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Synchronous Control of Second Order Linear System Based on Sliding Mode Method

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Abstract

In order to solve the uncertain problem that parameters of system model are not accurate, a synchronous control method which is based on sliding mode method is proposed in this paper. A synchronous system which has the similar structures with the controlled system is constructed, then a control law is design to make the synchronous system trace the controlled system. So the controlled system can be controlled by controlling the synchronous system. Finally, numerical simulation was done the testify the rightness of the proposed method.

***Keywords**: sliding mode control, synchronizing control, uncertainty, second order system, adaptive*

1. Introduction

In the real application, most models of controlled system are uncertain or not accurate which makes the controller design to be a complex problem [1-11]. Some model are certain in some conditions, for example the parameters and the model of the controlled system will change unpredictably when conditions change. PID controller can not behave well for system with big uncertainties in model's parameters. So it is meaningful to design a special control system to cope with the unpredictable change of the model.

Variable structure control (VSC) method was firstly proposed in the 1950's, it is a synthesis design method of modern control theory based on phase plane method. The characteristic of variable structure control（VSC） is that the robustness of the system is strong when systems face interference and parameter perturbation, so variable structure control can solve the problem of uncertain system [12-17]. But there are a lot of problems in the real application, such as system tremor, it needs a larger gain to offset interference and parameter perturbation. In addition the model of system should be accurate when variable structure control is used, so the method dependent on exact model.

A synchronous track control method is constructed for the situation that better quality of output is demanded when the parameters of second order system is uncertain. Since the parameters of synchronous system is known, it is easy to control the given accurate synchronous system [18-23].

For these reason, a new method combines synchronous control method and sliding mode control method was research in this paper, which will overcome the limitation of parameterization for uncertainty in the sliding mode design and also improve the robustness of the system.

2. Problem Description

The second-order system with a single control direction is a simple case in all of the second-order system. The control direction is the coefficient of the model inputs u which is called control coefficient. The model can be written as:

$$\dot{x} = Ax + bu \qquad (1)$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The parameters of model is unknown, the goal of adaptive backstepping control is to design an adaptive backstepping controller such that the system state x_1 can trace the expected value x_1^d, without loss of generality, assume the expected value $x_1^d = 1$.

3. Design Synchronizing System

Because the controlled system is uncertain, a synchronizing system is designed for the control the controlled system. When the controlled system is a second order system, the synchronizing can be designed as follows:

$$\dot{y}_1 = b_{11}y_1 + b_{12}y_2 + v_1 \qquad (2)$$

$$\dot{y}_2 = b_{21}y_1 + b_{22}y_2 + u + v_2 \qquad (3)$$

Where $v_i(i=1,2)$ is the control for synchronizing, and $b_{ij}(i=1,2\, j=1,2)$ is the control law parameter for synchronizing. Control parameter should be chosen as same as the parameter of controlled system.

4. Design Error System

In order to make synchronizing system track exactly controlled system,the error between the two systems should be eliminated. Define synchronizing error $e_i = y_i - x_1(i=1,2)$, so the error system can be got by subtracting the above two systems:

$$\dot{e}_1 = \dot{y}_1 - \dot{x}_1 \qquad (4)$$

$$\dot{e}_2 = \dot{y}_2 - \dot{x}_2 \qquad (5)$$

It is easy to get:

$$\dot{e}_1 = b_{11}y_1 + b_{12}y_2 - a_{11}x_1 - a_{12}x_2 + v_1 \qquad (6)$$

$$\dot{e}_2 = b_{21}y_1 + b_{22}y_2 - a_{21}x_1 - a_{22}x_2 + v_2 \qquad (7)$$

Arrive here, the problem can be transformed into designing synchronizing control law $v_i(i=1,2)$ and choosing synchronizing control law parameters $b_{ij}(i=1,2,\ j=1,2)$ to make the error of systems $e_i \to 0$, so the state of controlled system can track the expected state.

Choose synchronizing control:

$$v_i = -k_{pi}e_i - k_{si}\int e_i dt \qquad (8)$$

Choose a Lyapunov function as:

$$V = \tfrac{1}{2}e_1{}^2 + \tfrac{1}{2}e_2{}^2 \qquad (9)$$

By solving its derivative, it is easy to get:

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 \\ &= -k_{p1}e_1^2 - k_{p2}e_2^2 - k_{s1}e_1\int e_1 dt - k_{s2}e_2\int e_2 dt \\ &\quad + e_1(by_2 - a_{11}x_1 - a_{12}x_2) + e_2(-a_{21}x_1 - a_{22}x_2)\end{aligned} \qquad (10)$$

Unless synchronizing system turns out to be bounded, it is not guaranteed that systems can achieve synchronization. Systems are assumed that they are bounded, when k_{p1}, k_{p2} is large enough, there exists $\dot{V} < 0$. According to the principle of Lyapunov, the system is stable by now. So when choose the value of k_{p1}, k_{p2}, the value of k_{p1}, k_{p2} should be large enough.

5. Construct Sliding Surfaces

According to the design theory of sliding controller, there are two steps: one is choosing a proper sliding surface that can make the track of the error slide stably on sliding surface and meet the condition that $\lim\|e_i\| = 0, (i=1,2)$;the other is defining sliding control law,so that the track of the error can move to sliding surface.

Define error: $e = y_1 - y_1^d$

The sliding surface of the system is made of error and error intergral and error differential, the sliding can be written as:

$$s = ce + d\int edt + \dot{e} \qquad (11)$$

According to accessibility of sliding mode dynamics: when the state of system approach the sliding surface $s = 0$, there exists $S\dot{S} \le 0(s \to 0^+)$ and $S\dot{S} \ge 0(s \to 0^-)$, and differentiate the sliding mode surface:

$$\dot{s} = c\dot{e} + de + \ddot{e} \qquad (12)$$

Then:

$$\dot{s}=(b_{11}c+b_{11}^2+b_{12}b_{21})y_1+(b_{12}c+b_{11}b_{12}+b_{12}b_{21})y_2 \\ +de+b_{12}u+(c+b_{11})v_1+\dot{v}_1+b_{12}v_2 \quad (13)$$

Then the control law can be got:

$$u=-\frac{1}{b_{12}}\{(b_{11}c+b_{11}^2+b_{12}b_{21})y_1+(b_{12}c+b_{11}b_{12}+b_{12}b_{21})y_2 \\ +de+(c+b_{11})v_1+\dot{v}_1+b_{12}v_2+k\} \quad (14)$$

Where $k=k_1\,\mathrm{sgn}(s)+k_2s+k_3s/(|s|+0.2)$. Then the Synchronizing control system has been designed.

6. Numerical Simulation

Different second order linear systems have different parameters, it means that different array A represent different second order linear systems. Then, the problem of studying different second order linear system can be transformed into studying different array A. Then to discuss how to choose control parameters to make the state of the controlled system track the expected state when array A is different. Array A is chosen randomly in range of -10 to 10. Situations of choosing array A are as follows:

Table 1. The Situation of Choosing Array a (1)

	1	2	3	4	5	6
a11	-8.8422	-3.9077	-2.4325	6.3595	-3.1761	6.7699
a12	6.2633	-6.1314	7.0731	-3.1601	4.5423	-2.5917
a21	-2.9426	-6.2069	7.2002	3.2046	0.6816	1.3614
a22	-9.8028	3.6445	1.8713	-4.2055	-3.8142	4.0548

Because the controlled system is a single control system and the control u is in the second order subsystem, synchronizing system can be designed for the second order subsystem, choose synchronizing control law parameters $b_{11}=0, b_{12}=1, b_{21}=0, b_{22}=0$. the synchronizing system can be written as:

$$\dot{y}_1=b_{11}y_1+b_{12}y_2+v_1 \quad (15)$$

$$\dot{y}_2=b_{21}y_1+b_{22}y_2+v_2+u \quad (16)$$

Choose control parameters $c=2, d=5, k_1=10, k_2=10, k_3=1$. And the simulation results are as follows:

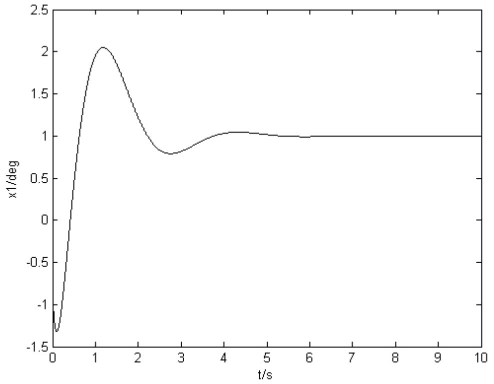

Figure 1. The First Result of State X1

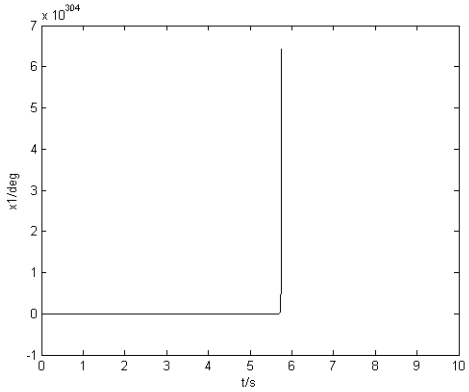

Figure 2. The Second Result of State X1

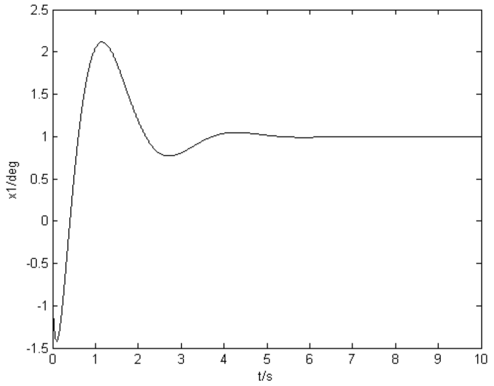

Figure 3. The Third Result of State X1

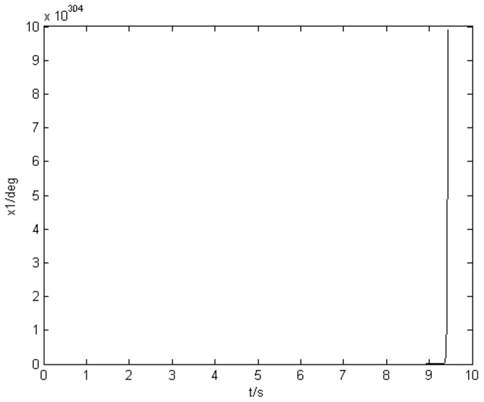

Figure 4. The Fourth Result of State X1

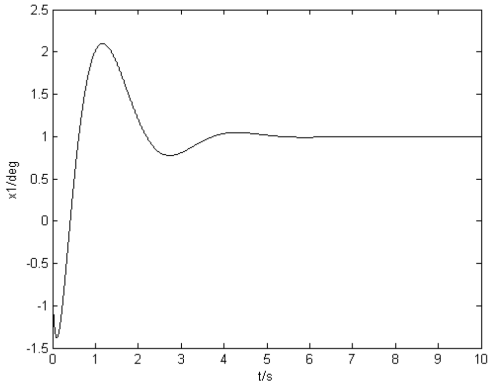

Figure 5. The Fifth Result of State X1

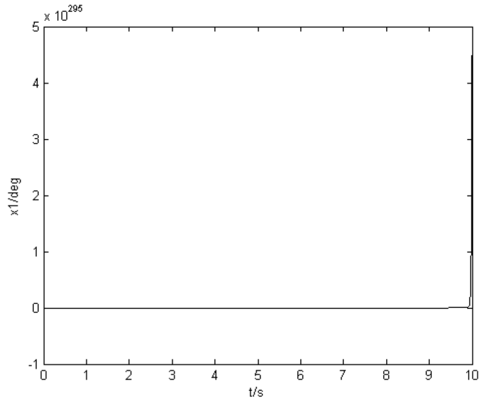

Figure 6. The Sixth Result of State X1

A phenomenon is discovered by analysing the above results. If controlled parameter $a_{12} < 0$, the controlled system is unstable when synchronizing control law paremeters are chosen as $b_{11} = 0, b_{12} = 1, b_{21} = 0, b_{22} = 0$. The situation of controlled system parameters $a_{12} < 0$ is discussed as follows:

Rearrange the error system:

$$\dot{e}_1=(b_{12}-a_{12})x_2+b_{12}e_2-a_{11}x_1+v_1=(b_{12}-a_{12})y_2+a_{12}e_2-a_{11}x_1+v_1 \tag{17}$$

Due to the goal of control is $y_i \rightarrow x_i(i=1,2)$, a_{12} should have the same sign as b_{12} in order to make the system stable. $b_{12}=-1$ Is chosen at the situation of $a_{12}<0$. Element a_{12} is chosen randomly in range of -10 to 0 and other elements is chosen randomly in range of -10 to 10. Situations of choosing array A are as follows:

Table 2. The Situation of Choosing Array A (2)

	1	2	3	4
a11	1.6218	1.6565	6.8921	2.2898
a12	-7.9428	-6.0198	-7.4815	-9.1334
a21	3.1122	2.6297	4.5054	1.5238
a22	5.2853	6.5408	0.8382	8.2582

the simulation results are as follows:

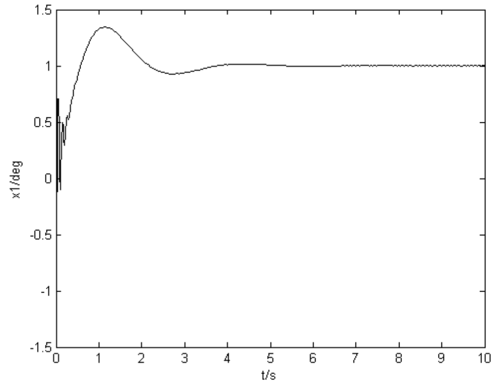

Figure 7. The First Result of State X1

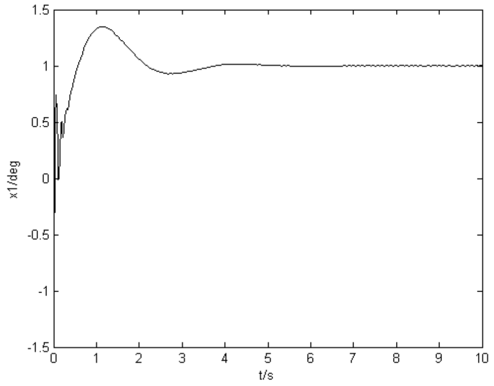

Figure 8. The Second Result of State X1

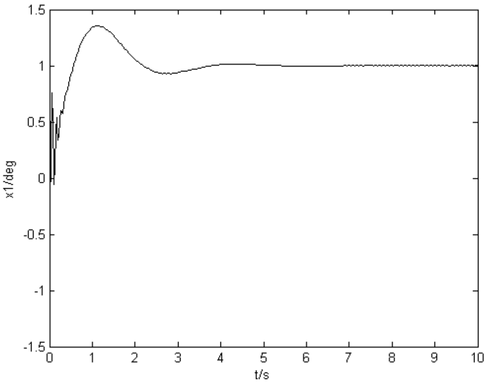

Figure 9. The Third Result of State X1

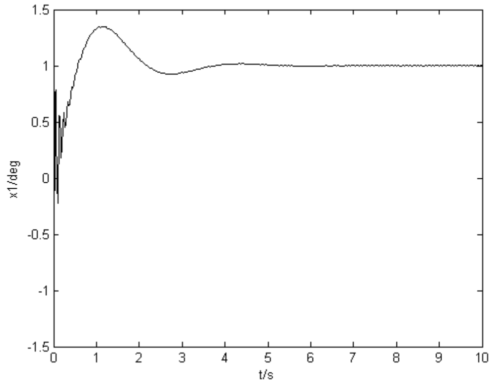

Figure 10. The Fourth Result of State X1

All the simulation results are stable, it indicates that the system can be stable when a_{12} and b_{12} have the same sign. But synchronizing control law parameters $b_{11}=0, b_{12}=1, b_{21}=0, b_{22}=0$ are chosen to aim at the second order subsystem, it has strongly particularity. Without loss of generality, synchronizing control law parameters can be chosen as $b_{11}=1, b_{12}=a_{12}/|a_{12}|, b_{21}=1, b_{22}=1$.

Consider controlled parameters $a_{ij}(i=1,2,\ j=1,2)$ is chosen randomly in range of -10 to 10. Situations of choosing $a_{ij}(i=1,2,\ j=1,2)$ are as follows:

Table 3. The Situation of Choosing Array A (3)

	a11	a12	a21	a22
1	-1.2	-2.4	5.3	5.9
2	-6.3	-2.2	-1.1	2.9
3	4.2	5.1	-4.8	3.6
4	3.1	-6.7	-7.6	0
5	9.2	-3.2	1.7	-5.5
6	7.8	9.2	0.9	-7.2

the simulation results are as follows:

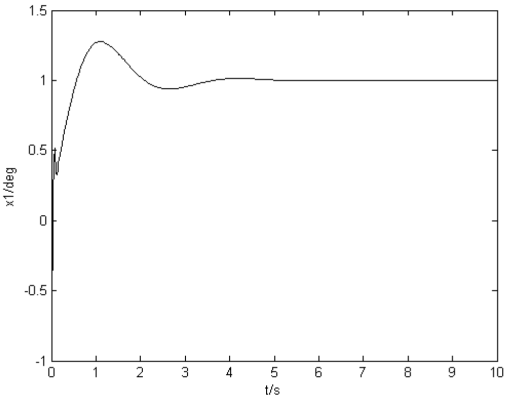

Figure 11. The First Result of State X1

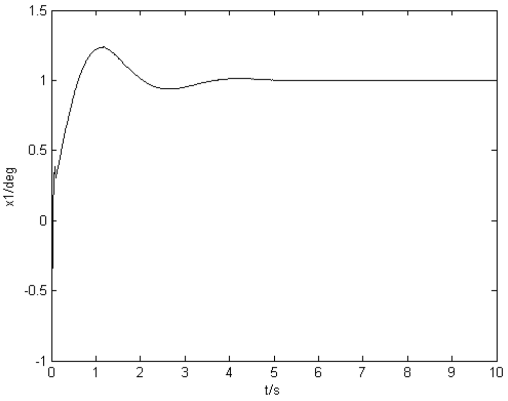

Figure 12.The Second Result of State X1

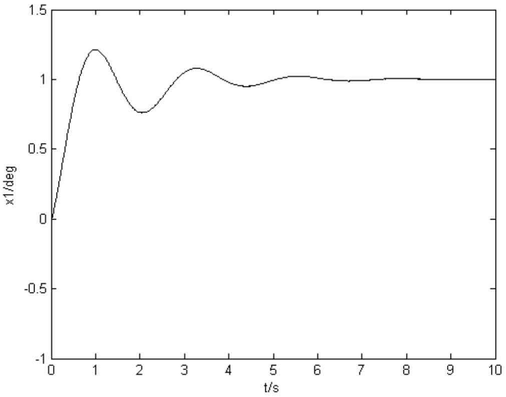

Figure 13. The Third Result of State X1

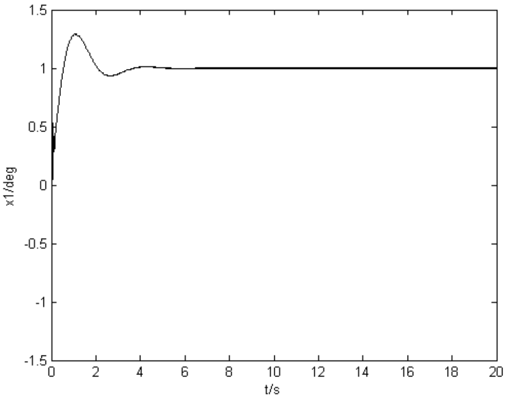

Figure 14. The Fourth Result of State X1

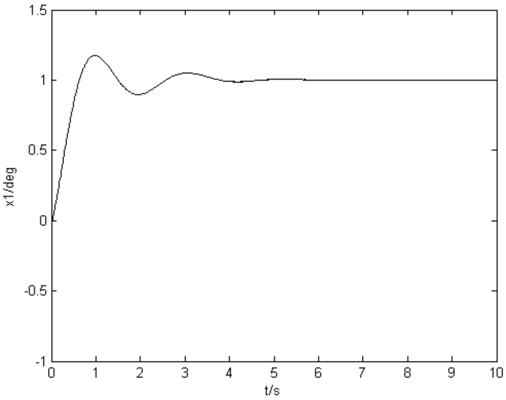

Figure 15. The Fifth Result of State X1

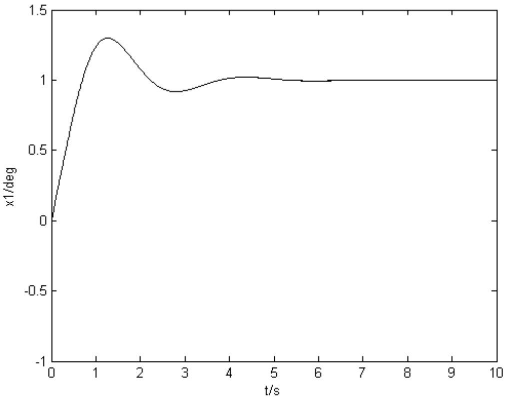

Figure 16. The Sixth Result of State X1

All the simulation results are stable, it indicates that the system can be stable when a_{12} and b_{12} have the same sign.

7. Conclusion

By analyzing the simulation results, the conclusion can be made as bellows that the sign of synchronizing control system parameters b_{12} should be same as the sign of controlled system parameters a_{12} when synchronizing control system is designed for the situation that the controlled system is a second order linear system with a single input. If the sign of synchronous control system parameters b_{12} is not the same as the sign of controlled system parameters a_{12}, the system will be unstable.

So the sign of synchronizing control system parameters b_{12} should be designed to be the same as the sign of controlled system parameters a_{12} if the synchronous control method is applied in second order linear system with known single input.

References

[1] L. Hsu and R.R, “Costa. Variable structure model reference adaptive control using only input and output measurement [J]”, International Journal Control, vol. 42, no. 2, **(1989)**, pp. 399-416.
[2] S.H. Yu and G. Guo, “Global fast terminal slding mode control for robotic manipulators [J]”, International Journal of Modeling, Identification and Control, vol. 1, no. 1, **(2006)**, pp. 72-79.
[3] S. Tal, I. Moshe and O.M. Golan, “Sliding mode control for integrated missile autopilot guidance [J]”, Journal of Guidance, Control, and Dynamics, vol. 29, no. 2, **(2006)**, pp. 250-260.
[4] Y. Zhou, Y. Wu and Y. Hu, “Robust backstepping sliding mode control of a class of uncertain MIMO nonlinear systems [C]”, 2007 IEEE International Conference on Control and Automation, Guangzhou, CHINA, **(2007)**.
[5] J. Li and D.M. Xu, “Adaptive sliding mode controller for nonlinear systems with mismatched uncertainties based on adaptive backstepping scheme [J]”, Control and Decision, vol. 14, no. 1, **(1999)**, pp. 46-50.
[6] A. Stansky, J.K. Hedrick and P.P. Yip, “The use of sliding modes to simplify the backstepping control method [C]”, Proceedings of the American Control Conference, **(1997)**, Albuquerque, New Mexico.
[7] G. Wheeler and C. Su, “A sliding mode controller with improved adaptation laws for the upper bounds on the norm of uncertainties [J]”, Automatica, vol. 34, no. 12, **(1998)**, pp. 1657-1661.
[8] C.X. Tong, Z.J. Wang and T.Q. Zhang, “Dual sliding mode variable structure control design for BTT missile autopilot [J]”, Journal of System Simulation, vol. 17, no. 3, **(2005)**, pp. 518-521.
[9] C.T. Lin, “Controlling chaos by GA-based reinforcement learning neural network [J]”, IEEE Transaction on Neural Networks, vol. 10, **(1999)**, pp. 846-859.
[10] J. Yan and C. Li, “On synchronization of three chaotic systems”, Chaos, Solitons and Fracrals, no. 23, **(2005),** pp. 1683-1688.
[11] E.M. Elabbasy, H.N. Agiza and M.M. El-dessoky, “Adaptive synchronization of lü System with uncertain parameters”,Chaos, Solitons and Fractals, vol. 21, **(2004),** pp. 657-667.
[12] M.T. Yassen, “Chaos synchronization between two different chaotic systems using active control”, Chaos, Solitons & Fractals, vol. 23, no. 4, **(2005)**, pp. 131-140.
[13] H.N. Agiza and M.T. Yassen, “Synchronization of Rossler and Chen chaotic dynamical systems using active control”, Physics Letters A, no. 278, **(2001)**, pp. 191-197.
[14] M.C. Ho and Y.C. Hung, “Synchronization of two different systems by Using generalized active control”, Physics Letters A, no. 30, **(2002)**, pp. 1424-428.
[15] A. Van-ec-ek and S. C-elikovesky, “Control systems: from linear analysis to synthesis of chaos”, **(1996)**, London: Prentice-Hall.
[16] X. Wu and J. Lu, “Parameter identification and backstepping control of uncertain Lv system”, Chaos, Solitons & Fractals, vol. 18, **(2003)**, pp. 721-9.
[17] Y. Yu and S. Zhang, “Adaptive backstepping control of the uncertain Lvu system”, Chin Phys, vol. 11, **(2002)**, pp. 1249-305.
[18] C. Wang and S. Ge, “Adaptive backstepping control of uncertain Lorenz system”, International Journal Bifurc Chaos, vol. 11, **(2001)**, pp. 1115-9.
[19] E.M. Elabbasy, H.N. Agiza and M.M. El-Dessoky, “Adaptive synchronization of a hyperchaotic system with uncertain parameter”, Chaos, Solitons and Fractals, vol. 30, **(2006)**, pp. 1133-1142.
[20] F. Tang and L. Wang, “An adaptive active control for the modified Chua’s circuit”, Physics Letters A, vol. 346, **(2005)**, pp. 342-346.
[21] M. Hu, Z. Xu, R. Zhang and A. Hu, “Parameters identification and adaptive full state hybrid projective synchronization of chaotic (hyper-chaotic) systems”, Physics Letters A, vol. 361, **(2007)**, pp. 231-237.
[22] Z.M. Ge and C.H. Yang, “Pragmatical generalized synchronization of chaotic systems with uncertain parameters by adaptive control”, Physica D , vol. 231, **(2007)**, pp. 87-89.
[23] T. Gao, Z. Chen, Z. Yuan and D. Yu, “Adaptive synchronization of a new hyperchaotic system with uncertain parameters”, Chaos, Solitons and Fractals, vol. 33, **(2007)**, pp. 922-928.

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