

## Modeling and Analysis of the Open Impeller Four Axis Machining System Composite Stiffness

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### Abstract

*Machining impeller parts complex curved surface, the overall stiffness of the machine tool, tool position and orientation changes, the stiffness distribution of the workpiece will affect the processing system, thereby affecting the quality of the machined surface. In this paper, four-axis NC machining centers as a example, based on multi-body small deformation theory, through the point transfer matrix, the Jacobian matrix, etc. complete the system stiffness field modeling, and the introduction of three-dimensional force ellipsoid. under the different tool spatial gesture to establish an integrated processing system stiffness field, by force ellipsoid beam analysis tools, tool position and orientation of the tool, the tool spindle with the department, blades, etc. for stiffness performance impact of the system, analysis of the distribution of the stiffness characteristics of the blade surface. Finally, experiments show that the theoretical model can effectively optimize the impeller blades processes.*

**Keywords:** *open impeller, surface, four-axis NC, force ellipsoid, stiffness field*

### 1. Introduction

In recent years, with the development of our country in the field of energy technology, national defense, aerospace and other high-tech, applied more and more widely of impeller parts, wind vane, air compressor impeller, turbine impeller and other important applications ranging from manufacturing electric power, aerospace, automotive, shipbuilding industry, etc. re the core components [1], This class is given priority to with blade impeller parts surface complex, must be the use of multi axis NC machining. Due to the complex structure of multi-axis CNC machine tools, cutting tool or workbench posture change will directly affect the comprehensive stiffness properties of machining system [2], and from the work piece movement space to tool movement space 's kinematic chain becomes longer, deformed parts of tools, machine tools and other joints, will be passed to the tool tip through the kinematic chain, and these deformations are usually magnified in the transfer process [3], influence of the stiffness of the system, especially when the high-speed machining, If the stiffness of the machining system insufficient can lead to over-cutting or make knives [4], affect the machining accuracy bring machining errors, In addition, due to the strength of the overall stiffness of the performance of the processing system will also affect the surface quality of the processed surface [5], and surface quality of the surface tend to have a direct impact on the corrosion resistance, wear resistance and anti-fatigue capability [6]. Therefore, according to the specific impeller and structure and movement of the chain of machining system and establish an integrated stiffness field model and analyze is particularly important.

As in recent years, domestic and foreign scholars study on the multi-body system stiffness modeling, Some progress has been made in the processing of CNC machine tool on the problem of dynamics [7] Modeling techniques can be divided into the

following five categories: Lumped mass, generalized coordinate method, finite element method, meshless method and substructure method, Budak [8-9], regarded the tool as a cantilever structure to calculate the stiffness matrix tool and it is used to evaluate the machining error caused by the deformation of the tool. C. Gosselin [10-11] considered the weak rigidity of the machine transmission parts, established the Jacobi matrix between the work piece coordinate system and the machine coordinates transmission parts, use the principle of virtual work to establish an integrated processing system stiffness model. Wan Min [12] considers the processing error due to the deformation of work piece and tool generated, contralateral milling process of the tool relative to the work piece at any sampling point deformation analyzed and calculated by finite element method, and the results are used for error compensation. Brecher [13] established the multi-body dynamics simulation model of three axis machining system. Luo Ming [14], Liang Ruijun [15] for the weak rigidity of multi axis NC machining of thin-walled parts and tools, Comprehensive consideration including machine tool, work piece, the processing system, through test extraction comprehensive dynamic stiffness performance of system. The above scholars through a variety of methods calculation and analysis part or the whole of the processing system stiffness characteristics, but for multi-axis machining tools or bench pose and position of the work piece impact on the overall dynamic stiffness field processing system has not thoroughly considered.

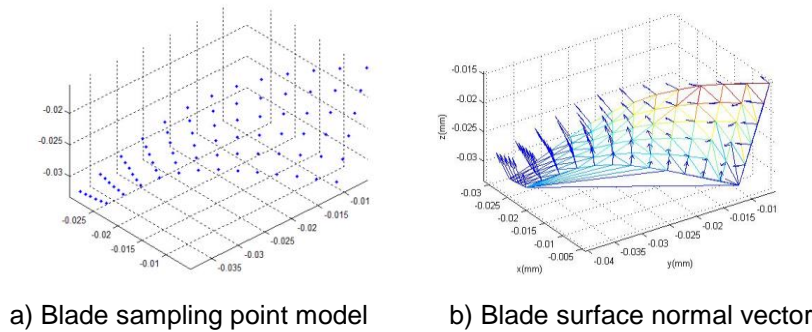
Liu Haitao [16], Considering the machine - the stiffness performance of the tool, established a multi-body dynamics finite element model of four-axis machining centers and generalized stiffness field function, by analyzing the dynamic stiffness of generalized functions, get the system integrated dynamic stiffness values under different tools pose, Chen Wei, Yan Rong, Peng Fangyu [17-19] for the seven five axis and five axis machine tool to establish a comprehensive closed loop stiffness model, by introducing three-dimensional force ellipsoid, integrated system stiffness characteristics were analyzed, in order for the tool position and orientation and cutting path optimize, but the curved surface shape of the workpiece has not been considered in the analysis process. Therefore, this article in its basic research on the application of the stiffness performance of the force generated by the ellipsoid more comprehensive study, consider cutting plane processing system and stiffness properties of the cutting plane normal, for the acquisition and analysis of composite stiffness field impeller machining system, and is of great significance to optimize its processing parameters.

## 2. Modeling of the Open Blade Stiffness Matrix

This chapter mainly studies the performance of four-axis machining rigidity open impeller, establish kinematic chain model of four axis machining of impeller system, solving the coordinate transformation matrix, then according to the general stiffness model to solve the comprehensive processing system stiffness matrix, finally, according to the different stiffness performance index to analyze system stiffness, put forward the corresponding method of stiffness performance optimization.

In the process of simulation, because there are multiple surfaces throughout the impeller, model complexity and direct analysis will lead to excessive number of samples, at the same time in the process of finite element analysis, complex grid will affect the efficiency FEM. Therefore, this paper surface reconstruction in ensuring the accuracy of the premise, analysis for only one of the blades of the impeller, based on the principle of bending, adoption adaptive sampling method [20] for sampling the surface of the blade, with the change in the surface curvature as a criteria, in the surface curvature change is more intense region, more intensive sampling points, on the contrary, in the surface curvature change is more soothing region, relatively sparse sampling points, were established 78 sampling points on the surface, Figure 1 (a)

shows, then, the sampling point use interpolation principle conduct surface reconstruction, established NURBS initial surfaces, to the surface as the research object, calculate normal vector of sampling points, Figure 1 (b) shows.



**Figure 1. Sampling Model and Normal Vector of Vane Surface**

Because in the process of machining an impeller, the impeller axis generally lies in the center of workbench. Therefore, in order to be more close to the actual processing situations, when establishing finite element model, not in the geometric center of the blade as the base coordinate, Instead, the impeller axis is set to the blade base coordinate, the workpiece coordinate system CSW, take a sampling point  $i$ , to the point  $i$  six directions are applied unit load, that  $F_x, F_y, F_z, M_x, M_y, M_z$ , Calculated in the six directions  $X, Y, Z, \theta_x, \theta_y, \theta_z$  of deformation under load per unit, finally get the transformation matrix of  $6 \times 6$ , because in the process of analysis, analyzes are carried out under the workpiece coordinate system of CSW, The transformation matrix is the sampling points of the workpiece coordinate system flexibility matrix  $S_i^{(w)}$ , for compliance matrix inverse matrix can be obtained sampling points stiffness matrix  $K_i^{(w)}$ , Similarly, to obtain compliance matrix for all points, then establish sampling points flexibility matrix database, save the coordinates of the sampling points and the compliance matrix. When needed, it can be achieved by compliance matrix inverse matrix to obtain the stiffness matrix of sampling points.

### 3. Four Axis Machining System General Stiffness Field Modeling

Assuming joint machine, tool, tool spindle junction, and the blade itself are elastic deformation and the deformation of the whole processing system must follow the small elastic deformation, by Jacobian matrix transformation can be obtained compliance matrix under the various subsystems of the workpiece coordinate system, then the compliance matrix sum by the basic principles of the deformation superposed and get comprehensive compliance matrix processing system:

$$\begin{aligned}
 S^{(w)} &= S_j^{(w)} + S_{cutter}^{(w)} + S_{touch}^{(w)} + S_i^{(w)} \\
 &= J_j^{(w)} \left( K_j^{(j)} \right)^{-1} \left( J_j^{(w)} \right)^T + J_t^{(w)} \left( K_{cutter}^{(t)} \right)^{-1} \left( J_t^{(w)} \right)^T + J_t^{(w)} \left( K_{touch}^{(t)} \right)^{-1} \left( J_t^{(w)} \right)^T + \left( K_i^{(w)} \right)^{-1} \quad (1)
 \end{aligned}$$

Then comprehensive compliance matrix inverse matrix, You can get a comprehensive four-axis machining system stiffness matrix  $K^{(w)}$ , Among them, the compliance matrix of each subsystem to the workpiece coordinate system transformation Jacobian matrix solver is as follows :

1) Jacobian matrix  $J_j^{(w)}$  on behalf of from the joint motion of space under the workpiece coordinate system processing system to the tool movement space transform the relationship between speed , for the  $6 \times 4$  order matrix:

$$J_j^{(w)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \cos(A) \cdot (L_{z,z} - L_{xy,z}) - s_y \sin(A) - s_z \cos(A) & \cos(A) & 0 & -\sin(A) \\ \sin(A) \cdot (L_{z,z} - L_{xy,z}) - s_z \sin(A) + s_y \cos(A) & \sin(A) & 1 & \cos(A) \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

2) Jacobian matrix  $J_t^{(w)}$  represents the cutting tool and the tool spindle fringe compliance matrix transformation relations from the tool site coordinate system CST to the workpiece coordinate system CSW, for the  $6 \times 6$  order matrix,

and  $J_t^{(w)} = \begin{bmatrix} R_t^w & 0 \\ 0 & R_t^w \end{bmatrix}_{6 \times 6}$ , then:

$$J_t^{(w)} = \begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & \cos(A) & -\sin(A) & & 0 & \\ 0 & \sin(A) & \cos(A) & & & \\ & & & 1 & 0 & 0 \\ & 0 & & 0 & \cos(A) & -\sin(A) \\ & & & 0 & \sin(A) & \cos(A) \end{bmatrix} \quad (3)$$

#### 4. Open Blade Four Axis Machining System General Stiffness Field Analysis

After establish the general stiffness field model of four axis machining system, selected sampling points on the impeller  $i$  were analyzed for the study object, position of the sample point as shown in Figure 2.

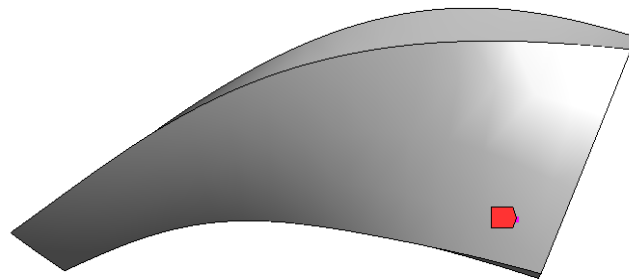
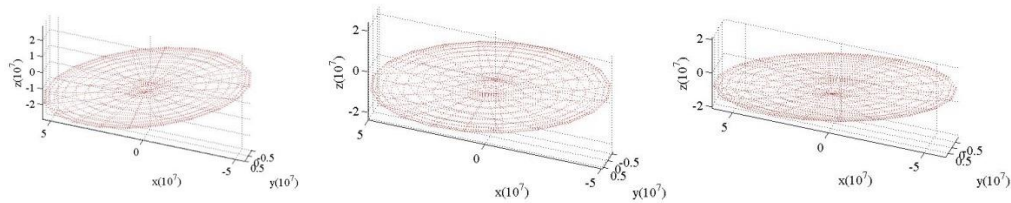


Figure 2. Position of Sampling Point

Sampling point coordinates under the workpiece coordinate system is  $(-1.86E-02, -6.76E-03, -5.35E-03)$ , after a review of the sampling points to obtain compliance matrix database sampling point  $i$  in the compliance matrix is:

$$S_i^{(w)} = \begin{bmatrix} 4.16E-10 & 1.89E-09 & 7.65E-10 & 4.35E-09 & 9.35E-09 & -1.72E-10 \\ -5.34E-09 & 1.07E-08 & -7.64E-09 & 1.23E-08 & -5.25E-08 & -1.21E-08 \\ 3.21E-09 & -4.97E-09 & 7.54E-09 & -8.91E-07 & 6.27E-07 & 5.89E-10 \\ -5.83E-09 & 1.05E-08 & -1.07E-07 & 1.46E-07 & -8.17E-07 & -1.22E-07 \\ 2.97E-09 & -5.30E-09 & 7.04E-09 & -8.68E-09 & 5.76E-08 & 6.14E-08 \\ 5.16E-09 & -9.39E-09 & 7.45E-09 & -1.14E-08 & 5.26E-06 & 1.08E-07 \end{bmatrix}$$

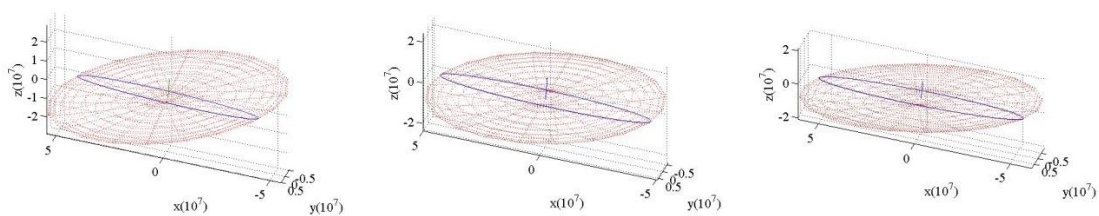
Then coordinates of sampling points and compliance matrix and tools, tool spindle with the department the machine joint flexibility matrix Substituting equation (1) obtained a comprehensive compliance matrix, establish a comprehensive stiffness field model to analyze, the A-axis angle is preset to 0°, 20°, 30° the remaining parameters fixed, calculation of sampling points  $i$  comprehensive stiffness matrix, for stiffness matrix integrated decoupling, then the matrix transformation, generate force ellipsoid under different angle, As shown in Fig.3, analysis to different A shaft angle to the blade system synthetical stiffness of the sampling points.

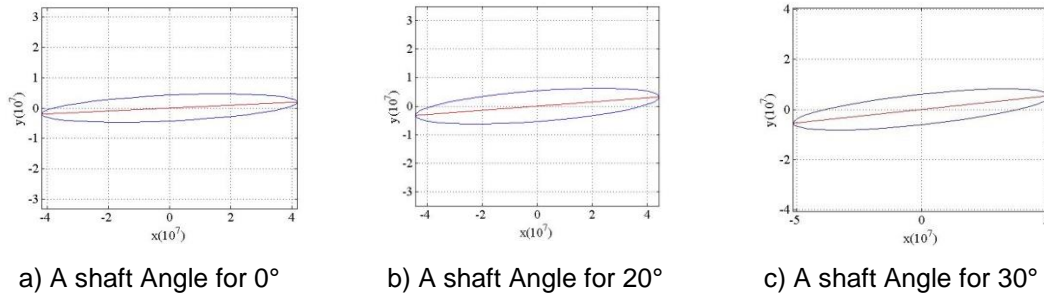


a) A shaft Angle for 0°      b) A shaft Angle for 20°      c) A shaft Angle for 30°

**Figure 3. Force Ellipsoid of Sampling Points on Different A-Axis Angle**

The minor axis of the ellipsoid, respectively 1.45e6, 2.21e6, 2.76e6 when through calculating the A shaft speed is 0°, 20°, 30°, the force ellipsoid shortest half shaft is the largest when the A shaft Angle is 30°, therefore, when the workbench Angle is A = 30°, comprehensive stiffness properties of the point is the best, then the surface shape of blade introduce into the force ellipsoid, by calculating get the point  $i$  the normal vector (-0.067,0.35,0.95), to the normal vector in force ellipsoid drawing this point in the different A shaft angle, intersection line of cutting plane and force ellipsoid, the line of intersection of an ellipse, then draw two-dimensional map of the surface of the ellipse, calculating the optimum cutting direction as is shown in Figure 4.

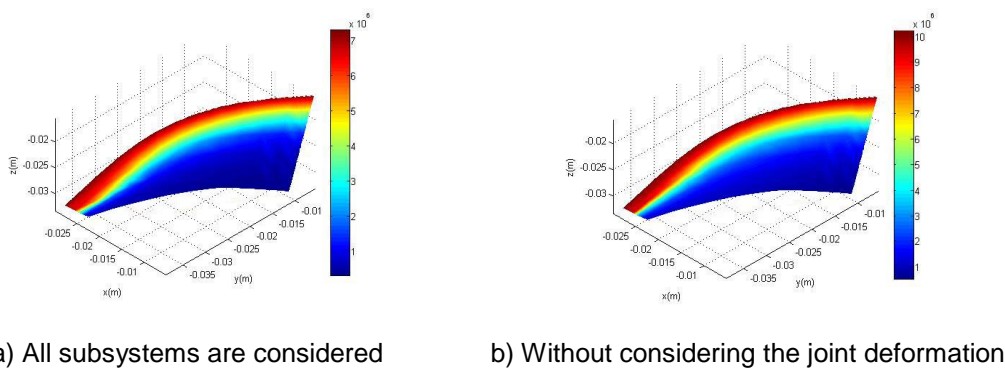


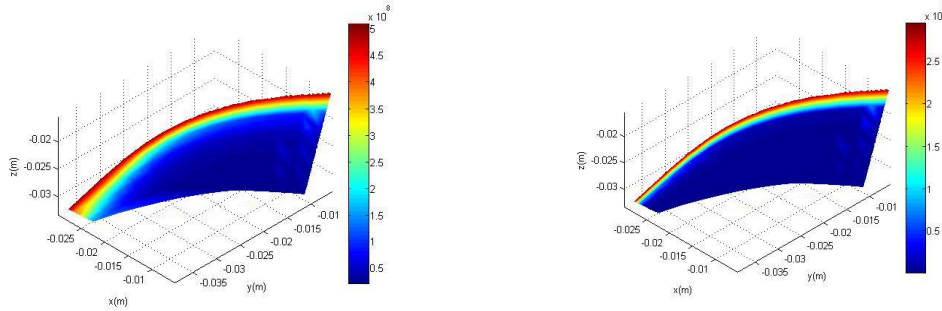


**Figure 4. Stiffness of Cutting Plane of Sampling Points on Different A-Axis Angle**

In Figure 4, point  $i$  is located in the coordinate system origin, blue arrow direction for surface in normal sampling points  $i$ , blue oval represent that the plane cutting stiffness performance, elliptical surface along a certain direction of axial length according to the sampling point  $i$  in the direction of cutting plane stiffness performance, the red solid line direction for the long axis of the oval face, represent processing along the direction of stiffness performance is best, the best feed direction. Due to the above have concluded that when the workbench angle is  $A = 30^\circ$ , the comprehensive stiffness performance of the sample point is best. As a result, by the calculation, when A shaft Angle of  $30^\circ$ , there is a parallel to the Z axis to the plane of the X axis Angle is  $6.1166^\circ$  and the elliptical surface line coincide with elliptical long axle load, In processing the sampling spots  $i$ , With the X axis angle of about  $6.1166^\circ$  in the vertical plane and the intersection of a cutting plane direction for that point the best feed direction, Along this direction processing system of comprehensive stiffness performance, the best processing and the most stable, the direction also available space vectors for (4.52, 1.01, 0.27).

In order to obtain the whole blade stiffness performance cloud, obtain the blade stiffness distribution of performance, the next build comprehensive stiffness field model of the whole leaves, calculate all sampling points of composite stiffness matrix. According to different problems, select different stiffness performance indicators, by the method of interpolation, Synthesize point to surface. Select ellipsoid short half shaft  $\lambda$  stiffness performance indicators, respectively, depending on the joints, the integration of a cutting tool, cutting tool spindle will not occur in the manufacturing process of deformation, namely, depending on the stiffness of infinity, through multiple sets of calculation, establish a comprehensive stiffness properties cloud, as shown in Figure 5, Looking for machining system integrated stiffness contribution to the larger part of the performance, then carries on the optimization and improvement.





c) Without considering the deformation of the tool

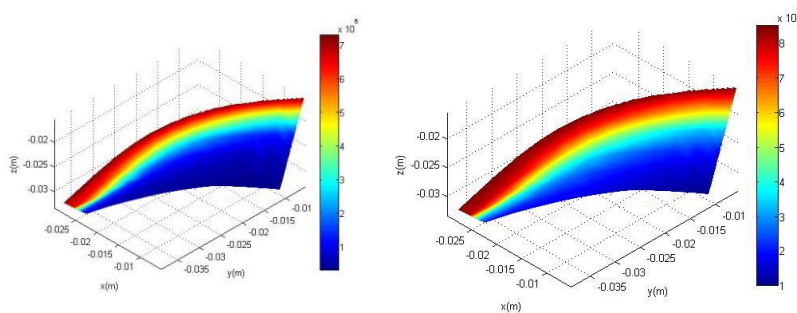
d) Without considering the tool spindle combined deformation

**Figure 5. Stiffness Nephogram without One of the Main Part**

By contrast, when not considering machine tool joint deformation, joint stiffness being infinite, the composite stiffness cloud of system and data and the distribution, which consider machine tool joint deformation, does not change much, showing that machine tool joint has a little contribution to composite stiffness field in processing system. However, when not considering cutter deformation, cutter stiffness being infinite, composite stiffness of system changes much, which is the same with the situation when not considering cutter spindle joint deformation. Therefore, cutter and spindle joint have a great contribution to composite stiffness field in processing system. This dissertation try to use carbide ball-nose milling cutter to machine open impeller of vane. Then the simulation study is made by replacing cutter of higher strength, in consideration of great contribution to composite stiffness field by cutter stiffness and the operability of replacing cutter. The data which is before and after replacement of the cutting is shown in Table 1, and the nephogram of composite stiffness of system is shown in Figure 6.

**Table 1. Parameter Table of Cutters**

	Tool type	Diameter (mm)	Overall Length (mm)	Blade length (mm)	Blade length (mm)	Ball radius (mm)	Pitch (mm)
Replace the front	R216.64-10030-A011G	6	100	3	30	3	35.50
After the replacement	R216.64-08030-A009G	8	100	3	36	4	45.00



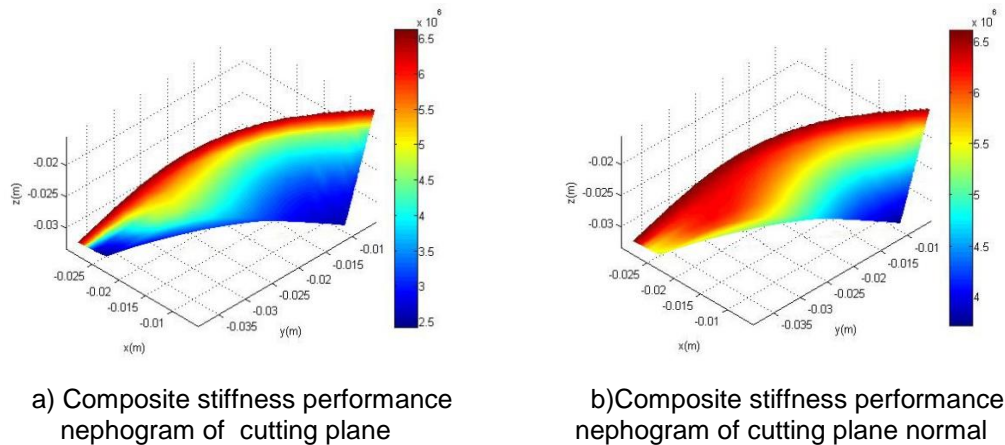
a) Before replacing the tool

b) After replacing the tool

**Figure 6. Stiffness Nephogram with Different Cutters**

By comparison, after replacing cutter of higher strength, we can see that it enlarges the range of strong stiffness of vanes, and the stiffness of the blade root increases by almost 21%, while, the blade tip which is relatively weak, increase less. It shows that

the option for cutter has a great affection for the blade root, but not for the blade tip. Then by calculating the normal vector of blades sampling points, the curved shape of the blade stiffness is introduced to composite stiffness, considering composite stiffness of blade curved surface in the cutting plane as well as the normal cutting plane. Taking respectively the semi-minor axis of ellipsoid which is intersected by cutting plane and force ellipsoid and the semi-minor axis length of cutting plane in ellipse ellipsoid as the stiffness index, it is made the composite stiffness performance nephogram, shown in Figure 7.



**Figure 7. Stiffness Nephogram of Cutting Plane and Normal Plan**

Analysis can be seen from the figure, during the process, because of the weak stiffness in cutter plane, it is not suitable for blade tip to increase the feed rate, which influences the processing efficiency. However, the stiffness performance of the blade tip closed to hub in the normal cutting plane is lower, which means this position is easily deformed and it greatly affect machining accuracy. Therefore, during the process, it is better to strengthen the clamping in the blade tip, constraining its deformation, thereby, enhancing the composite stiffness of the machining system.

## 5. Conclusion

This dissertation established a composite stiffness field model of impeller machining system, which was used to analyze the distribution of stiffness performance and to search the internal relations of the worktable pose of cutter, cutting paths and the stiffness characteristics of machining system. This model not only can be used for processing personnel to understand the anti-distortion capacity of impeller, but also can be used to evaluate the potential machine vibration that may affect the machining accuracy during machining procedure and then help personnel to take effective measures. They set the best stiffness performance of machining system as a goal by optimizing cutters, worktable poses, and cutting paths.

(1) Without considering the specific work pieces to analyze the stiffness performance of 4-axis machining system, taking them respectively at  $0^\circ$ ,  $20^\circ$ ,  $30^\circ$  horizontal and vertical surface in the A-axis. It is found that the stiffness performance of the system is the best when  $A=0^\circ$  in the horizontal and vertical surface.

(2) Considering the specific open blade to optimize the rotation angle of blade sampling points  $(-1.86E-02, -6.76E-03, -5.35E-03)$  in the A-axis during processing. It is found that the optimal stiffness performance of the A-axis when turning to  $30^\circ$  rotation angle, and then can calculate the optimal feed direction  $(4.52, 1.01, -0.27)$ .

(3) Based on the composite stiffness performance of the open blade nephogram, showing that machine tool joint has a little contribution to composite stiffness field in



processing system. While cutter and spindle joint have a great contribution to composite stiffness field in machining system. After replacing the cutter, we can see that the stiffness of the blade root increases by almost 21%, the blade tip which is relatively weak, increase less. It shows that the option for cutter has a great affection for the blade root, but not for the blade tip.

(4) Taking the curved shape of the blade to the analysis of stiffness composite field, and then considering composite stiffness of blade curved surface in the cutting plane as well as the normal cutting plane. We can see that during the process, because of the weak stiffness in cutter plane, it is not suitable for blade tip to increase the feed rate; while the stiffness performance of the blade tip closed to hub in the normal cutting plane is lower, which means this position is easily deformed and it greatly affect machining accuracy.

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