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A Structural Analysis and Design of an Engineering Control System in the Frequency Domain Over $F(z)$

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Abstract

Structural controllability is an important concept of linear systems, and problems relating to this concept have recently become the subject of a great deal of research. In this paper, two different structures of control are proposed that are based on the servo control system. An analysis is conducted on the structural controllability of two control systems, the differences of which relate to the structural controllability criterion in the frequency domain over F(z). Due to the fact that the system is not structurally controllable, the first servo control system is not completely manageable no matter what values the parameters take; therefore, the reasons for this are investigated. Conversely, the second system becomes structurally controllable by changing the structure of the control system. Thus, it is made fully stable by the feedback compensation even if it has an unstable mode. Accordingly, this research on the structural controllability of a system not only provides a deeper understanding of the system's nature, but is also important for explaining why a system is not structurally controllable and how changing its structure can resolve this.

Keywords: *A servo control system; structural controllability; rational function matrix; frequency domain*

1. Introduction

There is great interest in the security and reliability of system operations when linear systems are being designed; moreover the stability of a system is one of the most important conditions for safe operations, as for a system to work normally it must firstly be stable. Controllability and observability are closely related to stability; if a system is controllable and observable, it will be made fully stable by the state feedback compensation even if it has an unstable mode. Conversely, if a system has an unstable mode and is not controllable, it cannot be made stable by feedback. Therefore as long as the system is controllable, it is possible for it to be stable, and thus it is very important to analyze the controllability of systems. Subsequently the controllability theory of linear systems over the field R of real numbers has been the subject of a great deal of research [1]. This theory is useful for analyzing the characters which are determined by the value of physical parameters and the system structure; however, sometimes a practical system has a definite structure and an approximate (even unknown) parameter, because of limits to experimental conditions, manufacturing technology or observation errors.

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It is not known from the definition on complete controllability, that if a system does not meet the conditions of complete controllability whether this is due to the structure or to an inappropriate choice of parameter values. It is difficult when using a linear system over the field R of real numbers to analyze the structural characters of a physical system (e.g. structural controllability and structural observability). For example, consider the liquid level control system as shown in Figure 1.

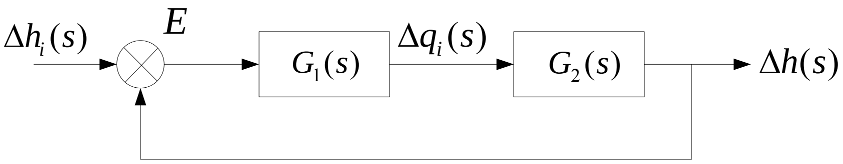

Figure 1. The Liquid Level Control System

where $G_1(s)=\dfrac{\Delta q_i(s)}{E(s)}=K_p(1+\dfrac{1}{T_i s})$, and $G_2(s)=\dfrac{\Delta h(s)}{\Delta q_i(s)}=\dfrac{K}{Ts+1}$.

The closed loop transfer function of this system is $\dfrac{\Delta h(s)}{\Delta h_i(s)}=\dfrac{b_1 s+b_0}{s^2+a_1 s+a_0}$. The block diagram representation of the system can be seen in Figure 2.

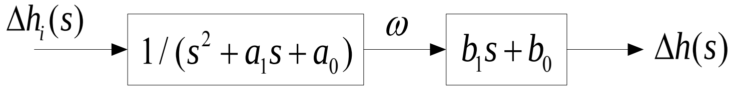

Figure 2. The Block Diagram Representation of the Liquid Level Control System

where

$$a_0=\frac{KK_p}{T_i T}, a_1=\frac{KK_p+1}{T}, b_0=\frac{KK_p}{T_i T}, b_1=\frac{KK_P}{T}. \tag{1}$$

Then the state equation of the feedback system can be denoted by

$$\dot{X}=AX+B\Delta h_i, \quad \Delta h=CX \tag{2}$$

where $X=(\omega,\dot{\omega})^T$, ω is the output signal of the left block, $\dot{\omega}$ is a derivative of ω, and $(\omega,\dot{\omega})^T$ denotes the transpose of $(\omega,\dot{\omega})$,

$$A=\begin{pmatrix} 0 & 1 \\ -\dfrac{KK_p}{TT_i} & -\dfrac{1+KK_p}{T} \end{pmatrix}, B=\begin{pmatrix} 0 \\ 1 \end{pmatrix}, C=\begin{pmatrix} \dfrac{KK_p}{TT_i} & \dfrac{KK_p}{T} \end{pmatrix} \tag{3}$$

For this system with given structure, only when all the physical parameters K_p, T_i, K and T take values, A, B and C are real number matrices and the system is real number system. When $K_p=2, T_i=1, K=3, T=1$ (this can be denoted by $z=(K_p,T_i,K,T)=\overline{z}=(2,1,3,1)$), substituting this into (3) yields

$$A=\begin{pmatrix} 0 & 1 \\ -6 & -7 \end{pmatrix}=, B=\begin{pmatrix} 0 \\ 1 \end{pmatrix}, C=\begin{pmatrix} 6 & 6 \end{pmatrix} \tag{4}$$

According to theory of real number systems, this system (4) is clearly not observable. However, system (3) is observable. The points (such as $\overline{z}=(2,1,3,1)$), which make this system unobservable over *R*, form a set $\{z \in R^4 \mid \det(C^T, A^T C^T)^T = 0\}$ that is only a hypersurface in parameter space R^4. Because K_p, T_i, K, T are physical parameters, it is not possible that their associated values are accurate enough to make $\det(C^T, A^T C^T)^T = 0$. As for the real number system (4) is unobservable, it can be understood that the man-made or mathematical constraint $z=\overline{z}=(2,1,3,1)$ of the parameters makes $T - T_i = 0$, that is $\det(C^T, A^T C^T)^T = 0$. Thus the analysis results of this real number system (such as the controllability of (A, B), observability of (A^T, C^T), reducibility of the characteristic polynomial $\det(\lambda I - A)$, and so on) are determined by two factors: physical system structure and parameter values. However, it is not possible to distinguish what the individual effect of the system structure is.

Structural controllability is an important concept, as it is a prerequisite that the system is controllable with constant parameters. Investigating the structural controllability of the system cannot only provide and in-depth understanding of the nature of the system, but also explain why the system is not controllable and can therefore lead to an improved approach [2].

In order to analyze the system structure, Lin [3] first introduced the concept of structural controllability in 1974. Subsequently many researchers have carried out studies on the properties of structural controllability. The authors [4-9] used a structured matrix (SM) for investigating the structural controllability of linear systems. Furthermore, the authors [10-11] introduced a one-degree polynomial matrix, whose entries are one-degree polynomials in independent parameters. The author [12] introduced a column-structured matrix (CSM) for when the different entries in a column of the matrix contain the same parameter factor, but the factors in different columns are independent of each other. The authors [13-14] defined and researched the mixed matrix (MM), and stated that $M=Q+T$; whereby if the nonzero entries of *T* are algebraically independent over the field *K*, *Q* is a matrix over the field of real numbers.

Clearly, the inverse matrix of the full rank square SM, CSM, one-degree polynomial matrix or MM is generally not a SM, CSM, one-degree polynomial matrix or MM. To overcome this problem, the authors [15-17] introduced a rational function matrix (RFM) in multi-parameters to describe the coefficient matrices of systems and networks, and used the description of systems and networks that were based on the RFM to analyze their structural properties.

From this work on RFMs the following properties were determined. Let $z_1, \cdots, z_q$ denote q independently variable parameters (can also be variables or indeterminates), but not constants or numerical values. Let $z=(z_1, \cdots, z_q) \in R^q$; R^q is the domain of the definition for z, and it can also be referred to as a parameter space. Let $F(z)$ denote the field of all rational functions with real coefficients in q parameters $z_1, \cdots, z_q$, and $F(z)[\lambda]$ denote the ring of all $F(z)$-coefficient polynomials in λ. A matrix is called a RFM or a matrix over $F(z)$ if each entry of the matrix is a member of $F(z)$. A linear system is a rational function system (RFS) or a system over $F(z)$ if all coefficient matrices of the system are RFMs.

The papers written by [3-13] are of mathematical significance; however, the matrices defined in them cannot normally fully describe the physical systems, and therefore they cannot be used alone to explore the structural properties of physical systems. If Figure. 1 is again considered as an example, the independent parameters of this system should be four physical parameters K_p, T_i, K and T. From the equations in (3) and the above definitions it is known that the matrix over the real field, SM, one-degree polynomial matrix in independent parameters, CSM and MM cannot completely describe the matrices *A*, *B* and *C*; however all these three matrices are RFMs and the system (*A*, *B*, *C*) is a RFS, where $z =$ （K_p, T_i, K, T）. The reason why an RFM can describe the structure of a physical system is that its conception is more general, and a matrix over the real field, SM, one-degree polynomial matrix, CSM and MM can be treated as special RFMs. Therefore, research that investigates the RFS has both mathematical and physical significance.

The author [18] has carried out studies on the controllability of linear systems over $F(z)$ and drew some meaningful conclusions; however, all of them were established in a time domain. In this paper, a servo control system was selected as the research object and its structural controllability was analyzed. Based on the frequency domain over $F(z)$, the reason why the system is not structurally controllable was investigated and the structure of the control system was redesigned; this led to the new servo control system being controllable in real fields everywhere. This method in a frequency domain is more concise than one in a time domain.

2. Preliminaries

Consider a linear system over $F(z)$

$$\dot{X} = AX + BU\ ,\ Y = CX + DU \tag{5}$$

where A, B, C and D are, respectively, $n \times n$, $n \times m$, $p \times n$, and $p \times m$ RFMs.

Let $T = (B, AB, \cdots, A^{n-1}B)$ and $T_0 = (C^T, A^T C^T,\ \cdots, (A^T)^{(n-1)} C^T)^T$ be its controllability and observability matrices respectively. Since A, B and C are the ones over $F(z)$, T and T_0 are also two matrices over $F(z)$ and may be denoted by $T(z)$ and $T_0(z)$. Let

$$N_C = \{z \in R^q \mid \mathrm{rank}T(z) = n\} \text{ or } N_C = \{z \in R^q \mid \det(T(z)T^T(z)) \neq 0\} \tag{6}$$

$$N_O = \{z \in R^q \mid \mathrm{rank}T_0(z) = n\} \text{ or } N_O = \{z \in R^q \mid \det(T_0^T(z)T_0(z)) \neq 0\} \tag{7}$$

where $\mathrm{rank}T(z) = \mathrm{rank}(T(z)T^T(z))$, $\mathrm{rank}T_0(z) = \mathrm{rank}(T_0^T(z)T_0(z))$, and $T^T(z)$ and $T_0^T(z)$ denote the transpose matrices of $T(z)$ and $T_0(z)$ respectively.

Let S be a point set and m^*S denote the Lebesgue measure of the set S.

Definition 1: RFS (5) is structurally controllable (SC) if $m^*N_C \neq 0$; otherwise, it is not. RFS (5) is structurally observable (SO) if $m^*N_O \neq 0$; otherwise, it is not.

Definition 2: Since $T(z)$ and $T_0(z)$ are two matrices over $F(z)$, $\det(T(z)T^T(z)) \in F(z)$ and $\det(T_0^T(z)\ T_0(z)) \in F(z)$. RFS (5) is controllable over $F(z)$ (or it is controllable for short) if $\det(T(z)T^T(z))$ is a nonzero member over $F(z)$; otherwise it is uncontrollable. RFS (5) is observable over $F(z)$ if $\det(T_0^T(z)T_0(z))$ is a nonzero member over $F(z)$; otherwise it is unobservable.

Lemma 1: Let $f(z) \in F(z)$. If $f(z)$ is a zero member over $F(z)$ (simply $f(z) = 0$), then $f(z) = 0$ for all $z \in R^q$; if $f(z)$ is a nonzero member over $F(z)$

(simply $f(z) \neq 0$), then $m^*\{z \in R^q \mid f(z) = 0\} = 0$ or $m^*\{z \in R^q \mid f(z) \neq 0\} \neq 0$. See paper [17] for a fuller explanation.

This Lemma is an obvious form of algebraic theory. If $f(z) \neq 0$, then the point set $\{z \in R^q \mid f(z) = 0\}$ is of a proper algebraic variety and therefore its Lebesgue measure is zero, or $f(z) \neq 0$, for almost all $z \in R^q$ and thus $m^*\{z \in R^q \mid f(z) \neq 0\} \neq 0$.

Remark 1: According to the above definitions and Lemma 1, SC (SO) is equivalent to the controllability (observability) over $F(z)$ for RFS (5).

Lemma 2: Let $P(\lambda) \in F(z)[\lambda], Q(\lambda) \in F(z)[\lambda]$. Then $P(\lambda)$ and $Q(\lambda)$ is co-prime if and only if $m*\{z \in R^q | (P(\lambda), Q(\lambda)) \neq 1\} = 0$ or $m*\{z \in R^q | (P(\lambda), Q(\lambda)) = 1\} \neq 0$, where $(P(\lambda), Q(\lambda)) = d(\lambda)$ denotes the greatest common divisor between $P(\lambda)$ and $Q(\lambda)$, and the leading coefficient of $d(\lambda)$ is 1.

Proof: See Lemma 2 in paper [15].

3. The Structural Controllability Criterion in a Frequency Domain

The PBH controllability criterion over R, which bridges the time domain and frequency domain and presents the linear time-invariant system, is controllable if and only if $\text{rank}[sI - A, B] = n$, $\forall s \in C$ where $A \in R^{n \times n}$, $B \in R^{n \times m}$. It is known that the controllability of a system over R relates to the coprimeness of its polynomial matrices. However, what is less clear is the relationship between the structural controllability and the properties of the polynomial matrices for the system over $F(z)$. The authors [19-21] have carried out studies on the controllability of linear systems in the frequency domain over $F(z)$ and their conclusions can be summarized as follows.

Conclusion 1: The linear system (5) is controllable over $F(z)$ if and only if for the set $E = \left\{ z \in \mathrm{R}^q \middle| \text{rank}[sI - A, B] = n, \forall s \in C \right\}$, $\mathrm{m}^* E \neq 0$.

Conclusion 2: The n-dimension linear system (5) is controllable over $F(z)$ if and only if the nonsingular polynomial matrix $sI - A$ and the matrix B over the ring of $F(z)[s]$ are left co-prime.

Conclusion 3: The n-dimension linear system (5) is controllable over $F(z)$ if and only if the Smith form of $(sI - A, B)$ is $(I, 0)$, where $(sI - A)$ a nonsingular polynomial matrix is.

Although the controllability of a system over $F(z)$ in a frequency domain also relates to the coprimeness of its polynomial matrices and the Smith form of $(sI - A, B)$, it should be emphasized that the conclusions regarding $F(z)$ are only determined by the structures of systems and not the values of z. This is because the conclusions about $F(z)$, which eliminate the value effect and only leave the structure effect, are not taken into account when the parameters are evaluated. Since the real field is a subfield of $F(z)$, the criterion over $F(z)$ is more universal.

From Definitions 1 and 2 and Lemmas 1 and 2, the following are known.

Theorem 1: Consider a linear system $\dot{x} = Ax + Bu$, where A and B are, respectively, $n \times n$ and $n \times m$ matrices over $F(z)$. Let $M = (sI - A, B)$ be a polynomial matrix in the ring $F(z)[s]$. Then the system is controllable over $F(z)$ if in M there is at least one $n \times n$ matrix M_1, which consists of n columns of M such that $\det M_1$ and $\det(sI - A)$ are two co-prime polynomials in $F(z)[s]$.

Proof: Since the two polynomials $\det M_1$ and $\det(sI-A)$ are co-prime, from Lemma 2 we have $m^*E \neq 0$, where $E=\{z \in R^q \mid (\det M_1(s,z), \det((sI-A(z)))=1\}$. Take one point $\bar{z} \in E$, then $\det(sI-A(\bar{z}))$ denotes a polynomial in s whose coefficients are all real numbers. Let $s_i(\bar{z})$ denote a root of $\det(sI-A(\bar{z}))$, $i=1,\ldots,n$. Since $\det M_1(s,\bar{z})$ and $\det(sI-A(\bar{z}))$ are co-prime, $\det M_1(s_i(\bar{z}),\bar{z}) \neq 0$ when $\det(s_i(\bar{z})I-A(\bar{z}))=0$. Thus $\text{rank}(s_i(\bar{z})I-A(\bar{z}),B(\bar{z}))=n$, $i=1,2,\ldots,n$, and $(A(\bar{z}),B(\bar{z}))$ are controllable for $z=\bar{z}$. Since $\bar{z} \in E$ and $m^*E \neq 0$, $(A(z),B(z))$ is structurally controllable by Definition 1. It is known by Remark 1 that the system $(A(z),B(z))$ is controllable over $F(z)$.

Theorem 2: Consider a linear system $\dot{x}=Ax+Bu$, where A and B are respectively, $n\times n$ and $n\times m$ matrices over $F(z)$. Let $M=(sI-A,B)$ be a polynomial matrix in the ring $F(z)[s]$; M_1, M_2,…, M_g in M are g different $n\times n$ sub-matrices (each of which consists of n columns of M) except $sI-A$, $g=C_{n+m}^{n}-1$. The system is not controllable over $F(z)$ if and only if (ⅰ) $(\det M_i(s,z), \det(sI-A(\bar{z})))=d_i(s,z)$ $d_i(s,z)$ denote the greatest common divisor, $\deg d_i(s,z)\geq 1$, $i=1,\cdots,g$, and (ⅱ) there is a common divisor $\bar{d}(s,z)$ among $d_1(s,z)$, $d_2(s,z)$,…, and $d_g(s,z)$, $\deg \bar{d}(s,z)\geq 1$.

Proof: Sufficiency. Since $\det M_i(s,z)$ and $\det(sI-A(z))$ have the greatest common divisor $d_i(s,z)$ and there is the common divisor $\bar{d}(s,z)$ among $d_1(s,z)$, $d_2(s,z)$,…, and $d_g(s,z)$, $m^*E\neq 0$, where the point set $E=\{z\in R^q \mid (\det M_i(s,z), \det(sI-A(z)))=d_i(s,z)$, $i=1,\cdots,g$; $d_1(s,z),d_2(s,z),\cdots,d_g(s,z)=\bar{d}(s,z)$. Arbitrarily take a point $\bar{z}\in E$. Let $s_l(\bar{z})$ be a root of $\det(sI-A(\bar{z}))=0, l=1,\cdots,n$. Then there is at least one root, such as $s_1(\bar{z})$, which is also a root of the common divisor $\bar{d}(s,\bar{z})=0$. Thus, when $\det(s_1(\bar{z})I-A(\bar{z}))=0$, $\bar{d}(s_1(\bar{z}),\bar{z})=0$, $d_i(s_1(\bar{z}),\bar{z})=0$ and $\det M_i(s_1(\bar{z}),\bar{z})=0$, $i=1,\cdots,g$. This implies $\text{rank}(s_i(\bar{z})I-A(\bar{z}),B(\bar{z}))<n$ and $(A(\bar{z}),B(\bar{z}))$ are not controllable. Since $m^*E\neq 0$, the system is not structurally controllable, that is, it is not controllable over $F(z)$.

Necessity. Conversely, if the condition (ⅰ) does not hold, that is, there exists at least one $\det M_i(s,z)$ such that $(\det M_i(s,z), \det(sI-A))=1$, then the system is controllable over $F(z)$ by Theorem 1. Suppose that (ⅰ) holds but (ⅱ) does not. Then each irreducible factor of $\det(sI-A)$ is not a common divisor among $d_1(s,z)$, $d_2(s,z)$,…, and $d_g(s,z)$, otherwise it contradicts that (ⅱ) does not hold. Suppose that $f(s,z)$ is any irreducible factor of $\det(sI-A)$. Since each irreducible factor of $\det(sI-A)$ is not a common divisor among $d_1(s,z)$, $d_2(s,z)$,…, and $d_g(s,z)$, there is at least one, such as $d_1(s,z)$, which does not contain the factor $f(s,z)$. Since $d_1(s,z)$ is the greatest common divisor between $\det(sI-A)$ and $\det M_1(s,z)$, $\det M_1(s,z)$ does not contain $f(s,z)$ also and $(\det M_1(s,z), f(s,z))=1$. Then we have $m^*E_1\neq 0$, where $E_1=\{z\in R^q \mid (\det M_1(s,z), f(s,z))=1\}$. Take $\bar{z}\in E_1$. Let $s_1(\bar{z}),\ldots,s_{n_1}(\bar{z})$ be

n_1 roots of $f(s,\bar{z})=0$, $n_1 \le n$. Since $\det M_1(s,\bar{z})$ and $f(s,\bar{z})$ are co-prime and $f(s_j(\bar{z}),\bar{z})=0$, $\det M_1(s_j(\bar{z}),\bar{z}) \ne 0$ and $\mathrm{rank}(s_j(\bar{z})I - A(\bar{z}), B(\bar{z})) = n$, $j=1,\cdots,n_1$. Since $m * E_1 \ne 0$, $\mathrm{rank}(s_i(z)I - A(z), B(z)) = n$ for almost all $z \in R^q$, $j=1,\cdots,n_1$. Moreover, since $f(s,z)$ is any irreducible factor of $\det(sI-A)$ and each irreducible factor of $\det(sI-A)$ is not a common divisor among $d_1(s,z)$, $d_2(s,z)$,..., and $d_g(s,z)$, $\mathrm{rank}(s_j(z)I - A(z), B(z)) = n$ for almost all $z \in R^q$, $j=1,\cdots,n_1$, which means that the system is structurally controllable, that is, it is controllable over $F(z)$.

Theorem 1 is a sufficient condition of the structure controllability of the system and has an essential internal connection with conclusions 2 and 3. In fact, $\det M_1$ and $\det(sI-A)$ are two co-prime polynomials, which means the nonsingular polynomial matrix $sI-A$ and the matrix B are left co-prime, namely, the Smith form of $(sI-A,B)$ is $(I,0)$. Theorem 2 is a necessary and sufficient condition of non- structure controllability of the system. In addition, the criterion in a frequency domain over $F(z)$ is simpler and more convenient than that in a time domain. By using the criterion in a frequency domain, only the co-prime between the polynomial should be judged so that the calculation is relatively simple and convenient when analyzing and designing the system structure. In the following section, a servo control system will be used as example to illustrate the above analysis.

4. A Structure Analysis and Design of a Servo System

A servo control system can be seen in Figure 3. θ_r and θ_c are the input and output of the system, respectively.

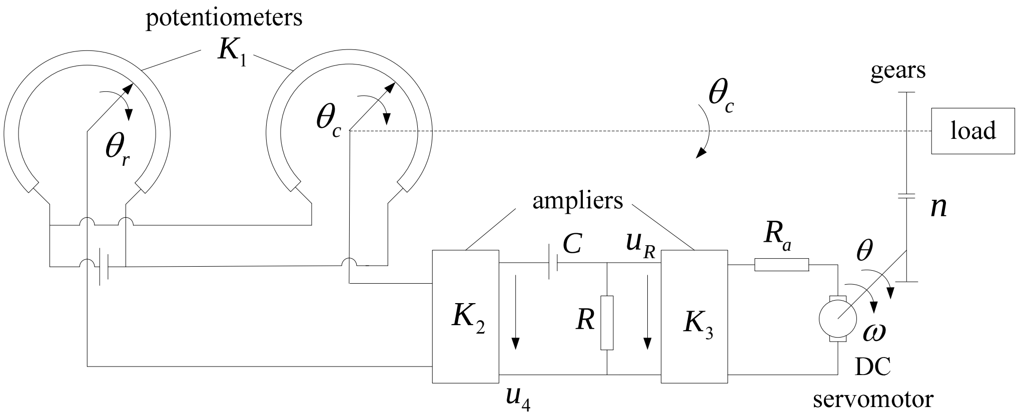

Figure 3. A Servo Control System

Firstly disregard the nonlinearities of the motor and the gears and then choose three state variables u_c, ω, θ as shown in Figure 3. Then

$$\begin{pmatrix} \dot{u}_C \\ \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} -a_0 & 0 & 0 \\ 0 & 0 & 1 \\ -a_1 & 0 & -a_2 \end{pmatrix} \begin{pmatrix} u_C \\ \theta \\ \omega \end{pmatrix} + \begin{pmatrix} a_0 d_1 \\ 0 \\ a_1 d_1 \end{pmatrix} \Delta\theta \qquad (8)$$

$$\theta_C = (n, 0)\begin{pmatrix} \theta \\ \omega \end{pmatrix}$$

where $\Delta\theta = \theta_r - \theta_c$, $a_0 = \frac{1}{RC}$, $d_1 = k_1 k_2$, $a_1 = \frac{f_m R_a + k_b C_m}{J_m R_a}$, $a_2 = \frac{k_3 C_m}{J_m R_a}$.

Now the controllability of the servo system in the real fieldcan be analysed.

Let $(C,R,k_1,k_2,J_m,R_a,f_m,k_b,C_m,k_3,n) = (1.8\times10^{-3},1000,2,45,1600,200,0.2,8,0.02,21,1500)$, then the controllability matrix is $\text{rank}T = \text{rank}(B, AB, A^2B) = 2$. Thus the servo system is not controllable. If $(C,R,k_1,k_2,J_m,R_a,f_m,k_b,C_m,k_3,n) = (2.7\times10^{-6},250,0.5,105,2427,158,0.8,10,1.4,10,1200)$, then the controllability matrix is $\text{rank}T = \text{rank}(B, AB, A^2B) = 2$, and the servo system is also not controllable. With these two different parameter values being selected the system still cannot be fully controlled; however it is necessary to establish whether this is due to the structure or the improper parameter being chosen. Thus the structural properties of the servo system are now analyzed.

The coefficient matrix of servo system $A = \begin{pmatrix} -\frac{1}{RC} & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{k_3 C_m}{J_m R_a} & 0 & -\frac{f_m R_a + k_b C_m}{J_m R_a} \end{pmatrix}$; according to the definitions in papers [2,3,9,11], A is not a SM, one-degree polynomial matrix in independent parameters, CSM and MM. Therefore it is appropriate to use a RFM in multi-parameters to describe the coefficient matrices of the servo systems, which are based on the description of RFM systems, and analyze their structural properties. Subsequently, the criteria of structure controllability in a frequency domain over $F(z)$ are applied to explore the properties of the servo control system.

Let $z = (z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}) = (C,R,k_1,k_2,J_m,R_a,f_m,k_b,C_m,k_3,n) \in R^{11}$, then

$$A = \begin{pmatrix} -\frac{1}{z_1 z_2} & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{z_3 z_4}{z_5 z_6} & 0 & -\frac{z_6 z_7 + z_8 z_9}{z_5 z_6} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{z_3 z_4}{z_1 z_2} \\ 0 \\ \frac{z_3 z_4 z_9 z_{10}}{z_5 z_6} \end{pmatrix} \qquad (9)$$

A, B is a RFM in multi-parameters, and (A, B) is an RFS. Then

$$M=(sI-A,B)=\begin{pmatrix} s+\frac{1}{z_1z_2} & 0 & 0 & \frac{z_3z_4}{z_1z_2} \\ 0 & s & -1 & 0 \\ \frac{z_3z_4}{z_5z_6} & 0 & s+\frac{z_6z_7+z_8z_9}{z_5z_6} & \frac{z_3z_4z_9z_{10}}{z_5z_6} \end{pmatrix} \tag{10}$$

Let $a_0=\frac{1}{z_1z_2}$, $d_1=z_3z_4$, $b_0=\frac{z_9z_{10}}{z_5z_6}$, and $a_1=\frac{z_6z_7+z_8z_9}{z_5z_6}$. Then $\det(sI-A)=s(s+a_0)(s+a_1)$. The matrix $M_1=\begin{pmatrix} s+\frac{1}{z_1z_2} & 0 & \frac{z_3z_4}{z_1z_2} \\ 0 & s & 0 \\ \frac{z_3z_4}{z_5z_6} & 0 & \frac{z_3z_4z_9z_{10}}{z_5z_6} \end{pmatrix}$ consists of the first, second and fourth columns of M. Its determinant $\det M_1=b_0d_1s^2$ and $\det(sI-A)$ have the greatest common divisor $d_1(s,z)=s$. The matrix $M_2=\begin{pmatrix} s+\frac{1}{z_1z_2} & 0 & \frac{z_3z_4}{z_1z_2} \\ 0 & -1 & 0 \\ \frac{z_3z_4}{z_5z_6} & s+\frac{z_6z_7+z_8z_9}{z_5z_6} & \frac{z_3z_4z_9z_{10}}{z_5z_6} \end{pmatrix}$ consists of the first, third and fourth columns of M. Its determinant $\det M_2=-b_0d_1s$ and $\det(sI-A)$ have the greatest common divisor $d_2(s,z)=s$. The matrix $M_3=0\begin{pmatrix} 0 & 0 & \frac{z_3z_4}{z_1z_2} \\ s & -1 & 0 \\ \frac{z_3z_4}{z_5z_6} & s+\frac{z_6z_7+z_8z_9}{z_5z_6} & \frac{z_3z_4z_9z_{10}}{z_5z_6} \end{pmatrix}$ consists of the second, third and fourth columns of M. Its determinant $\det M_3=a_0d_1s(s+a_1)$ and $\det(sI-A)$ have the greatest common divisor $d_3(s,z)=s(s+a_1)$. Clearly, $d_1(s,z)$, $d_2(s,z)$ and $d_3(s,z)$ have the common divisor $\bar{d}(s,z)=s$. Therefore the system is not controllable over $F(z)$ according to Theorem 2. The controllability matrix of the servo system is a not full-rank type, no matter what values the parameters take. In fact, the system may be considered to be a composite system containing three subsystems: Σ_1 with the input $\Delta\theta=\theta_r-\theta_c$ and the output u_R; Σ_2 with the input u_R and output θ_c; and Σ_3 in the feedback path with the input θ_c and the output θ_c. Choose three state variables u_c,ω,θ as shown in Figure 4. Then

$$\Sigma_1:\ \dot{u}_c=-a_0u_c+a_0d_1\Delta\theta,\quad u_R=-u_c+d_1\Delta\theta, \tag{11}$$

$$\Sigma_2: \begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -a_1 \end{pmatrix} \begin{pmatrix} \theta \\ \omega \end{pmatrix} + \begin{pmatrix} 0 \\ b_0 \end{pmatrix} u_R, \qquad \theta_C = (n, 0) \begin{pmatrix} \theta \\ \omega \end{pmatrix}, \tag{12}$$

$$\Sigma_3: \ \theta_C = \theta_C \tag{13}$$

The parameters of Σ_1 and Σ_2 are clearly independent of each other. Let $z^{(1)} = (C, R, k_1, k_2) \in R^4$ and $z^{(2)} = (J_m, R_a, f_m, k_b, C_m, k_3, n) \in R^7$. Σ_3 has no physical parameters. Then the total system Σ has eleven parameters $z = (z^{(1)}, z^{(2)}) \in R^{11}$. Clearly Σ_3 is SC. Since

$$G_1(s) = \frac{d_1 s}{s + a_0}, G_2(s) = \frac{b_0 n}{s(s + d_1)} \tag{14}$$

That is, Σ_1 and Σ_2 are SC, as the initial conditions of the energy storage elements of $\Sigma_i (i = 1,2)$ are independent. Thus the SC of the tandem composite system Σ_{12} is only dependent on the zeroes and poles at origin, however $G_1(s)$ have zero origin and $G_2(s)$ have pole origin. Subsequently Σ_{12} is not SC.

Since there is a cancellation at the origin, the total composite system Σ is not completely steady. Only if the physical structure of Σ is altered (for example Σ_1 can be changed into the structure shown in Figure. 4.), Σ may become steady.

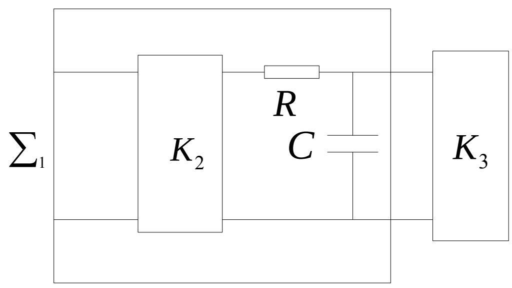

Figure 4. A New Network Structure

The state equation of the new structure servo control system is

$$\begin{pmatrix} \dot{u}_C \\ \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} -a_0 & 0 & 0 \\ 0 & 0 & 1 \\ a_1 & 0 & -a_2 \end{pmatrix} \begin{pmatrix} u_C \\ \theta \\ \omega \end{pmatrix} + \begin{pmatrix} a_0 d_1 \\ 0 \\ 0 \end{pmatrix} \Delta\theta \tag{15}$$

Then

$$M = (sI - A, B) = \begin{pmatrix} s + a_0 & 0 & 0 & a_0 d \\ 0 & s & -1 & 0 \\ -a_1 & 0 & s + a_2 & 0 \end{pmatrix} \tag{16}$$

$\det(sI-A)=\det\begin{pmatrix} s+a_0 & 0 & 0 \\ 0 & s & -1 \\ -a_1 & 0 & s+a_2 \end{pmatrix}=s(s+a_0)(s+a_2)$. The matrix M_1 consists of the first, third and the fourth columns of M. Its determinant $\det\begin{pmatrix} s+a_0 & 0 & a_0 d_1 \\ 0 & -1 & 0 \\ -a_1 & s+a_2 & 0 \end{pmatrix}=-a_0 a_1 d_1$ and $\det(sI-A)=s(s+a_0)(s+a_2)$ are co-prime. Therefore, the system is controllable according to Theorem 1.

5. Conclusion

In this paper, research has been carried out on two different structure servo control systems. The structure controllability of the two different servo systems has been analyzed based on the controllability criterion in the frequency domain over $F(z)$. This is simpler and more convenient when compared to the controllability criterion in a time domain. Furthermore, the method used to analyze the structure of the system can explain the reason why the servo control system is not controllable. It can be controlled in the real domain everywhere by changing the structure of the system. Similarly, this method can be applied to other control systems or electrical networks in engineering.

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