# Chaos Control of Permanent Magnet Synchronous Motor via Adaptive Sliding Mode Variable Structure Scheme

Shuo Zhou<sup>1, 2</sup>, Dazhi Wang<sup>1</sup>, Qingzhong Gao<sup>1</sup> and Yue Liu<sup>3</sup>

<sup>1</sup>College of Information Science and Engineering, Northeastern University, China <sup>2</sup>Department of Mechanical and Electronic Engineering, Guidaojiaotong Polytechnic Institute, China <sup>3</sup>College of Science, Shenyang Jian Zhu University, China 888zhoushuo@163.com

### Abstract

When system parameters falling into a certain area or under some working conditions, the permanent magnet synchronous motor (PMSM) will appear chaos phenomenon, which threatens the secure and stable operation of drive system. So this paper proposes a robust adaptive sliding mode control strategy aiming at the complex chaotic behavior of PMSM. Using the sliding mode control technique, a time varying surface is constructed. Then an adaptive gain of the robust control law is established considering uncertainties and external interference in PMSM. So the phenomenon of chaos in PMSM system will be eliminated using proposed controller. Finally, stability analysis will be given based on Lyapunov stability theory. Simulation results demonstrate the effectiveness of the proposed adaptive sliding mode control scheme.

Keywords: permanent magnet synchronous motor (PMSM), Chaotic system, Adaptive sliding mode control

### **1. Introduction**

The fundamental characteristic of a chaotic system is its sensitive dependence on initial conditions and systemic parameters. So a small shift in the initial states can lead to extraordinary and unpredictable perturbation in the system states. This will go against precise control of system. During the past two decades, chaos control study has attracted considerable attention among the scholars. Many different control methods of chaotic systems have been proposed such as linear state feedback control [1, 2], adaptive control [3], backstepping method [4, 5], sliding mode control [6-10], H $\infty$  approach[11], PID control [12-13], fuzzy logic control [14-16], *etc.* In recent years, chaotic systems are also applied in secure communication, biological systems, power quality, information processing and chemical reaction analysis.

Permanent magnet synchronous motors (PMSM) are intensively used in industrial applications due to their simple structure, high efficiency, high power density and large torque to inertia ratio. However, it is still a challenging problem to control the PMSM to get the perfect dynamic performance, because the dynamic model of PMSM is multivariable, nonlinear and parameters variable. Even PMSM system demonstrates chaotic behavior when systemic parameters fall into a certain area. The chaotic behavior in PMSM can destroy the stabilization of the motor even inducing the drive system to collapse. So the control of chaos in the PMSM have been an active research area in the field of nonlinear control of electric motors. Up to now, a lot of valid control methods have been successfully used to control chaos in PMSM. The OGY method [17] is the typical method of controlling chaos in PMSM. But the shortcoming of this method is not easy to determine system parameters. Time delay feedback control (TDFC) [18] was

International Journal of Control and Automation Vol. 9, No. 4 (2016)

successfully implemented to control the PMSM, but it is difficult to determine the time delay for the TDFC method given a specific target and is not suitable when the desired target is not an equilibrium or an unstable periodic orbit of the system. [19] used an adaptive fuzzy backstepping technology to control chaos in the PMSM. This method overcame the problem of "explosion of terms" caused by the repeated differentiations of virtual input, but the calculation process is more complicated. [20, 21] proposed sliding mode control (SMC) approach to realize the controlling of chaos in the PMSM. However, the uncertainties and external interference in PMSM were not taken into account.

In this paper, a robust adaptive sliding mode controller is developed to suppress chaos in permanent magnet synchronous motor drive systems. The proposed scheme will give sufficient consideration to uncertainties and external interference in PMSM. Furthermore, the stability of the propose method is guaranteed using Lyapunov stability theory. Finally, simulation results demonstrate the effectiveness and robustness against the chaotic in PMSM drive system.

### 2. Mathematical Model of Chaotic PMSM Drive System

The mathematical model of the permanent magnet synchronous motor with smooth air gap is described as follows [22]:

$$\begin{cases} \frac{d\omega}{dt} = \sigma(i_q - \omega) - T_L \\ \frac{di_q}{dt} = -i_q - i_d \omega + \gamma \omega + u_q \\ \frac{di_d}{dt} = -i_d + i_q \omega + u_d \end{cases}$$
(1)

where  $i_d$ ,  $i_q$  and  $\omega$  are the state variables, which represent d-q axis currents and motor angular frequency.  $u_d$  and  $u_q$  are the d-q axis stator voltage.  $T_L$  denotes the load torque.  $\sigma$  and  $\gamma$  are non-negative system parameters.

In system (1), the external inputs are set to zero,  $T_L = u_q = u_d = 0$ , and the system dynamic becomes an unforced system as follows[16]:

$$\begin{cases} \frac{d\omega}{dt} = \sigma(i_q - \omega) \\ \frac{di_q}{dt} = -i_q - i_d \omega + \gamma \omega \\ \frac{di_d}{dt} = -i_d + i_q \omega \end{cases}$$
(2)

The study found that the PMSM experiences chaotic behavior when the system parameters  $\sigma$  and  $\gamma$  fall in a certain range of values. For example, the PMSM displays chaos with  $\sigma = 5.45$  and  $\gamma = 20$ . A typical chaotic attractor is shown in Figure 1. Figure 2-4 are the time graph of  $\omega$ ,  $i_q$ ,  $i_d$  under chaos. For simplicity, the following notations are introduced:  $x = \omega$ ,  $y = i_q$  and  $z = i_d$ . Taking into account the uncertainties of system and adding the input u(t), the differential equation (2) is rewritten as

$$\begin{cases} \dot{x} = \sigma(y - x) + \Delta f(x, y, z) + u(t) \\ \dot{y} = -y - xz + \gamma x \\ \dot{z} = -z + xy \end{cases}$$
(3)

where u(t) is the input and  $\Delta f(x, y, z)$  is the uncertainty, the rang of uncertainty is in  $|\Delta f(x, y, z)| \le \delta$ .



Figure 1. The Chaotic Attractors of PMSM



Figure 2. The Time Graph of  $\boldsymbol{\omega}$  Under Chaos



Figure 3. The Time Graph of  $i_q$  Under Chaos



Figure 4. The Time Graph of  $i_d$  Under Chaos

### 3. Adaptive Sliding Mode Control Strategy

#### 3.1. Adaptive Sliding Mode Controller Design

The adaptive sliding surface is described as follow:

$$s(t) = x(t) + \Phi(t) \tag{4}$$

where  $\Phi(t)$  is given by

$$\dot{\Phi} = \gamma y + \sigma x + \eta x \tag{5}$$

where  $\eta > 0$ .

When the system reaches the sliding surface, Eq. (5) will satisfy the following equation:

$$s(t) = x(t) + \Phi(t) = 0$$
 (6)

The time derivative of (6) is given by

$$\dot{s}(t) = \dot{x} + \dot{\Phi} = 0 \Longrightarrow \dot{x} = -\dot{\Phi} = -\gamma y - \sigma x - \eta x \tag{7}$$

According to equations (3) and (7), the system of PMSM be rewritten as follows:

$$\begin{cases} \dot{x} = -\gamma y - \sigma x - \eta x \\ \dot{y} = -y - xz + \gamma x \\ \dot{z} = -z + xy \end{cases}$$
(8)

In order to analyze the stability of equations (8), the Lyapunov function can be defined  $\underset{V=}{\overset{\text{as}}{\overset{(x^2+y^2+z^2)}{2}}}$ . Taking the derivative of (8), we find

$$\dot{V} = x\dot{x} + y\dot{y} + z\dot{z}$$
  
=  $x(-\gamma y - \sigma x - \eta x) + y(-y - xz + \gamma x) + z(-z + xy)$   
=  $-(\sigma + \eta)x^2 - y^2 - z^2 \le 0$  (9)

According to Lyapunov stability theory, it can be concluded that system is asymptotically stable and all state variables would converge to the origin.

To guarantee the system state on sliding mode surface, the equivalent control law is designed as following:

$$u_{eq} = -\gamma y - \sigma x - \rho x - \Delta f(x, y, z)$$
<sup>(10)</sup>

The switching control law is designed as following:

$$u_{sw} = K \operatorname{sgn}(s) \tag{11}$$

where K is the switching gain, achieved by following adaptive law:

$$\dot{K} = -l|s| \tag{12}$$

where l is a positive constant number. So the control law of PMSM becomes

International Journal of Control and Automation Vol. 9, No. 4 (2016)

 $u = u_{eq} + u_{sw}$ 

$$= -\gamma y - \sigma x - \eta x - \Delta f(x, y, z) + K \operatorname{sgn}(s)$$
<sup>(13)</sup>

In order to control chaos in PMSM using Eq. (13), the function  $\Delta f(x, y, z)$  must be known. However,  $\Delta f(x, y, z)$  is the uncertainties and external disturbances in the system. To overcome this, the control law of PMSM is modified to

$$u = -\gamma y - \sigma x - \eta x + K \operatorname{sgn}(s)$$
<sup>(14)</sup>

#### **3.2. Stability Analyses**

In order to prove the stability of the scheme presented in (14), the Lyapunov function V can be defined as

$$V = \frac{1}{2}s^{2} + \frac{1}{2l}(\tilde{K} - K)^{2}$$
(15)

Taking the derivative of (15), we find

$$\dot{V} = s\dot{s} + \frac{1}{l}(\tilde{K} - K)(-\dot{K})$$

$$= s(\dot{x} + \dot{\Phi}) + (\tilde{K} - K)(-\frac{1}{l}\dot{K})$$

$$= s(\Delta f(x, y, z) + K\operatorname{sgn}(s)) + (\tilde{K} - K)(-\frac{1}{l}\dot{K})$$

$$\leq \delta |s| + K|s| + (\tilde{K} - K)(-\frac{1}{l}\dot{K})$$

$$\leq (\delta + \tilde{K})|s| + (\tilde{K} - K)(-\frac{1}{l}\dot{K} - |s|)$$
(16)

Considering  $-\frac{1}{I}\dot{K} - |s| = 0$ , the scalar  $\tilde{K}$  can be chosen in such a way that the value

of  $\delta + \widetilde{K} = -\xi$  remains negative. Inequality (16) can be rewrited as

$$\dot{V} \le -\xi |s| \tag{17}$$

Therefore, the condition of global asymptotic stability  $s(t)\dot{s}(t) < 0$  is satisfied.

### 4. Simulation Results

The parameters of the PMSM is listed in Tables 1. The overall block diagram of controlling chaos in PMSM is shown in Figure 5. In order to illustrate the effectiveness of the proposed method, the simulation will be conducted to control chaos in the PMSM drive system under the initial condition of  $[x, y, z]^T = [3, -4, 2]^T$  and system parameters of  $\eta = 7, l = 0.009$  . The uncertainty  $\sigma = 5.45, \quad \gamma = 20$  , term is  $\Delta f(x, y, z) = 0.5 - \sin(\pi x) \sin(2\pi y) \sin(3\pi z)$ . Figure 6 is the block diagram of chaos in PMSM with simulink. Figure 7 shows the state variables of PMSM system when the input control u is added to the system at the time t = 20 second.

Parameter	Value
Stator Resistance	0.7 Ω
Stator Inductor	0.00484 <i>H</i>
Flux	0.0556 <i>Wb</i>
Rotor Inertia	$0.74\text{e-}4kg\cdot m^2$
poles	4

# Table 1. Parameters of PMSM



Figure 5. Diagram of Controlling Chaos in PMSM



Figure 6. The Block Diagram of Chaos in PMSM with Simulink



![](_page_8_Figure_1.jpeg)

Figure 7. The State Variables of the Controlled PMSM System

It can be seen that the system will get rid of the chaotic motion and the state variables will gradually converge to equilibrium point after the time of 20s. The curve of control signal u is showed in Figure 8.

![](_page_8_Figure_4.jpeg)

Figure 8. The Curve of Control Signal u(t)

### 5. Conclusion

In this paper, an adaptive sliding mode control scheme was proposed for eliminating complex chaotic behavior of PMSM. The propose scheme will give sufficient consideration to uncertainties and external interference in PMSM and guarantee that the state variables of PMSM can converge to equilibrium point from chaotic. Finally, stability analysis will be given based on Lyapunov stability theory. Simulation results are provided to demonstrate the effectiveness and robustness against parameter uncertainties in the chaotic drive system.

## References

- H. P. Ren and D. Liu, "Nonlinear Feedback Control of Chaos in Permanent Magnet Synchronous Motor", IEEE Transactions on Circuits and Systems-II: Express Briefs, vol. 53, no. 5. (2006).
- [2] Y. Chen, X. Wu and Z. Gui, "Global Synchronization Criteria for a Class of Third-order Non-autonomous Chaotic Systems via Linearstate Error Feedback Control", Applied Mathematical Modelling, vol. 28, (2010), pp. 4161.
- [3] W. Xiang and F. Q. Chen, "An Adaptive Sliding Mode Control Scheme for a Class of Chaotic Systems with Mismatched Perturbations and Input Nonlinearities", Commun Nonlinear Sci Numer Simulat, vol. 16, no. 1, (2011).
- [4] N. J. Chen, F. Z. Song and G. P. Li, "An Sdaptive Sliding Mode Backstepping Control for the Mobile Manipulator with Nonholonomic Constraints", Commun Nonlinear Sci Numer Simulat., vol. 18, (2013), pp. 2885.
- [5] Z. Q. Miao and Y. N. Wang, "Robust Adaptive Radial Wavelet Neural Network Control for Chaotic Systems Using Backstepping Design", Acta Physica Sinica. vol., 61, no. 1, (2012).
- [6] M. R. Faieghi, H. Delavari and D. Baleanu, "A Note on Stability of Sliding Mode Dynamics in Suppression of Fractional-order Chaotic Systems", Computers and Mathematics with Applications, vol. 66, (2013), pp. 832.
- [7] C. Yin, S. M. Zhong and W. F. Chen, "Design of Sliding Mode Controller for a Class of Ractional-order Chaotic Systems", Commun Nonlinear Sci Numer Simulat., vol. 17, (2012), pp. 356.
- [8] C. C. Wang, N. S. Pai and H. T. Yau, "Chaos Control in AFM System using Sliding Mode Control by Backstepping Sesign", Commun Nonlinear Sci Numer Simul., vol. 15, (2010), no. 41.
- [9] T. Shen, D. Y. Chen, X. Y. Ma and H. T. Chen. "Chaos Analysis and Sliding Mode Control for Fluid Vibrations in Cylindrical Tanks under Limited Excitation", Water Resour Archit Eng., vol. 8, no. 76. (2010).
- [10] J. W. Feng, L. He, C. Xu, A. Francis and G. Wu, "Synchronizing the Noise-perturbed Genesio Chaotic System by Sliding Mode Control", Commun Nonlinear Sci Numer Simul., vol. 15, no. 46, (2010).
- [11] S. M. Lee, D. H. Ji, J. H. Park and S. C. Won, "H Synchronization of Chaotic Systems via Dynamic Feedback Approach", Physics Letters A. vol. 372, (2008), pp. 4905.
- [12] W. Chang, "PID Control for Chaotic Synchronization Using Particle Swarm Optimization", Chaos Soliton Fract. vol. 39, (2009), no. 910.
- [13] D. Davendra, I. Zelinka and R. Senkerik, "Chaos Driven Evolutionary Algorithms for the Task of PID control", Computers and Mathematics with Applications, vol. 60, (2010), pp. 1088.
- [14] T. C. Lin and M. C. Chen, "Adaptive Hybrid Type-2 Intelligent Sliding Mode Control for Uncertain Nonlinear Multivariable Dynamical Systems", Fuzzy Sets and Systems, vol. 171, no. 44, (2011).
- [15] N. S. Pai, H. T. Yau and C. L. Kuo, "Fuzzy Logic Combining Controller Design for Chaos Control of a Rod-type Plasma Torch System", Expert Syst Appl. vol. 37, (2010), pp. 8278.
- [16] M. Roopaei and M. Z. Jahromi, "Chattering-free Fuzzy Sliding Mode Control in MIMO Uncertain Systems", Nonlinear Analysis, vol. 71, (2009), pp. 4430.
- [17] E. Ott, C. Grebogi and J. A. Yorke, "Controlling Chaos", Physical Review Letters, vol. 64, (1990), pp. 1196.
- [18] H. Ren, D. Liu and J. Li, "Delay Feedback Control of Chaos in Permanent Magnet Synchronous Motor", Proceedings of the China Society Electronic Engineering Conference, vol. 23, (2003), pp. 175.
- [19] J. P. Yu, B. Chen, H. S. Yu and J. W. Gao, "Adaptive Fuzzy Tracking Control for the Chaotic Permanent Magnet Synchronous Motor Drive System via Backstepping", Nonlinear Analysis-Real World Applications, vol. 12, (2011), pp. 671.
- [20] H. Liu, "Control of Chaos in Permanent Magnet Synchronous Motor Drive System via Smooth Second Order Sliding Mode", Computer Engineering and Applications. vol. 48, (2012), pp. 222.
- [21] C. Ma, L. Wang and Z. Yin, "Sliding Mode Control of Chaos in the Noise-perturbed Permanent Magnet Synchronous Motor with Non-smooth Air-gap", Mining Science and Technology (China), vol. 21, (2011), pp. 835.
- [22] Y. Z. Zeng, "Adaptive Sliding Mode Control for Permanent Magnet Synchronous Motor", 2011 International Conference of Information Technology, (2011), pp. 356.

# Authors

![](_page_10_Picture_2.jpeg)

**Shuo Zhou**, he received the B.S. degree in electrical engineering from the Northeastern University, Shenyang, China, in 2003, and M. S. degree in system engineering from Shenyang University of Technology, Shenyang, in 2007, where he is currently working toward the Ph.D. degree in control theory and applications form Northeastern University.

![](_page_10_Picture_4.jpeg)

**Dazhi Wang**, he is professor and doctoral supervisor in Northeastern University, mainly engaged in the research of electric power system and electric power transmission.

![](_page_10_Picture_6.jpeg)

**Qingzhong Gao**, he received his B.Sc. from Northeastern University in 2005, and received his M. Sc. from Northeastern University in 2009. Now he is a doctoral candidate in Northeastern University. His main research interest is key technologies of adjustable permanent magnetic coupler. His research interests include Speed Permanent Magnet key technologies.

![](_page_10_Picture_8.jpeg)

**Yue Liu**, she received her B.Sc. from Northeastern University in 2005, and received her M.Sc. from Northeastern University in 2009. Her main research interest is key technologies of adjustable permanent magnetic coupler.

International Journal of Control and Automation Vol. 9, No. 4 (2016)