International Journal of Control and Automation
Vol. 9, No. 4 (2016), pp. 149-160
http://dx.doi.org/10.14257/ijca.2016.9.4.15

Robust Control of Vehicle Active Suspension System

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Abstract

In order to improve the road handling and passenger comfort of a vehicle, suspension is provided. An active suspension is considered to be better than a passive suspension. In this paper, 2 degree of freedom model of quarter car active suspension system is designed, which is subjected to road disturbance. Due to the fact that strong nonlinearity inherently exist in the spring ,damper and actuator components, therefore nonlinear effects must be taken into account in designing the controller for practical suspension systems. Since parametric uncertainty in the spring, damper and actuator has been considered, therefore robust control is used. H_∞*and* μ*- synthesis controllers are used to improve the ride comfort and road handling ability of the car as well as to check the robust performance of the system. The results shows that both controllers give good performance, but* μ *synthesis controller has superior robust performance as compared to* H_∞ *controller as well as settling time of body acceleration and suspension deflection is also minimum with* μ *synthesis controller.*

Keywords*: Quarter Vehicle Active Suspension System (QVASS), Linear Fractional Transformation (LFT),* H_∞ *Controller,* μ*- Synthesis Controller*

1. Introduction

The purpose of a car is to adequately support the physical body, to maintain tire contact with the ground and to manage the compromise between passenger comfort and vehicle road handling. This is important for the safety of the ride. Suspension consists of system of springs, shock absorbers and linkages that connects a vehicle to its wheels. Generally, there are three types of suspension systems, namely, passive, semi active and active suspensions. Passive suspension has the ability to store energy via a spring and dissipate it via a damper. Passive suspensions can only achieve good ride comfort or good road handling since these two criteria conflict each other and involve different spring and damper characteristics. Semi-active suspensions with their variable damping characteristics and low power consumption, offers a considerable improvement. A significant improvement can be achieved by using a active suspension system. The active suspension system able to inject energy into the vehicle dynamic system via actuator. The force actuator is able to add and dissipate energy from the system. This force may be function of several variables, which can be measured or sensed by sensors, so the flexibility can be greatly improved.

In recent past, a study of active suspension model and various types of controllers used had been reported [1].A comparison between passive and active suspension systems had been reported [2]. The aim is to achieve small amplitude value for suspension travel, wheel deflection and car body acceleration, LQR control is found to be better. It is noted that system parametric variation, which is caused by environmental changes or worn and

ISSN: 2005-4297 IJCA

torn factors, is a class of systems with unstructured uncertainty. The existing active suspension system is inefficient if there are changes in parameter of the system or of actuator, then controlling the suspension system becomes a big problem. Therefore H_∞ and μ synthesis control techniques are used. The analysis and synthesis of control systems using H_∞ methods had been reported [3]. H_∞ control effectively suppresses the vehicle vibrations in the sensitive frequency range of the human body[4-6].The desired robust performance is achieved in the closed loop system for a full vehicle model[7] and for a quarter vehicle model in the presence of structured uncertainties[8-10].

In the previous research, no comparison has been reported in the performance of H_∞ and μ synthesis controllers in the presence of both structured and unstructured uncertainties. In this paper, a comparison has been made regarding robust performance in the presence of road disturbance and controller performances in terms of vehicle body deflection, body acceleration and suspension deflection.

2. Mathematical Modeling

A. Dynamic Modelling

Figure 1 shows the quarter vehicle model for active suspension system. The sprung mass m_b represents the mass of the vehicle body, frame and internal components that are supported by the suspension. The unsprung mass m_w is mass of the assembly of the axle and wheel. k_S and b_S are respectively the spring and damper coefficients of the passive components. Tyre compressibility is K_t. The control force generated by the actuator is f_S..Where r denotes the road disturbance input acting on the unsprung mass.

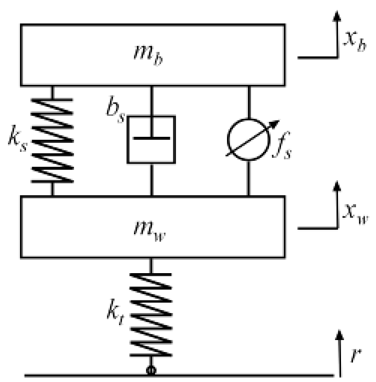

Figure 1. Quarter Vehicle Model of Active Suspension System

The vertical displacements of the sprung and unsprung masses are denoted as x_b and x_w respectively. Parameters of quarter active suspension system are shown in the Table 1. To develop the state space model of the system, the state variable are defined as

$$x_1 = x_b, x_2 = x_w, x_3 = \dot{x}_b, x_4 = \dot{x}_w$$

Equation of motion of the system for sprung and unsprung masses are as follow

$$m_b \ddot{x}_b = k_s (x_w - x_b) + b_s (\dot{x}_w - \dot{x}_b) + f_s \quad (1)$$

$$m_w \ddot{x}_w = k_t (r - x_w) - k_s (x_w - x_b) - b_s (\dot{x}_w - \dot{x}_b) - f_s \quad (2)$$

Table 1. Parameters of Quarter vehicle Model

Model parameters	symbol	Values
Vehicle body mass	m_b	300kg
Wheel assembly mass	m_w	60kg
Suspension stiffness	k_s	1600N/m
Suspension damping	b_s	1000N-s/m
Tyre stiffness	k_t	190000N/m

Dynamics of the system is described by the following state space model

$$\dot{X} = AX + BU + EW \tag{3}$$

$$A = \begin{Bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_b} & \frac{k_s}{m_b} & -\frac{b_s}{m_b} & \frac{b_s}{m_b} \\ \frac{k_s}{m_w} & -\frac{(k_s+k_t)}{m_w} & \frac{b_s}{m_w} & \frac{-b_s}{m_w} \end{Bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & \frac{1}{m_b} & -\frac{1}{m_w} \end{bmatrix}$$

$$E^T = \begin{bmatrix} 0 & 0 & 0 & \frac{k_t}{m_w} \end{bmatrix}$$

B. LFT Modelling

Considering the parametric uncertainty in spring and damper elements of active suspension system. The uncertainty in the spring and damper are represented as

$$KS = \bar{k}_S (1 + \delta KPK) \tag{4}$$

$$bS = \bar{b}_S (1 + \delta bPb) \tag{5}$$

Where $\bar{k}_S$ and $\bar{b}_S$ are the nominal values of the corresponding spring constant and damping coefficient respectively.P_K = 0.2 and P_b= 0.1 are the maximum relative uncertainty in each of them. Where

$-1 \leq \delta_K, \delta_b \leq 1$

The system block diagram with uncertain parameters is shown in the Figure 2.

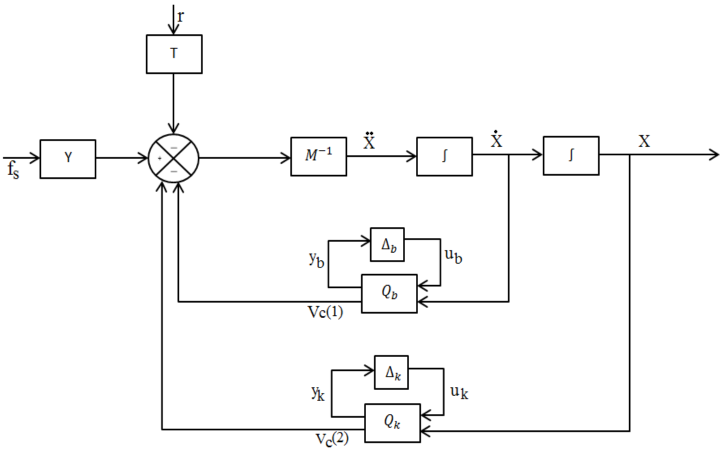

Figure 2. System Block Diagram with Uncertain Parameter

The unstructured uncertainty in the actuator is described in the form of input multiplicative perturbation as shown in the Figure 3.

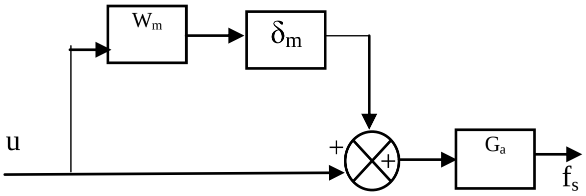

Figure 3. Uncertainty in the Actuator Model

The state and output equations are as follows.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \multicolumn{2}{c}{\left[-M^{-1}\bar{K}\right]_{2x2}} & \multicolumn{2}{c}{\left[-M^{-1}\bar{K}\right]_{2x2}}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \left[-M^{-1}B_{11}\right] & \left[-M^{-1}K_1\right] & \left[-M^{-1}T\right] & \left[-M^{-1}F\right] & & \end{bmatrix}\begin{bmatrix}u_b\\ u_k\\ r\\ f_s\end{bmatrix} \quad (6)$$

$$\begin{bmatrix}y_b\\ y_k\\ y_1\\ y_2\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & -1 & 0 & 0\\ \multicolumn{2}{c}{\left[-M^{-1}\bar{K}\right]} & \multicolumn{2}{c}{\left[-M^{-1}\bar{B}\right]}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \left[-M^{-1}B_{11}\right] & \left[-M^{-1}K_1\right] & \left[-M^{-1}T\right] & & \left[-M^{-1}F\right] & \end{bmatrix}$$

$$\begin{bmatrix}u_b\\ u_k\\ r\\ f_s\end{bmatrix} \quad (7)$$

The uncertain model of the whole system can be described by an upper LFT representation as shown in the Figure.4

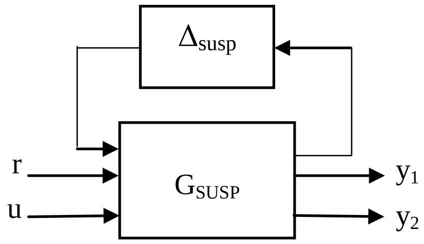

Figure 4. LFT Model of Active Suspension System

Thus the open loop active suspension system is eight input and eight output system.

$$\begin{bmatrix} y_b \\ y_k \\ y_a \\ y_1 \\ y_2 \end{bmatrix} = G_{susp} \begin{bmatrix} u_b \\ u_k \\ u_a \\ r \\ u \end{bmatrix}$$

Where G_{susp} and Δ_{susp} are as defined

$$G_{susp} = \begin{bmatrix} [A]_{4x4} & [B_1]_{4x4} & [B_2]_{4x2} \\ [C_1]_{4x4} & [D_{11}]_{4x1} & [D_{12}]_{4x2} \\ [C_2]_{4x4} & [D_{21}]_{4x4} & [D_{22}]_{4x2} \end{bmatrix}$$

$$\Delta_{susp=} \begin{bmatrix} \Delta_b & 0 & 0 \\ 0 & \Delta_k & 0 \\ 0 & 0 & \delta_m \end{bmatrix}$$

C. Open Loop Interconnected System

The structure of interconnected system is as shown in the Figure5.The feedback controller uses outputs y_1 and y_2 of suspension travel and body acceleration to compute the control (u) driving the actuator. There are three external sources of disturbance. The road disturbance r modelled as a normalised signal $d_1$shaped by a weighting function w_d.Sensor noise on both measurement, modelled as normalised signals d_2 and d_3 shaped by weighting function w_{d2} and $w_{d3.}$ Performance weights for comfort and road handling are w_{ab} and w_{sd}. The control objective can be interpreted to minimize the impact of disturbance inputs d_1, d_2, and d_3 on a weighted combination of control effort (u), suspension travel(s_d) and body acceleration (a_b)

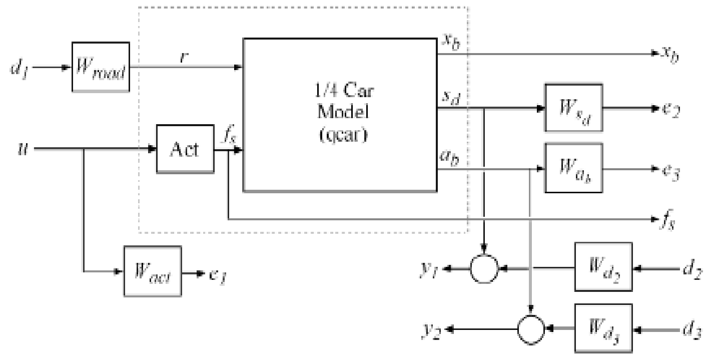

Figure 5. Structure of the Interconnected System

3. Robust Controller Design

A. H$_\infty$ Controller

H$_\infty$ optimization approach is an effective and efficient robust design method for linear, time invariant control systems. The robust design is to find a controller k for a given system such that, the closed loop system is robustly stable. For good tracking and disturbance attenuation, the design problem is to find a optimal controller which minimizes $\|(I+GK)^{-1}\|_\infty$ and for less control energy, $\|K(I+GK)^{-1}\|_\infty$ is to be minimized.

In order to have good tracking and disturbance rejection and to limit the control energy, we have to solve the mixed sensitivity problem. Its cost function can be described as

$$\left\| \begin{matrix} (I+GK)^{-1} \\ K(I+GK)^{-1} \end{matrix} \right\|_\infty \qquad (8)$$

The above cost function may be recast into a standard H$_\infty$ configuration shown in the Figure 6.

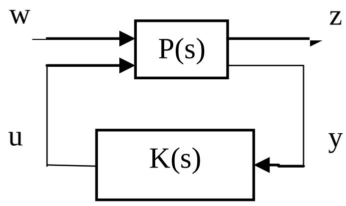

Figure 6. The Standard H$_\infty$ Configuration

P(s) is called the generalized plant/interconnected system.
w and z denotes the output signals to be minimized.
Y is the vector of measurements available to the controller. u is vector of control signals.
The objective is to find a stabilizing controller k to minimize the output z, in the sense of energy. Thus it is equivalent to minimize the H$_\infty$ norm of the transfer function from w to z.

The design objective now becomes min $\|F(P,K)\|_\infty$, it is referred to as the H$_\infty$ optimization problem.

B. μ Synthesis Controller

In standard M-Δ configuration as shown in the Figure 7
z = F_U (M,Δ)w and

$$||F_U(M,\Delta)||_\infty < 1 \tag{9}$$

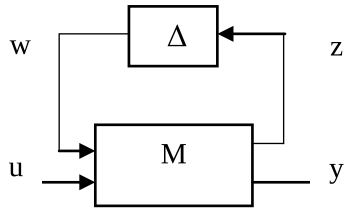

Figure 7. The Standard M-Δ Configuration

It denotes the stability of $F_u(M,\Delta)$ which means the stability with respect to the plant perturbation Δ. The relation between M and p can be obtained by

$$M(p,k) = F_L(p,k) \tag{10}$$

For robust stability and robust performance, it is required to find a stabilizing controller k such that

$$\sup \mu\ [M(p,k)] < 1 \tag{11}$$

For optimal robust stability and robust performance, the objective is to solve for k such that

$$\text{Inf}\ \sup \mu[M(p,k)] \tag{12}$$

An iterative method is used to solve (12). The method is called D-K iteration synthesis method. It is based on solving the following optimization problem (13) for a stabilizing controller k and a diagonal constant scaling matrix D.

$$\text{Inf}\ \sup\ \inf \bar{\sigma}[DMD^{-1}\ (jw)] \tag{13}$$

4. Simulation and Results

To investigate the suspension performance, a perfect road surface model is necessary to design the active suspension. In this study, the sine function is used to simulate the road disturbance. The road input is described by equation (14) and is as shown in the Figure 8.

$$r(t) = \begin{cases} a\{1 - \cos(8 * pi * t)\} & 0 \le t \le 0.25 \\ 0 & otherwise \end{cases} \tag{14}$$

Where a = 0.025

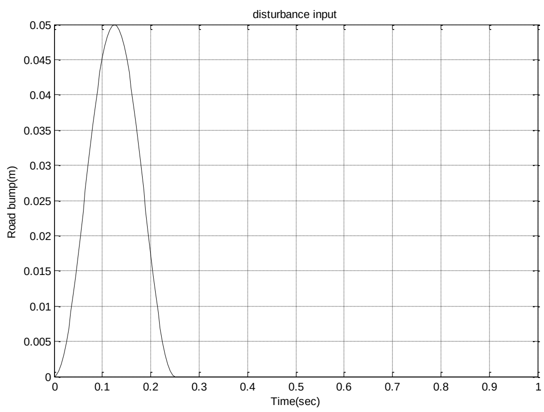

Figure 8. Road Disturbance

The responses of active suspension system without any controller, with road input are as shown in the Figure 9 (a), 9(b) and 9(c)

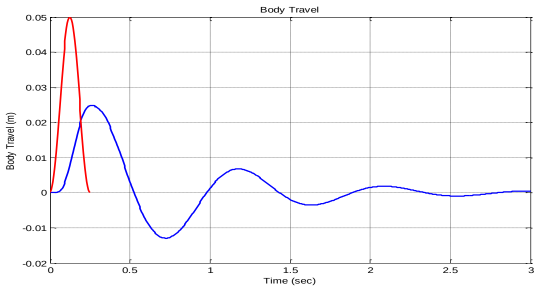

Figure 9. (a). Car Body Travel

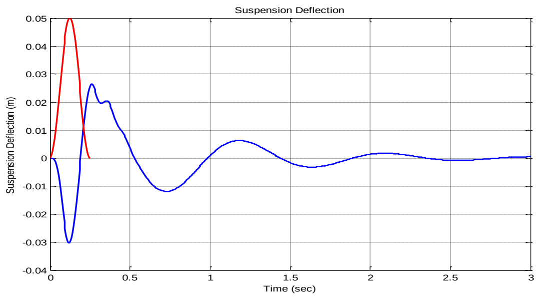

Figure 9. (b). Suspension Deflection

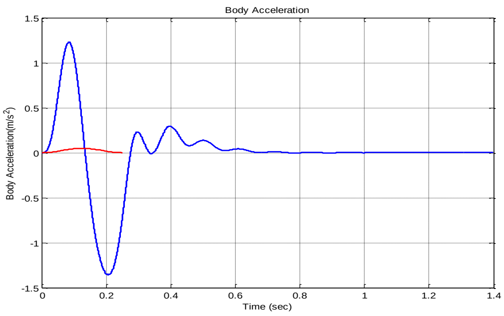

Figure 9(c). Car Body Acceleration

A. H$_\infty$ Control

Figure 10 (a), 10(b) and 10(c) shows the car body displacement, body acceleration and suspension deflection of the system with H$_\infty$ controller. The settling time for body acceleration and suspension deflection are 0.7sec and 1.0sec respectively. These are very small as compared to the system without controller. Thus the disturbance rejection response of the system has been improved with H$_\infty$ controller.

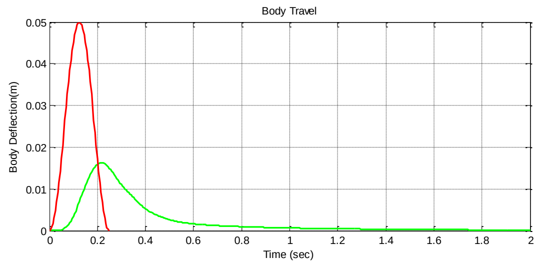

Figure 10. (a). Body Deflection

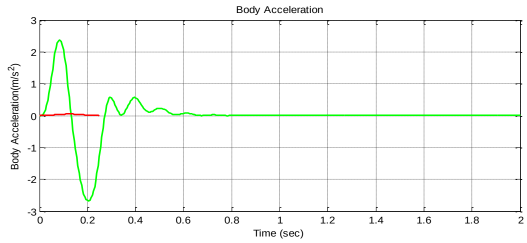

Figure 10. (b). Body Acceleration

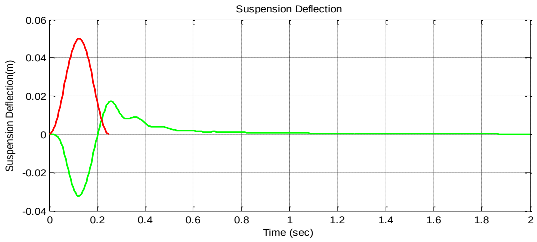

Figure 10. (c). Suspension Deflection

Figure 11 (a) and 11(b) shows the robust stability and robust performance of the system with H_∞ controller.

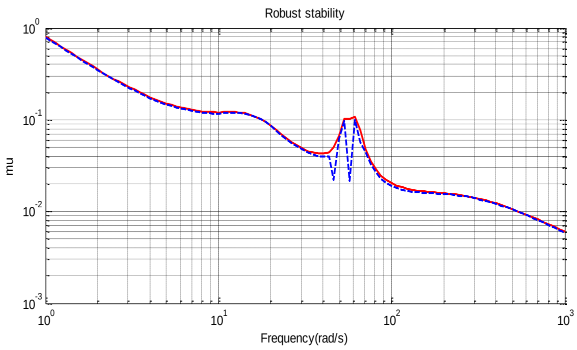

Figure 11. (a). Robust Stability

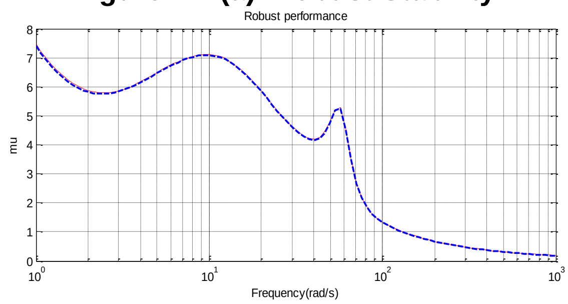

Figure 11. (b). Robust Performance

The maximum value of structured singular value is 0.8,thus satisfying the criteria for robust stability. Figure 11(b) shows that robust performance is achieved for frequency range from 150hz onwards, which is not satisfactory.

B. μ -Synthesis Control

Car body displacement, body acceleration and suspension deflection of the system with μ - synthesis controller are shown in the Figures 12(a), 12(b) and 12(c) respectively. The settling time for body acceleration and suspension deflection are 0.7sec and 0.6sec respectively; moreover peak overshoot is also reduced, which are better as compared to H_∞ controller.

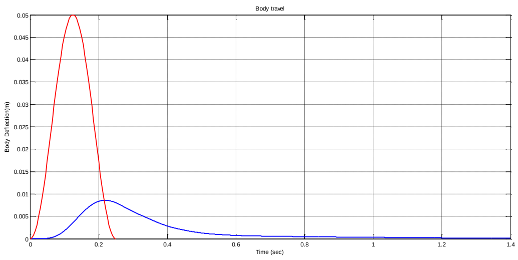

Figure 12. (a). Body Deflection

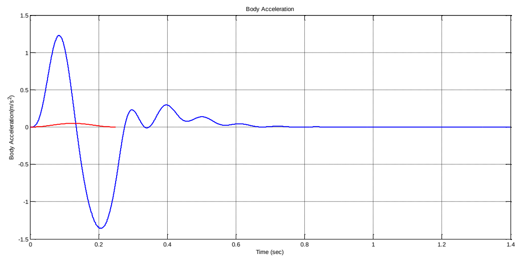

Figure12 (b). Body Acceleration

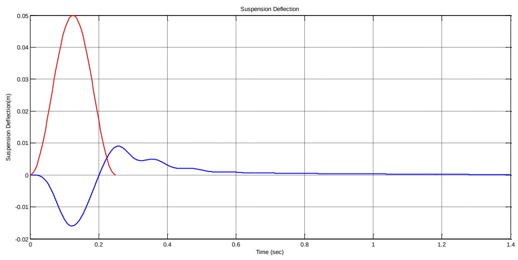

Figure 12. (c). Suspension Deflection

Figure 13 (a) and 13 (b) shows the robust stability and robust performance of the system with μ- synthesis controller. The maximum value of structured singular value is 0.1,thus satisfying the criteria for robust stability. Figure 13 (b) shows that the maximum value of μ is 0.9, thus it ensure that good robust performance is achieved.

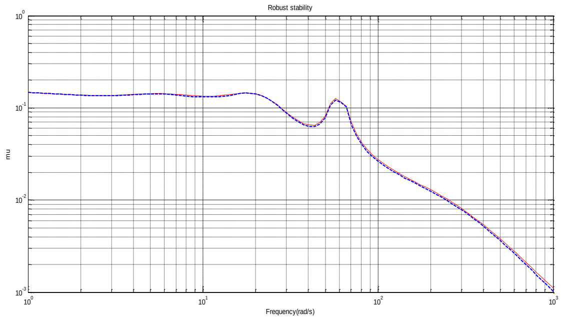

Figure 13. (a). Robust Stability

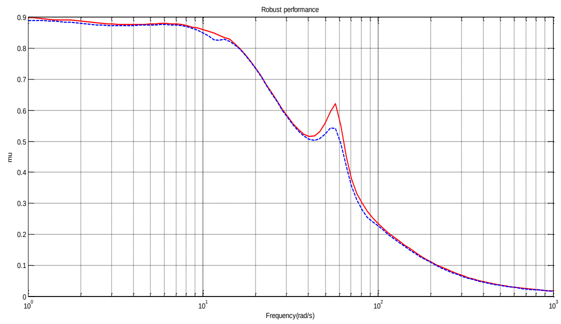

Figure 13. (b). Robust Performance

5. Conclusion

In this paper, H_∞ controller and μ- synthesis controllers are successfully designed using MATLAB for quarter car active suspension system. Both controllers are capable of stabilizing the suspension system very effectively, but the suppression of vibration is more effective with μ synthesis controller as compared to H_∞ controller. Both controllers ensures robust stability, but as far as robust performance is concerned, μ-synthesis controller provides the superior robust performance over the whole frequency range.

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