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H_∞ Tracking Control for a Class of Uncertain Time-delay Systems

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Abstract

This paper considers the H_∞ tracking control for a class of uncertain systems with time-delay and disturbance. The solvable sufficient conditions for the tracking control with state time delay are presented. The tracking controller is designed by using convex combination technique and Lyapunov functional method. The controller enables the system to meet the model reference H_∞ tracking performance index. The delay-independent H_∞ tracking control criteria is derived and formulated in the form of linear matrix inequality (LMI). A numerical simulation example is presented to verify the validity of the proposed method.

Keywords*: time-delay system, H_∞ tracking Control, Lyapunov function, linear matrix inequality (LMI)*

1. Introduction

Time-delays are frequently encountered in a variety of fields, such as manufacturing systems, biology, economy and other areas [1]. The delay in a practical system can induce instability, oscillation, poor performance, and so on [2]. The analysis of the time-delay systems is the difficult and hot points in control theory domain [3]. Therefore, the problem of time-delay systems has attracted the interest of many investigators for several decades and many significant results have been reported in the literature [4-10].

Tracking control is a basic problem in control theory and engineering. It is widely applied in aircraft control and industrial process. In order to achieve good tracking effect, different methods have been used to design the tracking controller. A great amount of researches have been conducted regarding it. In [11-13], the adaptive tracking problems have been studied. The neural network has been applied to the tracking control [14]. In [15, 16], the H_∞ tracking controller for nonlinear system has been derived via fuzzy system theory. The sliding mode control has been used to research the tracking control problems [17, 18].

This paper aims to explore the H_∞ tracking control problem for a class of systems with time delay. The sufficient conditions for the tracking control will be derived in the form of LMI. Based on Lyapunov method and LMI technique, the design method of tracking controller is given via LMI. A numerical example is given to demonstrate the effectiveness of the theoretical results.

This paper is organized as follows. Section 2 gives the problem formulation. The main results on designing the H_∞ controllers are presented in Section 3. Simulation results are shown in Section 4. Section 5 contains the main conclusions.

Notations. R^n denotes the n-dimensional Euclidean space. $R^{n\times m}$ denotes the set of $n\times m$ real matrices. The notation $P \geq Q$ ($P > Q$) means that matrix $P-Q$ is positive semi-definite (positive definite), where P and Q are symmetric matrices. I

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is an identity matrix of appropriate dimensions. The superscript T denotes the transpose. $\|\cdot\|$ refers to the Euclidean vector norm. $*$ represents the symmetric form of matrix, $\begin{pmatrix} P & W \\ * & Q \end{pmatrix} = \begin{pmatrix} P & W \\ W^T & Q \end{pmatrix}$.

2. Problem Formulation

Consider the following system with time-delay and disturbances:

$$\begin{cases} \dot{x}(t) = (A+\Delta A)x(t) + (A_1+\Delta A_1)x(t-d) + (B+\Delta B)u(t) + Cw(t) \\ y(t) = C_1 x(t),\ t \in [0,\infty) \\ \phi(\theta) = x(t+\theta),\ \theta \in [-\tau, 0],\ x(0) = \phi(0) = 0 \end{cases} \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the state vector, input vector of the system, respectively; $w(t) \in R^p$ is external disturbances; $y(t) \in R^q$ is the measured output vector; $\phi(\theta)$ is a continuous vector valued initial function; $d > 0$ is the constant delay. Here A, A_1, B, C and C_1 are real constant matrices of appropriate dimensions, ΔA, ΔA_1 and ΔB are unknown matrices representing time-varying parameter uncertainties in the system model. We assume that the uncertainties are norm-bounded and can be described as:

$$\begin{pmatrix} \Delta A & \Delta A_1 & \Delta B \end{pmatrix} = EF(t)\begin{pmatrix} H & H_1 & H_2 \end{pmatrix}$$

where E, H, H_1, H_2 are known real constant matrices, and $F(t) \in \mathbb{R}^{i\times j}$ is an unknown real and possibly time-varying matrix satisfying $F^T(t)F(t) \le I$ for any given t.

For given reference model

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + r(t) \\ x_r(0) = 0 \end{cases} \tag{2}$$

and performance index

$$\int_0^{t_f} e_r^T(t) e_r(t)dt < \gamma^2 \int_0^{t_f} e_r^T \bar{w}^T(t)\bar{w}(t)dt \tag{3}$$

where $x_r(t) \in R^n$ denotes the state of reference model system; $r(t)$ denotes bounded reference input; $e_r(t) \triangleq x(t) - x_r(t)$ is the error between the actual state of the system (1) and the state of the reference model (2); $\bar{w}(t) = \left(w^T(t), r^T(t)\right)^T$, t_f is the control termination, A_r is Hurwitz matrix, $\gamma > 0$ denotes the degree of inhibition of external disturbances.

Combining (1) and (2), we have the augmented system

$$\begin{pmatrix} \dot{x}(t) \\ \dot{x}_r(t) \end{pmatrix} = \begin{pmatrix} (A+\Delta A)x(t) + (A_1+\Delta A_1)x(t-d) + (B+\Delta B)u(t) \\ A_r x_r(t) \end{pmatrix} + \begin{pmatrix} Cw(t) \\ r(t) \end{pmatrix} \tag{4}$$

Definition 1. For the augmented system (4), if there exists control input

$$u(t) = Ke_r(t) \tag{5}$$

where $K \in R^{m\times n}$ is a constant gain matrix to be designed, such that the augmented system (4) is asymptotically stable with $w(t) = 0$, and the system (4) meets the performance index (3) under zero initial conditions with $w(t) \neq 0$, then the system (1) meets H_∞ model reference tracking performance.

The objectives of this paper are: Given a scalar $\gamma > 0$, derive the tracking controller $u(t) = Ke_r(t)$ such that the time-delay system (1) has H_∞ model reference tracking performance, that is:

(i) the system (4) with $w(t) = 0$ is asymptotically stable;

(ii) the H_∞ tracking performance index (3) is guaranteed.

Before concluding this section, we introduce two lemmas which are essential for the development of our results.

Lemma 1[19] Let M and N be any appropriate dimension matrices, for any constant matrix $Q > 0$ and a scalar $\gamma > 0$, the following inequality

$$MN + N^T M^T \le \gamma^{-1} M Q^{-1} M^T + \gamma N^T Q N$$

is always satisfied.

Lemma 2[20] For given appropriate dimensions matrices Y, G, H, and Y is symmetric, $Y + HF(t)G + G^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}} < 0$ for all matrices satisfying $F^{\mathrm{T}}(t)F(t) \le I$, if and only if existing a constant $\varepsilon > 0$, such that $Y + \varepsilon HH^{\mathrm{T}} + \varepsilon^{-1}G^{\mathrm{T}}G < 0$.

Lemma 3[21] For given symmetric matrix $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$, and matrix $S_{11} \in \mathbb{R}^{r\times r}$, then the following conditions are equivalent:

(i) $S < 0$;

(ii) $S_{11} < 0$, $S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$;

(iii) $S_{22} < 0$, $S_{11} - S_{12}^{\mathrm{T}} S_{22}^{-1} S_{12} < 0$.

3. Main Results

In this section, we first present the H_∞ tracking sufficient conditions obtained by means of Lyapunov method and LMI technique.

Substituting (5) to (4), the augmented system (4) can be formulated as

$$\begin{aligned} \begin{pmatrix} \dot{x}(t) \\ \dot{x}_r(t) \end{pmatrix} &= \left[\begin{pmatrix} A+BK & -BK \\ 0 & A_r \end{pmatrix} + \begin{pmatrix} \Delta A + \Delta BK & -\Delta BK \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} x(t) \\ x_r(t) \end{pmatrix} + \\ &\left[\begin{pmatrix} A_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \Delta A_1 & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} x(t-d) \\ x_r(t-d) \end{pmatrix} + \begin{pmatrix} C & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} w(t) \\ r(t) \end{pmatrix} \end{aligned} \tag{6}$$

Let

$$\bar{A} \triangleq \begin{pmatrix} A+BK & -BK \\ 0 & A_r \end{pmatrix}, \quad \Delta\bar{A} \triangleq \begin{pmatrix} \Delta A + \Delta BK & -\Delta BK \\ 0 & 0 \end{pmatrix},$$

$$\bar{A}_1 \triangleq \begin{pmatrix} A_1 & 0 \\ 0 & 0 \end{pmatrix},\ \Delta\bar{A}_1 \triangleq \begin{pmatrix} \Delta A_1 & 0 \\ 0 & 0 \end{pmatrix},\ \bar{C} \triangleq \begin{pmatrix} C & 0 \\ 0 & I \end{pmatrix}$$

and

$$\tilde{A} \triangleq \bar{A} + \Delta\bar{A},\ \tilde{A}_1 \triangleq \bar{A}_1 + \Delta\bar{A}_1$$

Equation (6) can be rewritten as

$$\dot{\bar{x}}(t) = \tilde{A}\bar{x}(t) + \tilde{A}_1\bar{x}(t-d) + \bar{C}\bar{w}(t) \tag{7}$$

where

$$\bar{x}(t) = \begin{pmatrix} x(t) \\ x_r(t) \end{pmatrix}.$$

Theorem 1. For the augmented system (7) and given scalar $\gamma > 0$, if there exist symmetric matrices $P, S > 0$ and real matrix K such that the following inequality is satisfied:

$$\begin{pmatrix} \tilde{A}^T P + P\tilde{A} + S + \bar{Q} & P\tilde{A}_1 & P\bar{C} \\ * & -S & 0 \\ * & * & -\gamma^2 I \end{pmatrix} < 0 \tag{8}$$

where $\bar{Q} = \begin{pmatrix} I & -I \\ -I & I \end{pmatrix}$ is weighted matrix, then the system (7) has H_∞ model reference tracking performance.

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$V(\bar{x}(t)) = \bar{x}^T(t)P\bar{x}(t) + \int_{t-d}^{t} \bar{x}^T(\sigma)S\bar{x}(\sigma)d\sigma$$

where P and S are definite symmetric matrices.

When $\bar{w}(t) = 0$, taking the derivative of $V(\bar{x}(t))$ with respect to t along the trajectory of (6) yields

$$\dot{V}(\bar{x}(t)) = \xi^T(t)\begin{pmatrix} \tilde{A}^T P + P\tilde{A} + S & P\tilde{A}_1 \\ * & -S \end{pmatrix}\xi(t)$$

where

$$\xi(t) \triangleq \begin{pmatrix} \bar{x}(t) \\ \bar{x}(t-d) \end{pmatrix}.$$

From (7), we can obtain

$$\dot{V}(\bar{x}(t)) < \xi^T(t)\begin{pmatrix} -Q - \gamma^{-2}P\bar{C}\bar{C}^T P & 0 \\ * & 0 \end{pmatrix}\xi(t) \le 0$$

So the system (7) with $\bar{w}(t) = 0$ is asymptotically stable.

Next we prove $\int_0^{t_f} e_r^T(t)e_r(t)dt < \gamma^2 \int_0^{t_f} e_r^T \bar{w}^T(t)\bar{w}(t)dt$ under zero initial conditions with $\bar{w}(t) \neq 0$.

When $\bar{w}(t) \neq 0$, taking the derivative of $V(\bar{x}(t))$ with respect to t along the trajectory of (6) yields

$$\dot{V}(\bar{x}(t)) = \xi^T(t)\begin{pmatrix} \tilde{A}^T P + P\tilde{A} + S & P\tilde{A}_1 \\ * & -S \end{pmatrix}\xi(t) + 2\bar{x}^T(t)P\bar{C}\bar{w}(t) \tag{9}$$

Use Lemma 1 to obtain

$$2\bar{x}^T(t)P\bar{C}\bar{w}(t) \leq \gamma^{-2}\bar{x}^T(t)P\bar{C}\bar{C}^T\bar{x}(t) + \gamma^2\bar{w}^T(t)\bar{w}(t) \tag{10}$$

Substituting (10) to (9) gives

$$\dot{V}(\bar{x}(t)) \leq \xi^T(t)\begin{pmatrix} \tilde{A}^T P + P\tilde{A} + S + \gamma^{-2}P\bar{C}\bar{C}^T P & P\tilde{A}_1 \\ * & -S \end{pmatrix}\xi(t) + \gamma^2\bar{w}^T(t)\bar{w}(t) \tag{11}$$

From (8), we can conclude

$$\xi^T(t)\begin{pmatrix} \tilde{A}^T P + P\tilde{A} + S + \gamma^{-2}P\bar{C}\bar{C}^T P & P\tilde{A}_1 \\ * & -S \end{pmatrix}\xi(t) < \xi^T(t)\begin{pmatrix} -\bar{Q} & 0 \\ * & 0 \end{pmatrix}\xi(t) \tag{12}$$

Considering (12), we get

$$\begin{aligned} \dot{V}(\bar{x}(t)) &< -\bar{x}^T(t)\bar{Q}\bar{x}(t) + \gamma^2\bar{w}^T(t)\bar{w}(t) \\ &= -e_r^T(t)e_r(t) + \gamma^2\bar{w}^T(t)\bar{w}(t) \end{aligned} \tag{13}$$

两边从积分

$$\int_0^{t_f}\dot{V}(\bar{x}(t))dt < \int_0^{t_f} -e_r^T(t)e_r(t)dt + \gamma^2\int_0^{t_f}\bar{w}^T(t)\bar{w}(t)dt$$

Considering zero initial conditions and the positive definite of $V(\bar{x}(t))$, we can derive

$$\int_0^{t_f} e_r^T(t)e_r(t)dt < \gamma^2\int_0^{t_f}\bar{w}^T(t)\bar{w}(t)dt$$

From above derivation combining definition 1, we can conclude that the system (7) has H_∞ model reference tracking performance.

Theorem 1 provides a sufficient condition of H_∞ tracking reference model for system (1), then we give the design method of the tracking controller based on LMI method.

Theorem 2. For given scalar $\gamma > 0$, if there exist symmetric matrices $\tilde{P}, \tilde{S} > 0$, a matrix Y and a scalar $\varepsilon > 0$ such that the following LMI is satisfied:

$$\begin{pmatrix} \tilde{P}\bar{A}^T + \bar{A}\tilde{P} + \tilde{S} + \varepsilon\bar{E}\bar{E}^T & \bar{A}_1\tilde{P} & \bar{C} & \tilde{P}\bar{H}^T \\ * & -\tilde{S} & 0 & \tilde{P}\bar{H}_1^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\varepsilon I \end{pmatrix} < 0 \tag{14}$$

where

$$\tilde{P}=\begin{pmatrix}\tilde{P}_1 & 0\\ * & \tilde{P}_1\end{pmatrix},\ \tilde{S}=\begin{pmatrix}\tilde{S}_1 & 0\\ * & \tilde{S}_2\end{pmatrix},\ \bar{E}=\begin{pmatrix}E & 0\\ 0 & 0\end{pmatrix}$$

$$\bar{H}=\begin{pmatrix}H+H_2K & -H_2K\\ 0 & 0\end{pmatrix},\quad \bar{H}_1=\begin{pmatrix}H_1 & 0\\ 0 & 0\end{pmatrix}$$

Then the state feedback control law can be obtained with

$$u(t)=Y\tilde{P}_1^{-1}e_r(t) \tag{15}$$

Proof. We first rewrite (8) as

$$\Xi+\begin{pmatrix}\Delta\bar{A}^TP+P\Delta\bar{A} & P\Delta\bar{A}_1 & 0\\ * & 0 & 0\\ * & * & 0\end{pmatrix}<0 \tag{16}$$

where

$$\Xi\triangleq\begin{pmatrix}\bar{A}^TP+P\bar{A}+S & P\bar{A}_1 & P\bar{C}\\ * & -S & 0\\ * & * & -\gamma^2I\end{pmatrix}.$$

By using the relation

$$\begin{pmatrix}\Delta\bar{A} & \Delta\bar{A}_1\end{pmatrix}=\bar{E}\bar{F}(t)\begin{pmatrix}\bar{H} & \bar{H}_1\end{pmatrix}$$

where

$$\bar{E}=\begin{pmatrix}E & 0\\ 0 & 0\end{pmatrix},\ \bar{H}=\begin{pmatrix}H+H_2K & -H_2K\\ 0 & 0\end{pmatrix},\quad \bar{H}_1=\begin{pmatrix}H_1 & 0\\ 0 & 0\end{pmatrix},\ \bar{F}(t)=\begin{pmatrix}F(t) & 0\\ 0 & 0\end{pmatrix}$$

(16) can be rewritten in the form

$$\begin{aligned}&\Xi+\begin{pmatrix}\left(\bar{E}\bar{F}\bar{H}\right)^TP+P\left(\bar{E}\bar{F}\bar{H}\right) & P\bar{E}\bar{F}\bar{H}_1 & 0\\ * & 0 & 0\\ * & * & 0\end{pmatrix}\\ &=\Xi+\begin{pmatrix}P\bar{E}\\ 0\\ 0\end{pmatrix}\bar{F}\begin{pmatrix}\bar{H} & \bar{H}_1 & 0\end{pmatrix}+\begin{pmatrix}\bar{H}^T\\ \bar{H}_1^T\\ 0\end{pmatrix}\bar{F}^T\begin{pmatrix}\bar{E}^TP & 0 & 0\end{pmatrix}<0\end{aligned} \tag{17}$$

Applying Lemma 2 in (17), the matrix inequality holds for all $\bar{F}(t)$ satisfying $\bar{F}^T\bar{F}(t)\le I$ if and only if there exists a scalar $\varepsilon>0$ such that

$$\Xi+\varepsilon\begin{pmatrix}P\bar{E}\\ 0\\ 0\end{pmatrix}\begin{pmatrix}\bar{E}^TP & 0 & 0\end{pmatrix}+\varepsilon^{-1}\begin{pmatrix}\bar{H}^T\\ \bar{H}_1^T\\ 0\end{pmatrix}\begin{pmatrix}\bar{H} & \bar{H}_1 & 0\end{pmatrix}<0 \tag{18}$$

It follows from the Lemma 3 that (18) is equivalent to

$$\begin{pmatrix} \bar{A}^T P + P\bar{A} + S + \varepsilon P\bar{E}\bar{E}^T P & P\bar{A}_1 & P\bar{C} & \bar{H}^T \\ * & -S & 0 & \bar{H}_1^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\varepsilon I \end{pmatrix} < 0 \quad (19)$$

Multiplying (19) by $diag\left\{P^{-1}, P^{-1}, I, I\right\}$, we have

$$\begin{pmatrix} P^{-1}\bar{A}^T + \bar{A}P^{-1} + P^{-1}SP^{-1} + \varepsilon\bar{E}\bar{E}^T & \bar{A}_1 P^{-1} & \bar{C} & P^{-1}\bar{H}^T \\ * & -P^{-1}SP^{-1} & 0 & P^{-1}\bar{H}_1^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\varepsilon I \end{pmatrix} < 0 \quad (20)$$

Let

$$P^{-1} = \tilde{P} = \begin{pmatrix} \tilde{P}_1 & 0 \\ * & \tilde{P}_1 \end{pmatrix}, \ P^{-1}SP^{-1} = \tilde{S} = \begin{pmatrix} \tilde{S}_1 & 0 \\ * & \tilde{S}_2 \end{pmatrix}, \ Y = K\tilde{P}_1$$

we find (20) is equivalent to (14). Specially, (14) can be formulated as

$$\begin{pmatrix} \Psi & -BY & A_1\tilde{P}_1 & 0 & C & 0 & \tilde{P}_1 H^T + Y^T H_2^T & 0 \\ * & \tilde{P}_1 A_r^T + A_r\tilde{P} + \tilde{S}_2 & 0 & 0 & 0 & I & -Y^T H_2^T & 0 \\ * & * & -\tilde{S}_1 & 0 & 0 & 0 & \tilde{P}_1 H_1^T & 0 \\ * & * & * & -\tilde{S}_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{pmatrix} < 0 \quad (21)$$

where $\Psi = \tilde{P}_1 A^T + A\tilde{P}_1 + BY + Y^T B^T + \tilde{S}_1 + \varepsilon EE^T$.

From $Y = K\tilde{P}_1$, we can conclude the controller gain

$$K = Y\tilde{P}_1^{-1}$$

If there exist no uncertainties in system's matrices A, A_1 and B, we can develop a more simple result. System (1) becomes

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1 x(t-d) + Bu(t) + Cw(t) \\ y(t) = C_1 x(t), \ t \in [0, \infty) \\ \phi(\theta) = x(t+\theta), \ \theta \in [-\tau, 0], \ x(0) = \phi(0) = 0 \end{cases}$$

Corollary 1. For given scalar $\gamma > 0$, if there exist symmetric matrices $\tilde{P}, \tilde{S} > 0$, and a matrix Y such that the following LMI is satisfied:

$$\begin{pmatrix} \Lambda & \bar{A}_1\tilde{P} & \bar{C} \\ * & -\tilde{S} & 0 \\ * & * & -\gamma^2 I \end{pmatrix} < 0$$

where

$$\tilde{P} = \begin{pmatrix} \tilde{P}_1 & 0 \\ * & \tilde{P}_1 \end{pmatrix}, \ \tilde{S} = \begin{pmatrix} \tilde{S}_1 & 0 \\ * & \tilde{S}_2 \end{pmatrix},$$

$$\Lambda = \begin{pmatrix} \tilde{P}_1 A^T + A\tilde{P}_1 + BY + Y^T B^T + \tilde{S}_1 & -BY \\ * & \tilde{P}_1 A_r^T + A_r \tilde{P}_1 + \tilde{S}_2 \end{pmatrix}$$

Then the state feedback control law can be obtained with

$$u(t) = Y\tilde{P}_1^{-1} e_r(t)$$

Remark 1. Inequality (21) is a LMI in matrix variables $\tilde{P}, \tilde{S}$ and Y. Hence, the H_∞ tracking control problem can be transformed to a feasible problem of a LMI, and the latter can be solved through feasp function in LMI Toolbox.

Remark 2. We can obtain minimum the degree of inhibition for given time-delay by solving the following optimization problem

$$\begin{aligned} &\min_{\tilde{P}, \tilde{S}, Y} \gamma \\ &s.t. (21) \qquad\qquad (22) \\ &\quad \tilde{P} > 0 \end{aligned}$$

Problem (22) is a convex optimization problem with LMI constraints. We can solve it by means of mincx solver in LMI Toolbox.

4. Numerical Example

In this section, we present a simulation example to illustrate the effectiveness of the proposed method.

Consider the system described by (1) with

$$A = \begin{pmatrix} 5 & 0 \\ 4 & -4 \end{pmatrix}, \ A_1 = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}, \ B = \begin{pmatrix} -1 \\ 3 \end{pmatrix},$$

$$C = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \ A_r = \begin{pmatrix} -1.5 & 1.2 \\ 2 & 1.2 \end{pmatrix}, \ I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$E = -0.2, \ H = \begin{pmatrix} 0.2 & 0 \\ 1 & 2 \end{pmatrix}, \ H_1 = \begin{pmatrix} -1 & 5 \\ 0 & -0.3 \end{pmatrix}, \ H_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

We choose $\gamma = 4$. Solving LMI (21) yields the feasible solutions

$$P_1 = \begin{pmatrix} -1.7869 & -2.3380 \\ * & 1.3523 \end{pmatrix}, \ S_1 = \begin{pmatrix} 23.4202 & -0.3268 \\ * & 5.0961 \end{pmatrix},$$

$$S_2 = \begin{pmatrix} 1.9696 & -0.6419 \\ * & 1.0736 \end{pmatrix},\ Y = \begin{pmatrix} -0.1674 & -0.0199 \end{pmatrix},\ \varepsilon = 23.6645$$

We have derived state feedback controller (5) with gain

$$K = \begin{pmatrix} 0.0346 & 0.0451 \end{pmatrix}$$

We can obtain the minimum degree of inhibition γ^* for given time-delay system (1) by solving the optimization problem (22). The minimum degree of inhibition γ^* is

$$\gamma^* = 1.9$$

In case of when $\gamma^* = 1.9$, solving LMI (21) yields the feasible solutions

$$P_1 = \begin{pmatrix} -1.3524 & -1.4333 \\ * & 0.9049 \end{pmatrix},\ S_1 = \begin{pmatrix} 14.0490 & -0.6593 \\ * & 3.1827 \end{pmatrix},$$

$$S_2 = \begin{pmatrix} 0.6645 & -0.2420 \\ * & 0.3837 \end{pmatrix},\ Y = \begin{pmatrix} -0.0666 & -0.0015 \end{pmatrix},\ \varepsilon = 14.6852$$

Then the sate feedback controller gain with $\gamma^* = 1.9$ is

$$K = \begin{pmatrix} 0.0190 & 0.0285 \end{pmatrix}$$

From above numerical example, it can be found that the method in this paper is effective.

5. Conclusions

The problem of H_∞ tracking control for a class of uncertain time-delay systems with disturbance has been addressed. Based on Lyapunov functional method and LMI technique, the solvable sufficient conditions for the tracking control problem have been presented. The controller has been designed by using convex combination technique and LMI method. The delay-independent H_∞ tracking control criteria has been derived and formulated in the form of linear matrix inequality (LMI). A numerical simulation example has illustrated the effectiveness of the proposed method.

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