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Modeling & Controlling the Dynamics of Tri-rotor UAV Using Robust RST Controller with MRAC Adaptive Algorithm

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Abstract

This paper presents a mathematical approach of the controlling of Unmanned Aerial Vehicle (UAV) translational and rotational movement dynamics by using classical regulation, pole-placement & tracking (RST) controller with model reference adaptive control (MRAC). Identification of a black box is done by using the Least Square Estimation (LSE) method while MIT rule is used as a control algorithm in conjunction with the RST controller with MRAC. The desired result of 6-degrees of freedom (DOF) via Multiple Input and Multiple Output (MIMO) model comprises the equation of motions which utilizes UAV to stabilize the nonlinear, translational and rotational movement implements the adaptive approach via initial conditions at the start of vertical takeoff. The numerical approach simplifies the control equations of the tri-rotor and stabilizes towards linearity. The resulted control strategy is affected over nonlinear simulation for each control dynamic and gives more stability with less power requirements. Hence the adaptive algorithm is ensuring the controlling and stability of the system.

Keywords: *Unmanned Aerial Vehicle, Tri-rotor Controlling, RST Controller, Least Square Estimation*

1. Introduction

In this fast developing world, where mobile engineering is penetrated in every arena of research, Investment and research into the field of UAV is a hot topic among the top class researchers of the field which is mainly because of its applications. With the advancement in aircraft technology, UAV control gets lots of attention. A micro helicopter UAV with VTOL capability with fast maneuvering attracts lot of attention among civilian and military community for surveillance, firefighting and environmental monitoring [1]. They also find application in traffic monitoring, forest fire patrolling and rescue. There are numerous advantages of rotor crafts over fixed wing aircrafts such as VTOL and payload they can carry [2] UAVs are classified into two categories, fixed and rotary wing types. In practical operations, fixed wing. UAVs have been used for years in routine surveillance missions, but they lack the hovering flight capability [3]. Rotor crafts are multi input multi output (MIMO) under actuated systems. Rotorcraft (UAV) is one of the most complex types of the flying machine.

In Urban areas, VTOL vehicles become more attractive in civilian and military organizations. Recently, autonomous VTOL crafts with stationary flight capacity become more focused area of research. Autonomous Helicopter drones with high payload to power ratio is studied in (Frazzoli *et al.*, 2002, Saripalli *et al.*, 2002 and Amidi *et al.*, 1999). The major issue with Helicopter is its exposed rotors [4]. When dealing with Autonomous helicopter, ground constraints, flight control and dynamics of rotor must be dealt accordingly while landing and takeoff.

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UAV's are best suited to work in these situations. Furthermore, UAV's must have to fulfill some basic preliminaries in order to work in such a difficult conditions. They must be smaller in size to ensure free motion as well as fast and rapid movements to avoid collisions [5]. To address the control problem, variety of control algorithms was designed such as mobile robot dynamics. Adaptive and Intelligent control schemes were used including fuzzy and neural networks schemes. Different UAV types and comparing the outcomes for performance indexes. The framework is based on optimal control formulation and enables quantifying influences of numerous individual Intel Robot System (2011) Guidance performance evaluation frameworks. The guidance problem is formulated as an optimal control problem such that merging the influence of different vehicle design, system components (sensors, actuators and airframe) and mission requirement. On the basis of optimal formulation, spatial behaviors are characterized as SCTG maps and trajectories, when analyzed yields situated perspective necessary for a comprehensive evaluation of the system [6].

A tri-rotor aerial robot acquired three Cartesian coordinates (X, Y, Z) and 6-DOF via four control parameters, dynamic inputs referring (Col, Lat, Lon, Ped.) are Altitude, Lateral, Longitudinal and Angular moments. (p, q, r), (u, v, w) and (θ, φ, ψ) are the output of the tri-rotor vehicle and called as Rotational velocities, Translational velocities and Rotational Angles. UAV exhibit a number of important physical effects such as aerodynamic effects, inertial counter torques, gravity effects, gyroscopic effects and friction, etc., which makes it difficult to design a real time-time controls for them. A tri-rotor UAV is a highly nonlinear, multivariable, strongly coupled, and under the actuated system having six degrees of freedom (6 DOF) with three actuators [7]. To organize the strategy of nonlinear sequential control implemented in the drive 6-DOF model, constitute of rotational and translational subsystems. The research phase pointed towards an error occurred in a tri-rotor aircraft because the yaw moment induces due to the unpaired rotor reaction and produce torque. To rectify this difficulty, the number of designs in a tri-rotor aircraft has been made to their own specifications. Now a BLDC motor is installed, which tilts the angle to nullify the moment. The main advantage of this design is better movement, especially for a quicker turn, by tilting one of the rotor's axes.

A detailed mathematical model, including aerodynamics, is obtained via the Newton-Euler formulation. In terms of control, we propose a control algorithm that achieves stability for the longitudinal under-actuated dynamics during vertical flight [8]. If an autonomously controlled UAV dynamic model parameters change during flight for any reason, the controller performance quality might reduce if not fail. However, if a method can estimate the values of the changing parameters online and update the controller with the new values, an increase in the performance quality of the controller is expected. The Least square estimation method is used for identifying the black box model of the system.

The RST controller is a classic controller and by testing its performance and restrictions in our complex system with two different ways the first is applied the Robust RST controller directly to the system and after that Robust RST controller with MIT adaptive algorithm is applied and take the difference of their robustness between both controller results. The RST is a distinguish controller due to its good compromise between performance and complexity. RST controller has two degree-of-the freedom attained by means of an input–output model-based pole placement method, which infers the resolution of a Diophantine equation. R–S–T polynomials are obtained by design procedure. The degree of freedom consists of a feed forward side defined by T/R, and has a feedback side defined by S/R [9]. The RST polynomial regulator seems to be an interesting alternative to the regular PI controller, because it allows a good compromise between speed and performances. Based on the pole placement theory, it is possible to impose poles in the closed loop and to carry out in a separate way the objectives for tracking and for regulation [10].

To specify the desired control-loop performances. To have a dynamic model of the system to be controlled (this can be obtained from real data by identification). To possess a suitable controller design methodology, compatible with the desired performance and the corresponding system model. To have a procedure for controller validation and on-site re-tuning. To have appropriate software packages with real-time capabilities for data acquisition, system identification, control design and on-site commissioning [11]. This proposed design method ensures a steady state error between the reference and controlled output close to zero. PID is less robust than the RST controller against the noise and disturbance of the system. The method also ensures that there is no steady state error between the referred and controller output [10].

This paper presents an adaptive algorithm based on the RST controller for the stabilization of a tri-rotor Unmanned Aerial Vehicle. It has three propellers driven by a BLDC motor. The controlling of dynamics and altitude of the UAV with the help of propellers because al forces acting on it. Now RST control strategy is proposed to control the system elevation, Euler angles and velocity responses of the UAV.

The division of this manuscript is as follows. In section 2 equations of motions of a rigid body of UAVs explained. Section 3 defines the dynamics of UAV along with the control mechanism of the system. In addition to this, Section 4 covers the main engine model. Moreover, the identification of system and control algorithm is introduced in Section 5. The last section covers the simulation results and discussion presented for input error control as well as the translational & rotational control of the system.

2. The Equation of Motion of Rigid Body

Newton's second law defines the rigid body equations of motion which state's that the sum of all external forces substitutes in a body is equal to the time frame of change of the momentum of the body. Furthermore, the addition of external instants behaving on the body is equal to the rate of change of angular momentum. Linear and angular momentum change of time rates are referred as an inertial reference frame. The aircraft will also have a capability of same velocity, components, forces & moments which can develop into a six-degree of freedom nonlinear equations of motion. The equations of motion of a rigid body are expressed in differential equations describing all the motions of an aircraft rotary motion, kinematics and its translator motion.

Above-mentioned notations and its denotations are used in the equations 1 to 12 below. The translational position and translational velocity of the coordinate system is defined by (X, Y, Z) and (u, v, w).The external moments of the UAV is represented by (L, M, N) the fix body frame and (θ, φ, ψ) are Euler angles. That represent pitch, roll and yaw for describing the rotation of the local coordinate system and angular rates of the system define by (p, q, r).

Aero Dynamics Force Equations

$$X - mg\, sin\theta = m(\dot{u} + qw - rv) \quad (1)$$
$$Y + mg\, cos\theta\, sin\varphi = m(\dot{v} + ru - pw) \quad (2)$$
$$Z + mg\, cos\theta\, cos\varphi = m(\dot{w} + pv - qu) \quad (3)$$

Aero Dynamic Moments Equations

$$L = I_x\dot{p} - I_{xz}\dot{r} + qr\left(I_z - I_y\right) - I_{xz}pq \quad (4)$$
$$M = I_y\dot{q} + rp\,(I_x - I_z) + I_{xz}(p^2 - r^2) \quad (5)$$
$$N = -I_{xz}\dot{p} + I_z\dot{r} + pq\left(I_y - I_x\right) + I_{xz}qr \quad (6)$$

Angular Rates

$$p = \dot{\varphi} - \dot{\psi} sin\theta \quad (7)$$
$$q = \dot{\theta}\, cos\varphi + \dot{\psi} cos\theta\, sin\varphi \quad (8)$$
$$r = \dot{\psi}\, cos\theta\, cos\varphi - \dot{\theta}\, sin\varphi \quad (9)$$

Euler angles and body angular velocities

$$\dot{\theta} = q\, cos\varphi - r\, sin\theta \quad (10)$$
$$\dot{\varphi} = p + q\, sin\varphi\, tan\theta + r\, cos\,\varphi\, tan\theta \quad (11)$$
$$\dot{\psi} = (q\, sin\varphi + r\, cos\varphi) sec\theta \quad (12)$$

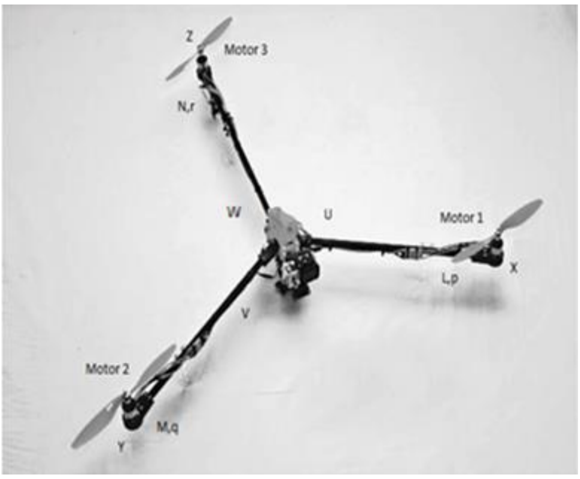


Figure 1. Diagram of Tri-rotor UAV

2.1. Tri-Rotor UAV Dynamics

The Dynamics of tri-rotor vehicles are highly coupled and nonlinear, which makes the control design of these vehicles, the key for successful flight and operations [12]. According to Newton's law of momentum theory, Stems and advanced on the bases of the axial velocity U of the fluid through the actuator is generally higher than the speed V with which actuator is advancing through the air. The developing thrust by the actuator is equal to the physique of the air passing through the circle in a unit time. The aerodynamic forces and moments are derived from a combination of momentum and blade element theory.

The hovering of tri-rotor blades on the propeller become inefficient with the increase in number of blades in the propeller and limit its forward flight speed. By using helicopter aerodynamics conditions using momentum theory is similar to tri-rotor theory. A tri-rotor UAV has three motors with propellers. The coefficients of non-dimensional power, thrust and torque are used to define rotor characteristics in a form that is independent of rotor size, where

R: Radius of the blade.
Ω: Angular velocity.
A: Blade area.
ρ: Density of air.
Q: Rotor shaft.
T: Thrust.
$C_{(t)}$: Thrust of aerodynamic coefficient.

$$C_{(t)} = \frac{T}{\rho A(\Omega R)^2} \quad (13)$$

$$C_{(q)} = \frac{Q}{\rho A(\Omega R)^2 R} \quad (14)$$

$$C_{(p)} = \frac{P}{\rho A(\Omega R)^3} \quad (15)$$

Equation (16) represents the power and torque coefficients shown by the equation.

$$P = Q\Omega \quad (16)$$

By substituting the power coefficient in the equation (14) we have.

$$C_{(p)} = C_q = \frac{Q\Omega}{\rho A(\Omega R)^3} = \frac{Q}{\rho A(\Omega R)^2 R} \quad (17)$$

Hovering of an UAV, the uniform inflow and a constant profile drag coefficient Cdo is assumed as 0.015, the approximation known as the momentum modified theory. In which σ is the rotor solidity ratio. Power induced in the flight climbing is usually two or three times greater than the power of a profile.

$$C_{(p)} = K\sqrt{\frac{C_t}{2}} * C_{(t)} + \frac{1}{8} * \sigma * Cdo \quad (18)$$

3. Dynamic Representation of a System

Euler angles define the orientation of tri-rotor UAV pitch (θ), yaw (ψ) and roll (φ) rotating along three axes respectively (x, y, z). The tri-rotor UAV rotate and translate in to the dimensional space and the rigid body dynamics are derived from Newton's law. Furthermore, forces, moments, and velocity components experience by an aircraft is defined in a Table 1.

Table 1. Tri-rotor UAV Dynamic Constants

x , y ,z Axis system	Roll (φ)	Pitch (θ)	Yaw (ψ)
Components of Aerodynamic Force	X	Y	Z
Components of Aerodynamic Moment	L	M	N
Velocity Components	U	V	W
Angular Rates	p	q	r
Inertia about each axis	Ix	Iy	Iz

Tri-Rotor UAV Control: The tri-rotor flight dynamics is nearly similar to any other aircraft, in which the orientation and flight control is a product of pitch, yaw and roll. The control strategy is the same as any traditional helicopter. Control strategies of tri-rotor UAV include Roll, Yaw, Pitch and altitude and also a tilt angle play a vital role to control the parameters of UAV.

Mechanism of Altitude Control: The speed of all rotors (1), (2) and (3) is same. Increasing the speed constantly will ultimately increase the altitude of the UAV's. The angular velocity of all three rotors is same.

Mechanism of Roll Control: Regulating rotors and advancing the speed of two rotors will produce a roll. By decrementing the speed of the rotor (1) the tri-rotor UAV will roll to the leftward and rotor speed (2) roll to the rightward.

Mechanism of pitch control: To Vary the rotor speeds from forward-facing and behind rotors will produce a pitch. After decreasing the speed of the rotors (1) and (2) increasing the rotor speed (3). The system pitches down and stabilizes advancing the tri-rotor UAV. Decreasing the speed of an actuator (3) and increasing the speed of an actuator (1) and (2) tri-rotor UAV will pitch up and fly in reverse.

Mechanism of yaw control: By changing the angle of the actuator (3) to vector the thrust to product a torque moment, which will yaw tri-rotor UAV left and right. To maintain lift, the rotor speed increases while thrust angle changes.

4. Main Engine Model

Brushless direct current (BLDC) motors are prevailingly used in high performance drive applications such as machine tools, robotics, space crafts, and medical applications, owing to their superior speed-torque characteristics, high efficiency, less maintenance, and wide operating speed range [13]. The BLDC motor has been in demand in small UAV's because of high efficiency, the desired torque versus speed characteristics. They have more complex control algorithm, quite efficient and precise as compared to any other motor.
The mechanical and electrical equations of BLDC are as follows

$$V = Ri + (L - M)\frac{di}{dt} + E \tag{19}$$

$$E = K_e \omega_m F(\theta_e) \tag{20}$$

$$T = K_t i_a F(\theta_e) \tag{21}$$

$$T_e = J\frac{d^2\theta_m}{dt^2} + \beta\frac{d\theta_m}{dt} \tag{22}$$

$$\theta_e = \frac{P}{2}\theta_m \tag{23}$$

$$\omega_m = \frac{d\theta_m}{dt} \quad \backslash \tag{24}$$

Where

V: Applied Voltages.
R: Resistance.
E: Back Emf.
L: The motor inductance.
M: Mutual inductance. K_e = Bck emf constant.
ω_m : Angular speed of the rotor.
θ_e : The Electrical angle of the rotor.
T_e : Electrical torque produced by BLDC.

5. Control Algorithm

A. Identification of the Black Box Model

Built the mathematical model of the system using input and output this technique is called system identification, which is being widely applied to various fields of production and life. Different conventional and unconventional techniques are available in the literature for modelling the dynamics of nonlinear systems. But in this paper we are using

Identification of a black box and it is done using a Least Square Estimation implemented in Matlab (Simulink) shown in Fig 2(a) and the least square estimation.

On Equation (α).

$$\hat{\theta} = [\varphi^T * \varphi]^{-1} \varphi^T * Y$$

Where
Θ: Vector of Coefficient of the desired system.
Φ: The matrix of the input Coefficient over time.
Y: Represents the output Coefficient.

By solving the matrix we acquire the coefficients of an unknown system matrix "θ ".
The virtual model provides accuracy of pitch, yaw and roll to 60 percent. While the black box model was only limited to the accuracy of 60 percent of the pitch only.

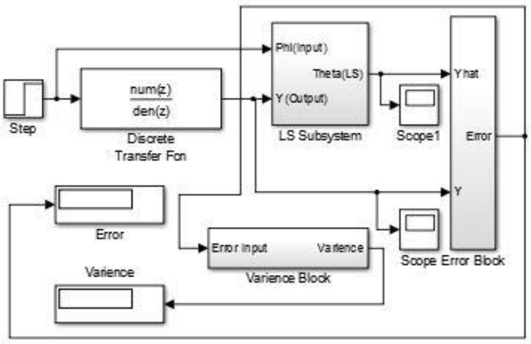


Figure 2(a). The Least Square Estimation

Figure 2 (b) represents the black box response and its coefficients that.

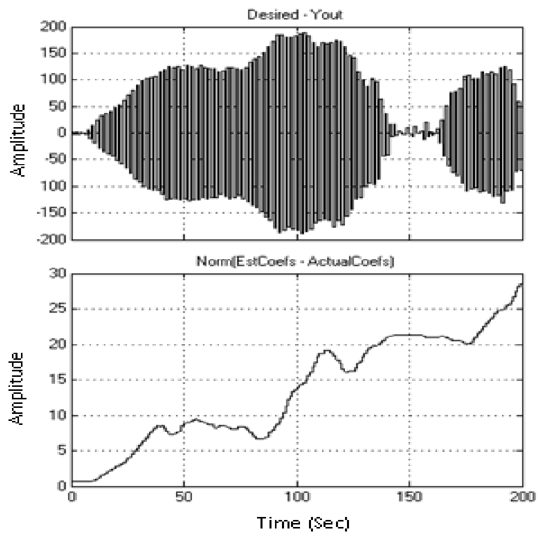


Figure 2(b). The Black Box Output Response and Norms

Table 2. Black Box Prediction

No.	% of the output	P (Roll)	Q (Pitch)	Y (Yaw)
1	The Black Box model	45.7	47.08	46.07
2	The Black Box Virtual Model	51.1	47.29	48.31

Black Box, Actual Vs Black Box Virtual Prediction

B. Control Technique

Once we have identified the model than derive the control algorithm for the model and for that we used MIT technique. The equation (α) represents the actual model identified by using the least square estimation.

$$G_{\theta_1(q)} = \frac{B\,(q)}{A(q)} = Y \tag{25}$$

The Degree of [B (q)] & degree of [A (q)] is found 1 & 3 respectively.

$$B = (B^- * B^+)$$
$$B^- = Constant\ of\ B, B^+ Is\ variable\ of\ B.$$

Equation (26) represents the desired response or location for the Tri-rotor to which it will hover.

$$G_{\theta_2(q)} = \frac{Bm(q)}{Am(q)} = Y_{(m)} \tag{26}$$

Now implementing MRAC technique based on gradient theory.

$$degA_c = 2 * degA - 1 \tag{27}$$

From equation 27, we can identify the degree of designing the controller, which is found to 5. Now to implement the 2nd order RST Controller, 25 and 26, respectively, for R, S and T as shown in the equation (28), (29) and (30).

$$R = (q)^2 + r_0 * q + r_1 \tag{28}$$
$$S = s_0(q)^2 + s_1 * q + s_2 \tag{29}$$
$$T = (t_0 q + t_1)\, A_0 \tag{30}$$

Remark 1. In [14] the pole placement technique disturbance is obsolete. Because in control law, reference model and observer polynomial disturbance included as a constraint.

Remark 2. $G_{\theta_1(q)}$ & $G_{\theta_2(q)}$ have the numerator and denominator of the entire system. On the other side the proposed model is generated an unstable control signal with minimum phase angle. Our proposed controller is not flexible for this case. In [15] can find the suitable controller for that case. If the system has minimum phase angle on the complex plain and have some absolutes damping values. As a result to evade the cancellation of zeros in the model which is located inside the region which will cancelled [14].

Remark 3. The proposed controller only design an RST controller at period of sampling time (NT). It reduces the complication in the designing stage and controller completion.

Lemma: Information depict for close loop system at sampling time NT period is defines as.

$$\frac{[Y^T]^{NT}}{R^{NT}} = \frac{\left[G_{\theta1}^T G_{\theta2}^T H^{T,NT}\right]^{NT} G_{\theta1}^{NT}\left[\frac{T^{NT}}{R^{NT}}\right]}{1+\left[G_{\theta1}^T G_{\theta2}^T H^{T,NT}\right]^{NT} G_{\theta1}^{NT}\left[\frac{S^{NT}}{R^{NT}}\right]}$$

* Where $H^{T,NT}$ Rate converter located after expanding depends upon the reference signal.

Proof. To consider RST controller in [16]. The output of the non-uniform controlled system will be.

$$Y^T = G_{\theta1}^T G_{\theta2}^T H^{T,NT}\left[G_{\theta1}^{NT}\right]^{NT}\left[\frac{T^{NT}}{R^{NT}} R^{NT} - \frac{S^{NT}}{R^{NT}}\left[Y^T\right]^{NT}\right]^T$$

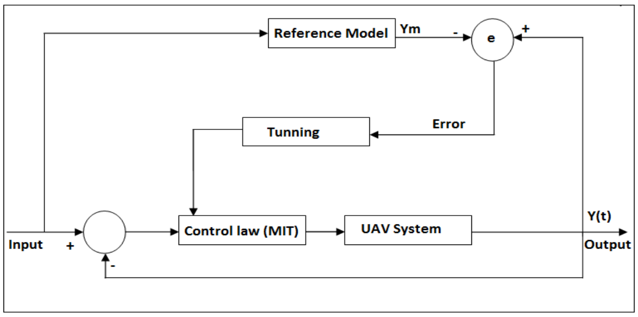


Figure 3. The Model Reference Adaptive Control System

For MIT rule, it is imperative to identify the error as the MIT rule is based on optimal control algorithm. Error response can be identified from the equation (31).

$$e = y_{(Actual)} - y_{(m)(Desired)} \tag{31}$$

Equation (32) represents the sensitivity derivative, replacing in the equation (31) to get the equation (34).

$$Y = \frac{BT}{(AR+BS)} \tag{32}$$

$$U_c = \frac{AR+BS}{BT} * y \tag{33}$$

Now to identify the sensitivity derivative of the parameters $(t_0, t_1, r_0, r_1, s_0, s_1, s_2)$
Applying sensitivity derivative in the equation (31) of MIT rule.

$$e = \frac{BT}{AR+BS} * U_c - Y_m \tag{34}$$

Putting the value of $T = (tq + t_1)A_0$ in the equation (34)

$$e = \frac{B(t_0 q + t_1)A_0}{AR+BS} * U_c - (Y_m) \tag{35}$$

Taking the derivative of the equation (35) w.r.t "t_0"

$$\frac{\delta e}{\delta(t_0)} = \frac{BqA_0}{(AR+BS)} \tag{36}$$

The Equation (37) represents the Diophantine equation.

$$AR + BS = A_0 A_m \tag{37}$$

$$\frac{\delta e}{\delta(t_0)} = (\frac{Bq}{(A_m)}) * Uc \tag{38}$$

Taking derivative of the equation (35) w.r.t "t_1"

$$\frac{\delta e}{\delta(t_1)} = (\frac{B}{(A_m)}) * U_c \tag{39}$$

Replacing the value of R =$(q)^2 + r_0 q + r_1$ in the equation (34)

$$e = \frac{BT}{A(q^2+r_0q+r_1)+BS} * U_c - (\mathrm{Y_m}) \tag{40}$$

By differentiating the equation (35) w.r.t "r_0"

$$\frac{\delta e}{\delta(r_0)} = (\frac{B}{(A_m)}) * (U_c) \tag{41}$$

By differentiating the equation (35) w.r.t "r_1"

$$\frac{\delta e}{\delta(r_1)} = -\frac{AqY}{A_m A_0} \tag{42}$$

Putting the value of the S = $(s_0 * q^2 + s_1 * q + s_2)$
Equation (34)

$$e = \frac{BT}{AR+B(s_0q^2+s_1q+s_2)} * U_c - (Y_m) \tag{43}$$

Taking derivative of the equation (43) w.r.t "s_0"

$$\frac{\delta e}{\delta(s_0)} = -\frac{(B*q^2*Y)}{(A_m*A_0)} \tag{44}$$

Taking derivative of the equation (43) w.r.t "s_1"

$$\frac{\delta e}{\delta(s_1)} = -\frac{BqY}{(A_{m*}A_0)} \tag{45}$$

Taking derivative of the equation (43) w.r.t "s_2"

$$\frac{\delta \mathrm{e}}{\delta(\mathrm{s_2})} = -\frac{\mathrm{BY}}{(\mathrm{A_{m*}A_0})} \tag{46}$$

Implementing the MIT Rule on the desired system, the equation (47) represents the cost function and the equation (48) represents the MIT rule. Whitaker defines the change in a system parameter as a function of the system error and the gradient of the system error with respect to the system parameter. The gradient of the error is the partial derivative of error with respect to the parameter. If the parameter of interest is the inertia estimate J, and the velocity error between the model and plant [17]. For optimal control of the system we must require a cost function through which the error response should be minimized over time to attain the desired response.

$$J(\theta) = \frac{1}{2}e^2(\theta) \tag{47}$$

$$\frac{d\theta}{(dt)} = -\gamma'\frac{k}{k_o}y_{(m)}e = -\gamma * y_{(m)} * e \tag{48}$$

After that Applying MIT rule. To take differentiate RST controller variables $(S_0, S_1, S_2, r_0, r_1, T_0 \text{ and } T_1)$ and place in the main controller equation.

$$\frac{d(S_0)}{dt} = -\gamma_e\frac{\delta e}{\delta S_0}$$

$$\frac{d(S_0)}{dt} = \frac{\gamma_e q^2 By}{A_m A_0} \tag{a}$$

$$\frac{d(S_1)}{dt} = -\gamma_e\frac{\delta e}{\delta S_1}$$

$$\frac{d(S_1)}{dt} = \frac{\gamma_e Bqy}{A_m A_0} \tag{b}$$

$$\frac{d(S_2)}{dt} = -\gamma_e\frac{\delta e}{\delta S_2}$$

$$\frac{d(S_2)}{dt} = \frac{\gamma_e By}{A_m A_0} \tag{c}$$

$$\frac{d(r_0)}{dt} = -\gamma_e\frac{\delta e}{\delta r_0}$$

$$\frac{d(r_0)}{dt} = \frac{\gamma_e Aqy}{A_m A_0} \tag{d}$$

$$\frac{d(r_1)}{dt} = -\gamma_e\frac{\delta e}{\delta r_1}$$

$$\frac{d(r_1)}{dt} = \frac{\gamma_e Ay}{A_m A_0} \tag{e}$$

$$\frac{d(T_0)}{dt} = -\gamma_e\frac{\delta e}{\delta T_0}$$

$$\frac{d(T_0)}{dt} = -\frac{\gamma_e Bq}{A_0} \tag{f}$$

$$\frac{\mathrm{d}(\mathrm{T}_1)}{\mathrm{dt}} = -\gamma_\mathrm{e}\frac{\delta \mathrm{e}}{\delta \mathrm{T}_1}$$

$$\frac{d(T_1)}{dt} = -\frac{\gamma_e}{(A_{(m)})} \tag{g}$$

Now, the Main Controller Equation RST becomes,

$$U_{(t)} = \left(\frac{((T_0 q + T_1)A_0)}{q^2 + r_0 q + r_1}\right) * (Uc(t)) - \left(\frac{S_0 q^2 + S_1 q + S_2}{q^2 + r_0 q + r_1}\right) y$$

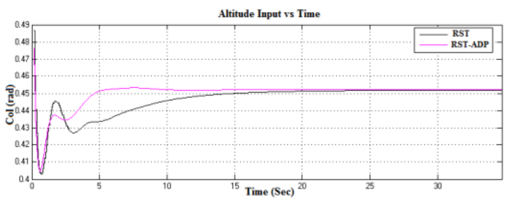
In Figure 4 rate changes as the direction of the vehicle changes. Although there are some oscillations in system response, but it settles down much faster towards stability through adaptive algorithm. The same effect can also be seen in fig and (respectively, using Simulink (MATLAB) to check the simulation of the controller.

6. Simulation Results & Discussion

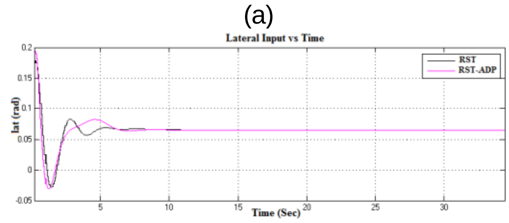
The verification of the proposed control algorithm is present in this section along with the parameters of tri-rotor nonlinear simulations shown in the Table 3. All the figures shows the comparisons of the classical Robust RST controller with our proposed MRAC the adaptive based Robust RST controller. Figure 3 shows the Altitude, Lateral, Longitudinal & Angular controlling of nonlinear dynamic inputs and as compared to the classical controller are proposed the controller is able to stabilize the tri-rotor within 6 seconds a fair time and they converge to zero degrees, without any overshoot it means that the settling time response is better than the classical controller. Figure 4 defines the angular rate responses of UAV. The response in Figure 5 shows the angular system by using the proposed the controller with initial conditions $x = 1\text{m}$ and $y, z, \psi = 0$ at the sampling time take 0.2 seconds. The proposed controller succeeded to control the pitch, yaw and roll angles of tri-rotor UAV and settled the system by eliminating the steady state error and overshoot as shown in figure. In Figure 6 shows the vertical velocity and altitude responses.

Table 3. Parameters of Tri-rotor UAV

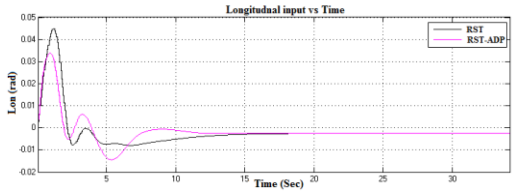
Parameters	Value	SI Units
Ix	0.3105	$\text{kg}m^2$
Iy	0.2112	$\text{kg}m^2$
Iz	0.2215	$\text{kg}m^2$
Mass	0.785	kg



(a)



(b)



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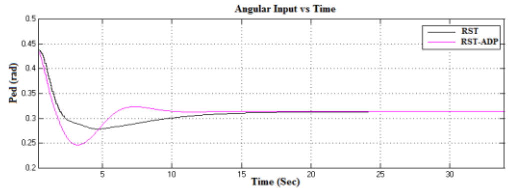
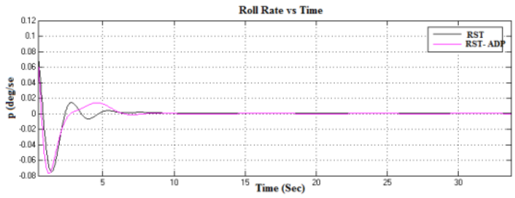
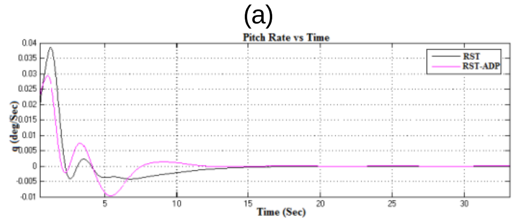


(d)

Figure 4. Comparison of RST- Adaptive based MRAC with Robust RST for Control Commands



(a)



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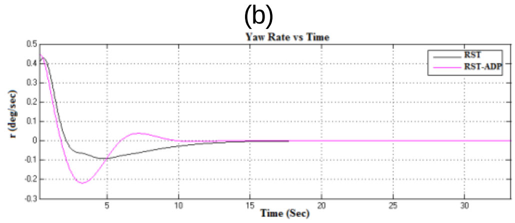
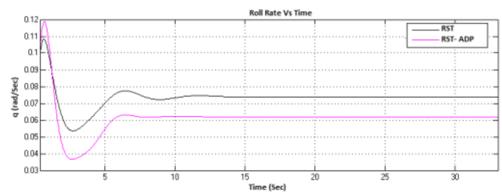
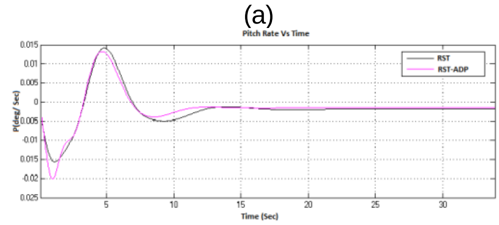


(c)

Figure 5. Comparison of RST- Adaptive based MRAC with Robust RST for Rotational Velocities



(a)



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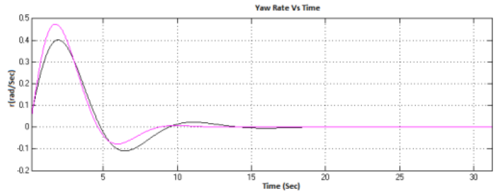
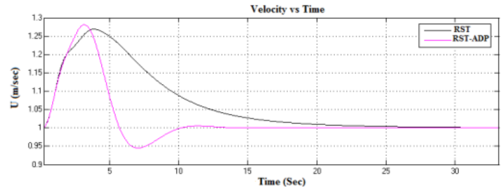
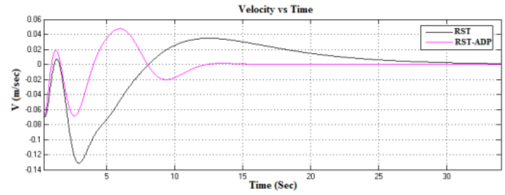


(c)

Figure 6. Comparison of RST- Adaptive based MRAC with Robust RST for Rotational Angles



(a)



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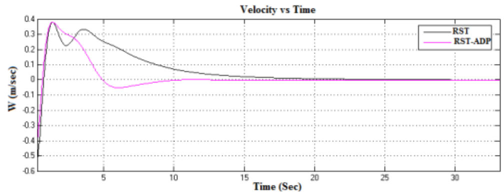

(c)

Figure 7. Comparison of RST- Adaptive based MRAC with Robust RST for Translational Velocities

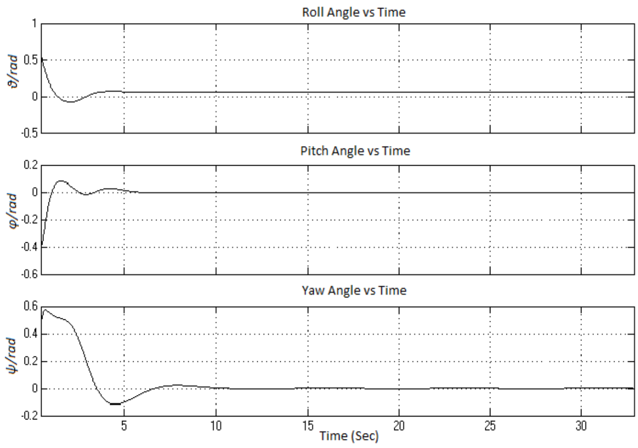

Figure 8.Proposed Controller Euler Responses

In Figure.7 the proposed control algorithm shows the robustness successfully show their responses in which the settling and rise time is settled with the desired rate as well as no overshoot and system shows completely stability.

7. Conclusion

In this article classical regulation, pole placement & tracking a controller have been implemented to control our tri-rotor translational and rotational dynamics although their angles are stabilized and controlled mathematically. The predefined model was used to figure out the actual parameters for the black box system that is used in RLS algorithm, once it identified the system, then all did was to implement it and it performed according to our expectation. Our proposed designed can be used in all kinds of rigid environment where it may be impossible for manned vehicle to reach or may be harmful to human physiology. It can be used in many fields such as assisting law and order situation, disaster management, or for mass media coverage as well.

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