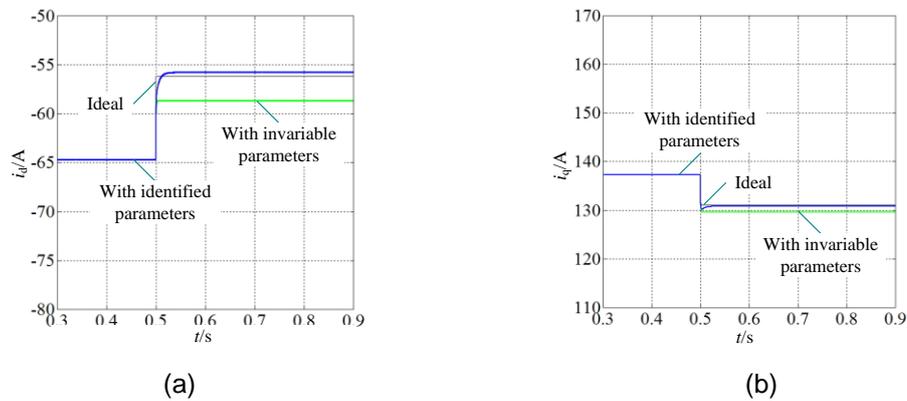


Figure 10. Comparisons of D-Q Axis Currents with Ψ_f Variance. (A) I_d . (B) I_q

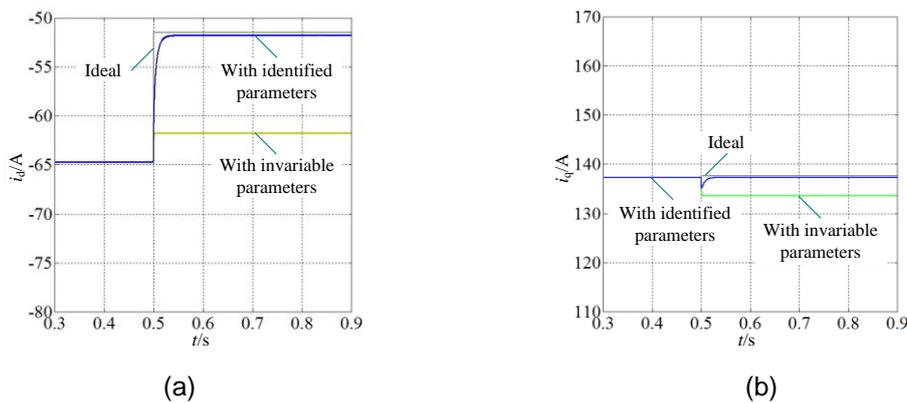


Figure 11. Comparisons of D-Q Axis Currents with Both L_q and Ψ_f Variances. (A) I_d . (B) I_q

7. Conclusion

To solve the problem of the MTPA trajectory deviations with parameter variances, a practical and effective parameter identification method based on MRAS is introduced to improve the tracking effect of the MTPA trajectory. To weigh between the deviations of MTPA trajectories with parameter variances and

the rank deficiency problem of the MRAS model, an improved adaptive identification method with the q-axis inductance and the rotor flux linkage estimator is analyzed and designed. Simulation results show that the designed method improves the MTPA tracking responses with parameter variances.

Acknowledgments

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