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Strap-down Inertial Navigation System Initial Alignment Based on Iterative Square-Root Cubature Kalman Filter

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Abstract

This paper presents an improved Cubature Kalman filter (CKF) for strap-down inertial navigation system (SINS) initial alignment. In order to enhance the stability and precision of the filter, Newton iteration algorithm and square-root strategy are blended into CKF. Iterative Square-root Cubature Kalman Filter (ISRCKF)makes full use of latest measurement information and directly employs the square root of covariance matrix for recursive update in the iterative process. ISRCKF can effectively avoid non-positive covariance matrix and improve the estimation accuracy. Simulation results show that ISRCKF performs well for SINS initial alignment with large azimuth misalignment.

Keywords*: SINS, Initial alignment, CKF, Iteration, Square-root*

1. Introduction

With the rapid development of modern technology, SINS has been quickly developed and widely used for navigation in recent years. Initial alignment is one of the key problem which would influence SINS properties. The error of initial alignment is the main error sources of SINS. When the azimuth misalignment angle is large, error model of SINS does not satisfy linear condition, thus nonlinear filter is needed [1]. As a commonly-used nonlinear filter, EKF works as the system model is approximate linear and could reach first order accuracy only. The estimation accuracy of Unscented Kalman Filter (UKF) is better than that of EKF. But the accuracy decreases when the dimension of system increases. CKF needs not to calculate the Jacobian matrix. It could reach higher precision than EKF and (UKF). However, when the filter works, non-positive covariance matrix may appear, which affects the stability and accuracy. In order to solve this problem, ISRCKF is proposed. The novel CKF integrated with square-root and iterative algorithm can ensure positive definiteness of covariance matrix and improve the efficiency of filter process [2-4]. The performance of ISRCKF for initial alignment is better than CKF.

2. Nonlinear Model of SINS

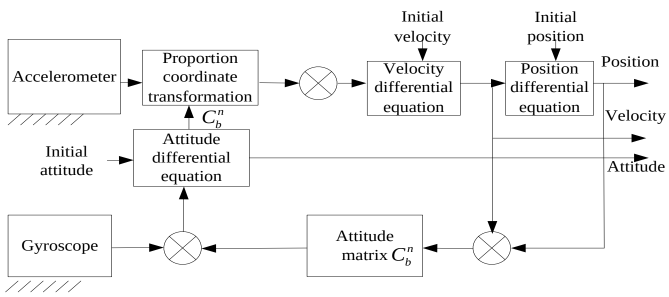

Figure 1. Algorithm Flow of SINS

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Initial alignment should be made before SINS works. The precision and speed of filtering process for initial alignment are directly related to the SINS accuracy and starting performance. The purpose of initial alignment is to establish the initial value of the attitude matrix. And the estimation of misalignment angle is the core. Error equation of SINS reflects the relationship between inertial components error and positioning error. The algorithm flow of SINS is shown in figure 1.

On the basis of literature, SINS error model is [5]:

$$\delta\dot{\boldsymbol{v}} = [\boldsymbol{I} - (\boldsymbol{C}_t^{t'})^T]\boldsymbol{C}_b^{t'}\hat{\boldsymbol{f}}^b + \boldsymbol{C}_b^t\delta\boldsymbol{f}^b - (2\hat{\boldsymbol{\omega}}_{ie}^t + \hat{\boldsymbol{\omega}}_{et}^t)\times\delta\boldsymbol{v} - (2\delta\boldsymbol{\omega}_{ie}^t + \delta\boldsymbol{\omega}_{et}^t)\times\boldsymbol{v} + \delta\boldsymbol{g} \quad (1)$$

$$\dot{\boldsymbol{\phi}} = (\boldsymbol{I} - \boldsymbol{C}_t^{t'})\hat{\boldsymbol{\omega}}_{it}^t + \delta\boldsymbol{\omega}_{it}^t - \boldsymbol{C}_b^{t'}\delta\boldsymbol{\omega}_{ib}^b \quad (2)$$

$\delta\boldsymbol{v}$ is velocity error, $\boldsymbol{\phi}$ is misalignment angle, $\boldsymbol{C}_b^t$ is direction cosine matrix from carrier coordinate system to geography coordinate system. $\boldsymbol{C}_b^{t'}$ is direction cosine matrix from carrier coordinate system to calculated geography coordinate system. $\hat{\boldsymbol{f}}^b$ is real proportional output, $\delta\boldsymbol{f}^b$ and $\delta\boldsymbol{\omega}_{ib}^b$ are measurement error, $\hat{\boldsymbol{\omega}}_{ie}^t$ is earth rotation angle rate in calculated geography coordinate system, $\hat{\boldsymbol{\omega}}_{et}^t$ is rotation angle rate relative to geography coordinate system, $\hat{\boldsymbol{\omega}}_{it}^t$ is rotation angle rate relative to inertial coordinate system, $\delta\boldsymbol{g}$ is error of gravity acceleration.

Suppose $\delta\boldsymbol{\omega}_{ib}^b$ is gyroscope measurement error, $\boldsymbol{\varepsilon}^b$ is constant drift, $\boldsymbol{w}_g^b$ is Gaussian white noise with nonzero mean, the measurement error of accelerometer is constant zero-bias $\boldsymbol{\nabla}^b$ and white noise with zero mean $\boldsymbol{w}_a^b$. $\delta\boldsymbol{g}$ is neglected. The output of accelerometer is $[f_x\ f_y\ f_z]^T = \boldsymbol{C}_b^{t'}\hat{\boldsymbol{f}}^b, \boldsymbol{v} = 0$, the nonlinear model of SINS initial alignment is:

$$\begin{aligned}\delta\dot{v}_x = &-f_x(\cos\phi_z - 1) + f_y\sin\phi_z - f_z(\phi_y\cos\phi_z + \phi_x\sin\phi_z) + \\ &2\omega_{ie}\sin\varphi\delta v_y + C_{11}(\nabla_x^b + w_{ax}^b) + C_{12}(\nabla_y^b + w_{ay}^b)\end{aligned} \quad (3)$$

$$\begin{aligned}\delta\dot{v}_y = &-f_x\sin\phi_z - f_y(\cos\phi_z - 1) - f_z(\phi_y\sin\phi_z - \phi_x\cos\phi_z) \\ &-2\omega_{ie}\sin\varphi\delta v_x + C_{21}(\nabla_x^b + w_{ax}^b) + C_{22}(\nabla_y^b + w_{ay}^b)\end{aligned} \quad (4)$$

$$\begin{aligned}\dot{\phi}_x = &-\sin\phi_z\omega_{ie}\cos\varphi + \phi_y\omega_{ie}\sin\varphi - \delta v_y / R_m + \\ &C_{11}^{'}(\varepsilon_x^b + w_{gx}^b) + C_{12}^{'}(\varepsilon_y^b + w_{gy}^b) + C_{13}^{'}(\varepsilon_z^b + w_{gz}^b)\end{aligned} \quad (5)$$

$$\begin{aligned}\dot{\phi}_y = &(1 - \cos\phi_z)\omega_{ie}\cos\varphi - \phi_x\omega_{ie}\sin\varphi + \delta v_x / R_n + \\ &C_{21}^{'}(\varepsilon_x^b + w_{gx}^b) + C_{22}^{'}(\varepsilon_y^b + w_{gy}^b) + C_{23}^{'}(\varepsilon_z^b + w_{gz}^b)\end{aligned} \quad (6)$$

$$\begin{aligned}\dot{\phi}_z = &(-\phi_y\sin\phi_z + \phi_x\cos\phi_z)\omega_{ie}\cos\varphi + \delta v_x\tan\varphi / R_n + \\ &C_{31}^{'}(\varepsilon_x^b + w_{gx}^b) + C_{32}^{'}(\varepsilon_y^b + w_{gy}^b) + C_{33}^{'}(\varepsilon_z^b + w_{gz}^b)\end{aligned} \quad (7)$$

$$\dot{\varepsilon}_i^b = 0 \quad (i = x, y, z) \quad (8)$$

$$\dot{\nabla}_i^b = 0 \quad (i = x, y) \quad (9)$$

The state equation and the measurement equation are:

$$\begin{cases} \dot{X} = f(X) + G(X)W \\ Z = HX + V \end{cases} \tag{10}$$

Where, the state variable is $X = [\delta v_X \ \ \delta v_Y \ \ \Delta\alpha_E \ \ \Delta\alpha_N \ \ \Delta\alpha_U \ \ \varepsilon_X \ \ \varepsilon_Y \ \ \varepsilon_Z \ \ \nabla_X \ \ \nabla_Y]^T$, $W = [w_{aX} \ \ w_{aY} \ \ w_{gX} \ \ w_{gY} \ \ w_{gZ} \ \ 0_{1\times5}]^T$.

Use velocity error δv_X, δv_Y to be observation variable. $H = [I_{2\times2} \ \ 0_{2\times8}]$, V is observation noise.

3. ISRCKF

ISRCKF is also based on Cubature rule. Spherical-radial cubature criterion is used to calculate the integration. The calculation of mathematical expectation of the integration with same dimension is in this form [6]:

$$I(f) = \int_{R^n} f(x)\exp(-x^T x)dx \tag{11}$$

Where, $f(\cdot)$ is arbitrary function, R^n is integral region. The computing values of the integration above are not easy to get, so approximate method is needed.

Spherical-radial cubature criterion uses spherical radial transform to calculate equation (11). Take $x = ry(y^T y = 1, r \in [0,\infty))$, in spherical-radial coordinate, equation (11) can be written:

$$I(f) = \int_0^\infty \int_{U_n} f(ry) r^{n-1} e^{-r^2} d\sigma(y) dr \tag{12}$$

Where, U_n is unit sphere of n dimension, $\sigma(\cdot)$ is the element of U_n.

The integration can be calculated by Spherical-radial rule:

$$I_N(f) \approx \frac{1}{\sqrt{\pi^n}}(\frac{\sqrt{\pi^n}}{2n}\sum_{i=1}^{2n} f(\sqrt{2}\sqrt{\frac{n}{2}}[1]_i) = \sum_{i=1}^{m}\omega_i f(\xi_i) \tag{13}$$

Where: ξ is cubature point, $\xi_i = \sqrt{\frac{m}{2}}[1]_i, m = 2n$. ω_i is the corresponding weights, $\omega_i = \frac{1}{m}, (i = 1,...m)$. So CKF selects $2n$ cubature points and Gauss integration with the same weight.

The Square-root strategy for CKF is as follows [7-8]:

(1) Triangular decomposition

The covariance matrix can be written in the form: $P = BB^T$. The orthogonal decomposition of B:

$$B^T = QR \tag{14}$$

$$P = BB^T = R^T QQR^T = R^T R = S^T S \tag{15}$$

(2)Time update

Square-root CKF makes the volume point:

$$x_i(k-1|k-1) = S(k-1|k-1)\zeta_i + \hat{x}(k-1|k-1) \tag{16}$$

Where $S(k-1|k-1)$ is the square-rooting matrix of error covariance at the moment $k-1$, $\hat{x}(k-1|k-1)$ is the optimal estimation.

The transmission of the volume point is:

$$\tilde{x}_i(k-1|k-1) = f(x_i(k-1|k-1)) \tag{17}$$

The estimation of state vector is:

$$\hat{x}(k|k-1) = \frac{1}{m}\sum_{i=1}^{m} \tilde{x}_i(k|k-1) \tag{18}$$

Where: $\zeta_i = \begin{cases} \sqrt{m/2}\,\varepsilon_i, i = 1, 2 \cdots n \\ -\sqrt{m/2}\,\varepsilon_i, i = n+1, n+2 \cdots 2n \end{cases}$ $(i = 1, 2, \cdots m)$ and $m = 2n$. n is the dimension of state vector..

The square root of forecast error covariance is :

$$S(k|k-1) = tri[\tilde{X}(k|k-1)S_Q(k)] \tag{19}$$

QR decomposition method is used and tri represents triangle operation. S_Q is the square root of process noise covariance.

Where:

$$\tilde{X}(k|k) = \frac{1}{\sqrt{m}}[\tilde{X}_1(k|k-1) - \hat{x}(k|k-1), \tilde{X}_2(k|k-1) - \hat{x}(k|k-1), \cdots \tilde{X}_m[(k|k-1) - \hat{x}(k|k-1)]^T.$$

(3) Measurement update

Volume point calculating is:

$$x_i(k|k-1) = S(k|k-1)\zeta_i + x(k|k-1) \tag{20}$$

$$z_i(k|k-1) = M(k)x_i(k|k-1) \tag{21}$$

Measurement forecast is:

$$\hat{z}(k|k-1) = \frac{1}{m}\sum_{i=1}^{m} z_i(k|k-1) \tag{22}$$

Square root of forecast covariance matrix is:

$$S_{zz}(k|k-1) = tri[z(k|k-1)S_R(k)] \tag{23}$$

Where: $z(k|k-1) = \frac{1}{\sqrt{m}}[z_1(k|k-1) - \hat{z}(k|k-1), z_1(k|k-1) - \hat{z}(k|k-1), \cdots z_m(k|k-1) - \hat{z}(k|k-1)]$ and S_R is the square root of measurement noise covariance.

Cross-covariance matrix is:

$$P_{xz}(k|k-1) = x(k|k-1)z^T(k|k-1) \tag{24}$$

Where: $$x(k|k)=\frac{1}{\sqrt{m}}[x_1(k|k-1)-\hat{x}(k|k-1),x_2(k|k-1)-\hat{x}(k|k-1),\ldots x_m(k|k-1)-\hat{x}(k|k-1)].$$

Gain matrix is:

$$K(k)=P_{xz}(k|k-1)[S_{zz}^T(k|k-1)]^{-1}\times S_{zz}(k|k-1)^{-1} \qquad (25)$$

State update is:

$$\hat{x}(k|k)=\hat{x}(k|k-1)+K(k)[z(k)-\hat{z}(k|k-1)] \qquad (26)$$

Square root of calculated covariance matrix is:

$$S(k|k)=tri[x(k|k-1)-K(k)z(k)-\hat{z}(k|k-1)]\times K(k)S_R(k) \qquad (27)$$

Newton iteration algorithm is introduced into Square-root CKF(SRCKF).When SRCKF comes into measurement update, Newton iteration is used to determine the maximum likelihood estimation. The likelihood function of variable x_k and z_k is:

$$p(z_k|x_k)=\exp\{-\frac{1}{2}[(x_k-x_{k|k-1})^T P_{k|k-1}^{-1}(x_k-x_{k|k-1})+ (z_k-h(x_k))^T R_k^{-1}(z_k-h(x_k)]\} \qquad (26)$$

Cost function is:

$$C(x_k)=(x_k-x_{k|k-1})^T P_{k|k-1}^{-1}(x_k-x_{k|k-1})+(z_k-h(x_k))^T R_k^{-1}(z_k-h(x_k)) \qquad (27)$$

The minimum of the cost function can be calculated by Newton iteration algorithm:

$$\overset{i+1}{x}_{k|k-1}=x_{k|k-1}+P_{k|k-1}(H_k^i)^T(H_k^i P_{k|k-1}(H_k^i)^T+ R_k)^{-1}[\,z_k-h(\overset{i}{x}_{k|k})-H_k^j(x_{k|k-1}-\overset{i}{x}_{k|k})] \qquad (28)$$

Auto-correlation covariance P_{zz}^i and cross-correlation covariance P_{xz}^i can be calculated by SRCKF. Measurement update matrix is:

$$H_k^i=(P_{xz,k|k-1}^i)^T(P_{k|k-1})^{-1} \qquad (29)$$

Put equation(29)into equation(28) can finish iteration update [9-10].

4. Simulation and Experiment

The parameters of simulation is: initial value of system state $\boldsymbol{X}(0)=0$.Gyro drift is $0.1^\circ/h$. Accelerometer bias is $1\times10^{-4}g$. Initial variance matrix is $1\times10^{-4}g$. System noise matrix is $\boldsymbol{Q}$. Measurement noise matrix is $\boldsymbol{R}$:

$$\boldsymbol{P}(0)=diag\left\{(0.1m/s)^2(0.1m/s)^2(1^\circ)^2(1^\circ)^2(15^\circ)^2(0.1^\circ/h)^2 (0.1^\circ/h)^2(0.1^\circ/h)^2(100ug)^2(100ug)^2\right\},$$

$$\boldsymbol{Q}=diag\left\{(0.05^\circ/h)^2\ (0.05^\circ/h)^2(0.05^\circ/h)^2(50ug)^2\ (50ug)^2\ 0_{5\times1}\right\},$$

$$\boldsymbol{R} = \left\{ (0.1m/s)^2 \ (0.1m/s)^2 \right\}.$$

The simulation results are shown in figure 2-4:

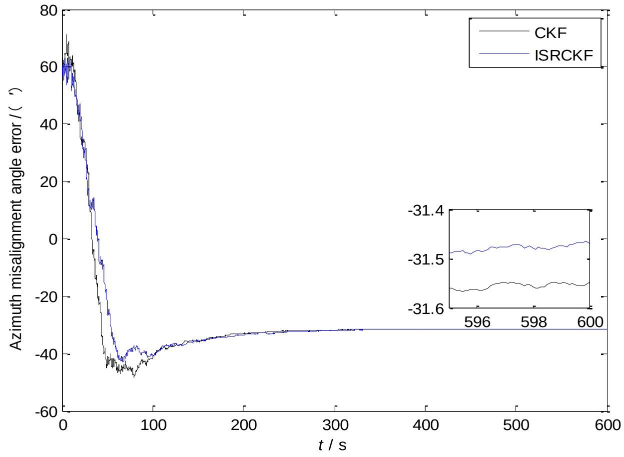

Figure 2. Azimuth Misalignment Angle Error

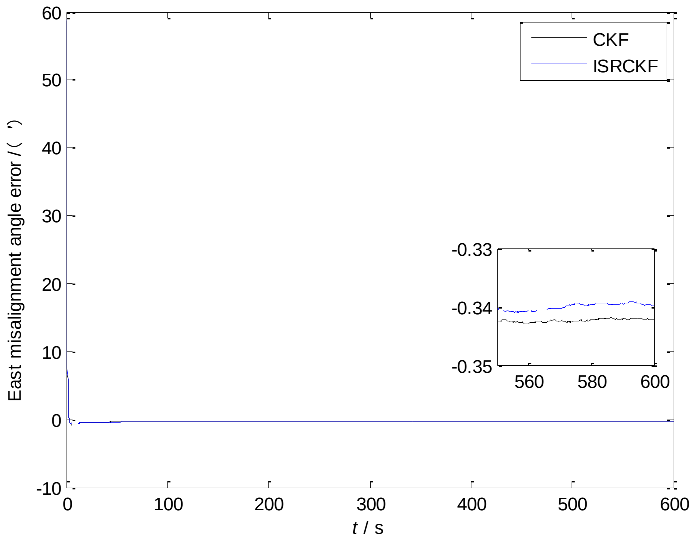

Figure 3. East Misalignment Angle Error

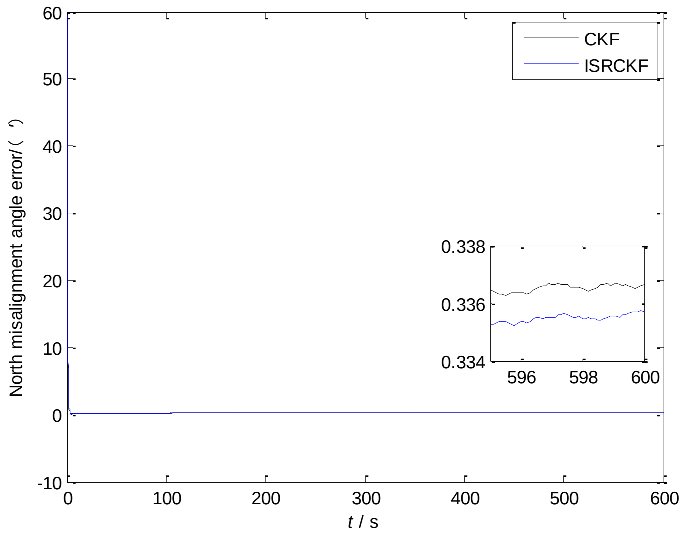

Figure 4. North Misalignment Angle Error

Both ISRCKF and CKF can estimate the misalignment angle. The convergence rate of ISRCKF and CKF is nearly the same. Results for comparison are shown in table1:

Table 1 Error of Misalignment Angle (')

	East	North	Azimuth
ISRCKF	-0.2557	0.2951	-28.63
CKF	-0.2709	0.2807	-31.58
Theoretical value	-0.2671	0.3123	-26.39

The experiment uses inertial measurement unit (IMU) with three fiber optic gyroscope and three quartz accelerometer. The gyro precision is $0.01^{\circ}/h$. The accelerometer precision is $5\times10^{-5}g$ .IMU is shown in figure 5:

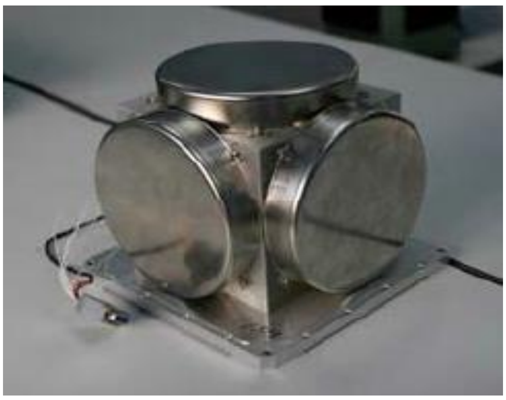

Figure 5. Inertial Measurement Unit

The IMU is put on the inertial testing turntable. In the experiment, the East and North misalignment angle converges fast, and the estimation accuracy is high. The azimuth misalignment angle estimation error is large and converges slow. The azimuth misalignment angle error curve is shown in figure 6:

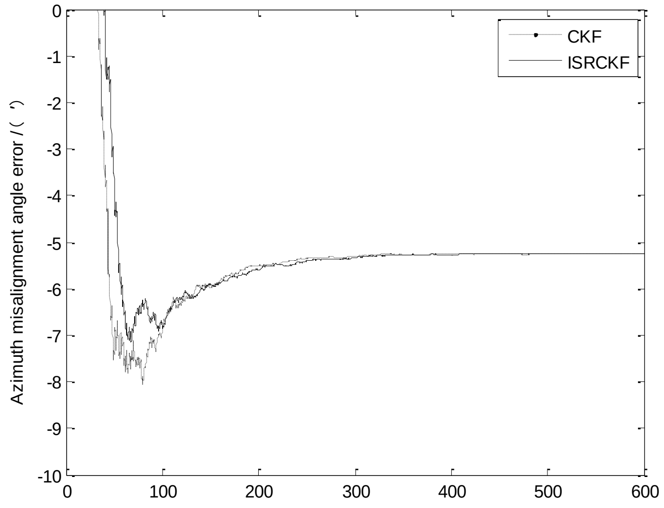

Figure 6. Azimuth Misalignment Angle Error

Experiment results also show that the azimuth misalignment angle error of ISRCKF is less than that of CKF.

5. Conclusion

Initial alignment is made respectively by CKF and SRCKF. Simulation and experiment shows that ISRCKF can reduce the error of misalignment angle and has a better performance than CKF. ISRCKF is based on cubature rule and it calculates the mean and covariance of nonlinear function directly. By introducing the triangular decomposition and Newton iteration algorithm into CKF, ISRCKF avoids non-positive covariance matrix, which enhances the stability of the filter. It makes full use of measurement information and it improves the estimate accuracy. ISRCKF is a better filter for initial alignment than CKF.

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