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Nonlinear Estimation and Control during Pipe Connection in a Drilling Operation

Roshan Sharma

Department of Electrical Engineering, IT and Cybernetics, Telemark University College, Porsgrunn, Norway
roshan.sharma@hit.no

Abstract

The bottom hole pressure should be controlled during the pipe connection procedure in a drilling operation to prevent the possibility of oil well blow out, to avoid the collapse of the well and to prevent the fracturing of the bore walls. It should be maintained within a window defined by the collapse pressure as the lower limit and by the fracture pressure as the upper limit. This paper describes the use of a nonlinear model predictive controller (NMPC) to maintain the bottom hole pressure. In addition to a topside choke valve, backpressure pump is also used as a control input. While connecting an additional pipe segment, the injection of the drill fluid is completely stopped. The drill fluid pulse telemetry system that is used in the measurement and transmission of the bottom hole pressure may not function properly and there will be absence of signal while the pipe is being connected. A nonlinear estimator, the unscented Kalman filter (UKF) is used for the continuous estimation of the fluid flows and pressures at different sections of the well being drilled. Simulation results show that the bottom hole pressure can be managed effectively during the drilling of an oil well with nonlinear estimation and control.

Keywords: *Model predictive control, Unscented Kalman filter, drilling, pipe connection.*

1. Introduction

Oil well drilling is performed to create wells that extend several kilometers into the ground (in offshore drilling, below the seabed). A simplified diagram of an oil well drilling system is shown in **Error! Reference source not found.**. Drill bits are attached to the end of the drill string. The drill bits are rotated using a drive system at the top side, and they cut out material from the surface (rock, soil etc.) being drilled. Drill fluid circulation system that consists of the drilling mud pump as one of the components is used to inject the drill fluid (also known as drill mud) down through the drill string. The mud then flows into the annulus through the drill bit and returns upwards. Finally the mud flows out of the wellbore through the choke valve and then to the mud pit (not shown in Figure 1) normally through open flow channels. The drill bit contains a non-return valve or check valve which prevents the backward flow of fluid into the drill string from the annulus. An important function of the drill fluid is to maintain a certain pressure gradient in the annulus along the length of the well. The drill fluid is also used to transport the drill cuttings from the bottom of the well up to the surface [1, 2]. After the fluid exits the annulus through the choke valve, it is filtered (the drill cuttings are separated from the drill mud) and then the clean drill mud is recycled as the inflow to the mud pump.

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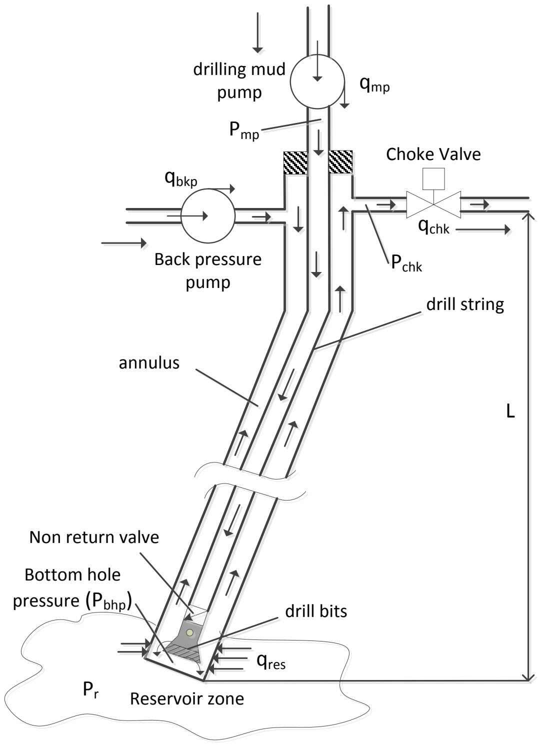

Figure 1. Simplified Diagram of an Oil Well Drilling System

During drilling, it is necessary to keep the bottom hole pressure of the annulus within the available drilling window. The fracture pressure as the upper limit and the collapse pressure as the lower limit define this drilling window. If the bottom hole pressure is greater than the fracture pressure, the mud weight will fracture the rocks and some of the drilling fluid will enter into the well pores. The drilling fluid will block the oil well perforations (well openings) and as a result, the productivity from the well will be lost when it is set into operation. If the bottom hole pressure is smaller than the collapse pressure, the walls of the well bore will fall off or collapse. The drill string may be stuck and a new well may have to be drilled again. While drilling in the reservoir zone, if the bottom hole pressure is smaller than the reservoir pressure, the reservoir fluid will start to flow into the well. If the reservoir influx is not controlled, oil well blow out might occur leading to environmental damage and possible loss of lives. Moreover, the well may be lost and this significantly increases the cost for drilling. Therefore, control of the bottom hole pressure for a proper management of the drilling operation is very important.

During drilling, many disturbances occur and these can fluctuate the bottom hole pressure at the annulus [1]. A source of such a disturbance that is primarily considered in this paper is the pipe connection procedure. The drill pipes come in stands of about 27 m. As the drilling proceeds, the well will become longer. New pipe segments should be regularly added to form the drill string (the drill string consists of several pipe segments that are joined together). The pipe connection procedure severely fluctuates the bottom hole pressure of the annulus. In this paper, a nonlinear model predictive controller is developed and implemented to control the bottom hole pressure during the pipe connection and during normal operation.

Logging While Drilling (LWD) sensors and gauges are located at the bottom of the well bore to measure various information including pressure, flow rate, rotational speed of the drill string, temperature etc. The most common method of transmitting this information is by using a mud-pulse telemetry system. The transmission rate of a typical mud-pulse telemetry system is 40 bit/sec. At greater depths, the data transmission rate reduces significantly. Furthermore, when additional pipes are being connected, the drill mud circulation is low or even zero as the drilling mud pump is stopped. Under such conditions, the mud-pulse telemetry system does not function properly and in the worst case, no signal is received at the surface. The bottom hole pressure is mainly due to the hydrostatic and frictional pressure drops of the drilling fluid column in the annulus. It is dependent on the rheological properties like viscosity and density of the drilling fluid in the annulus. There are various uncertainties and inaccuracies associated in measuring and calculating the rheological properties of the drilling fluid continuously in real time. When the drill cuttings mix with the drilling fluid in the annulus, the rheology of the drilling fluid will be changed. This introduces uncertainties in measuring the amount and the properties of the drill cuttings produced by the drill bits during drilling. Therefore, instead of using such unreliable measurement data of the bottom hole pressure (or in the absence of such a data), it is better to estimate it using the more reliable top side measurements. In this paper, an unscented Kalman filter, which is a nonlinear estimator, is developed and used for the purpose.

Nonlinear estimation of the bottom hole pressure during drilling have been studied by [2, 4, 5]. Automatic control of the bottom hole pressure in a drilling operation has been studied by [1, 6, 7]. However, in these works, the backpressure pump is assumed to be absent in the drilling rig. Similarly, in many literatures, the flow of fluid into the annulus from the reservoir (while drilling in/near reservoir zone) has not been considered. In this paper, the bottom hole pressure in the annulus is controlled by manipulating the flows through the choke valve as well through the back pressure pump simultaneously. The drilling operation is considered to be occurring in the reservoir zone. The flow of reservoir fluid into the annulus has therefore been taken into account.

The paper is organized as follows: In section 2, a mathematical model of the drilling operation is discussed. The consequences of not controlling the bottom hole pressure during pipe connection and the need for estimation and control are highlighted in section 3 through simulation results. Section 4 deals with the development of a nonlinear estimator (the Unscented Kalman filter). The use of the filter for estimating the bottom hole pressure even in the absence of the measurement data from the mud pulse telemetry system is discussed. In section 5, a nonlinear model predictive controller (NMPC) is designed to maintain the bottom hole pressure within the drilling window defined by the fracture pressure and the collapse pressure.

2. Simple Model of a Drilling Operation

A simplified model of the drilling operation that is readily used for designing control systems and for estimation can be found in the literature as in [2, 3] and [4]. A similar model has been used in this paper, however, with some modifications. The model is developed using first principles modeling method with mass and momentum balances. Detailed steps of the modeling are not shown and only the final model equations are presented.

Assume that the vertical depth of the drill string and the annulus are equal. This is denoted by L in Figure 1. From the mass balance in the drill string, dynamics of the pressure at the well head of the drill string, P_{mp} (pressure downstream the drilling mud pump) can be written as,

$$\frac{dP_{mp}}{dt} = \frac{\beta_d}{A_d L}(q_{mp} - q_{bhp}) \qquad (1)$$

Here, q_{mp} is the volumetric flow rate of the drill mud through the mud pump, q_{bhp} is the volumetric flow rate of the drill mud into the annulus from the drill string through the drill bit at the bottom hole, β_d is the bulk modulus at the drill string and A_d is the cross sectional area of the drill string.

From the momentum balance, the dynamics of q_{bhp} is written as,

$$\frac{dq_{bhp}}{dt}=\frac{1}{\frac{\rho_l L}{A_d}+\frac{\rho_{mix}L}{A_a}}\left(P_{mp}+\rho_l gL-\Delta P_f^d-P_{bhp}\right) \tag{2}$$

Here, ρ_l is the density of the drill mud which is assumed to be constant at all sections of the drill string, ρ_{mix} is the density of fluid flowing in the annulus, g is the acceleration due to gravity, ΔP_f^d is the pressure loss in the drill string due to friction and P_{bhp} is the bottom hole pressure in the annulus. The drill bit contains a non-return valve to prevent the flow of fluid from the annulus back into the drill string. When $q_{bhp}=0$, the equation for the dynamics of q_{bhp} is written as

$$\frac{dq_{bhp}}{dt}=\max\left\{0,\frac{1}{\frac{\rho_l L}{A_d}+\frac{\rho_{mix}L}{A_a}}\left(P_{mp}+\rho_l gL-\Delta P_f^d-P_{bhp}\right)\right\} \tag{3}$$

From the mass balance at the annulus, the differential equation for the well head pressure at the annulus (P_{chk}) is given by,

$$\frac{dP_{chk}}{dt}=\frac{\beta_a}{A_a L}(q_{bhp}+q_{res}+q_{bkp}-q_{chk}) \tag{4}$$

Here, β_a is the bulk modulus at the annulus, A_a is the cross sectional area of the annulus, q_{res} is the volumetric flow rate of the reservoir fluid (mixture of oil and water, production of gas from the reservoir is not considered), q_{bkp} is the volumetric flow rate from the back pressure pump which is used to inject drill fluid back into the annulus to change P_{bhp} in addition to controlling the choke valve and q_{chk} is the volumetric flow rate of fluid through the choke valve.

The bottom hole pressure at the annulus (P_{bhp}) is given by,

$$P_{bhp}=P_{chk}+\rho_{mix}gL+\Delta P_f^a \tag{5}$$

Here, ρ_{mix} is the density of fluid in the annulus (mixture of drill fluid and reservoir fluid) and ΔP_f^a is the pressure loss due to friction in the annulus. The equation for ρ_{mix} is given by,

$$\rho_{mix}=\rho_w W_c+(1-W_c)\rho_l \tag{6}$$

Here, ρ_w is the density of water and W_c is the water cut of the reservoir fluid. Water cut denotes the amount of water present in a unit volume of the reservoir fluid. For e.g. $W_c=0.1$ denotes that 10% of the reservoir fluid is water and 90% of the reservoir fluid is oil. When there is no flow of reservoir fluid into the annulus i.e. when $q_{res}=0$, then $W_c=0$ which means that under such condition $\rho_{mix}=\rho_l$. The effect of the drill cuttings on the density of the fluid in the annulus has not been considered in this paper.

The volumetric flow rate of the reservoir fluid flowing into the annulus (q_{res}) is given by the *Productivity Index* (P_I) model [8] of the well as,

$$q_{res} = \max\{P_I\left(P_{res} - P_{bhp}\right), 0\} \;\; for\ P_{bhp} < P_{frac} \tag{7}$$

Here, P_{res} is the pressure of the reservoir and P_{frac} is the fracture pressure (the upper limit of the drilling window). It is assumed that there will be no flow of drill mud from the annulus into the reservoir pores until $P_{bhp} < P_{frac}$ and $P_{bhp} > P_{coll}$. Here, P_{coll} is the collapse pressure (the lower limit of the drilling window). When $P_{bhp} > P_{res}$, Equation 7 makes it sure that there is no flow of drill mud into the reservoir. The flow of fluid when $P_{bhp} > P_{frac}$ has not been modeled (the purpose of the paper is to try to avoid this from happening by using NMPC).

The volumetric flow rate of the fluid through the choke valve (q_{chk}) can be expressed using the standard flow equation ANSI/ISA S75.01 developed by Instrument Society of America [9] as,

$$q_{chk} = \overline{N}_6 Z_c(u_c)\sqrt{\frac{\max\left(P_{chk} - P_0, 0\right)}{\rho_{mix}}} \tag{8}$$

Here, $\overline{N}_6 = N_6/(3600\sqrt{10^5})$ with $N_6 = 27.3$. The valve characteristics as a function of its opening, $Z_c(u_c)$ is modeled using three linear equations by fitting the data supplied by the choke supplier as,

$$Z_c(u_c) = \begin{cases} 0 & u_c < 5 \\ 0.111u_c - 0.556 & 5 \le u_c \le 50 \\ 0.5u_c - 20 & u_c > 50 \end{cases} \tag{9}$$

Finally, the equation representing the dynamics of the height of the well bore being drilled is given by,

$$\frac{dL}{dt} = v_{pen} \tag{10}$$

Here, v_{pen} is the rate of penetration or rate of drilling. In this paper, the uncertainty in the estimation of frictional losses along the annulus due to changes in the properties of the drill cuttings has not been considered. The pressure loss due to the flow of fluid through a pipeline in general, can be calculated by using Darcy-Weisbach formula [10] as,

$$\Delta P_f^x = \frac{f_d L \rho v|v|}{2D_h} \tag{11}$$

Here, *x = d (for drill string) & a (for annulus),* ρ is the density of the fluid in the pipeline, v is the velocity of the fluid in the pipeline and D_h is the hydraulic diameter of the pipeline. The velocity can be calculated as $v = q/A$ with q being the volumetric flow rate and A being the cross sectional area of the pipeline.

The Darcy friction factor f_d for turbulent flow can be evaluated using Coolebrook-White equation [10] as,

$$\frac{1}{\sqrt{f_d}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D_h} + \frac{2.51}{N_{\mathrm{Re}}\sqrt{f_d}}\right) \tag{12}$$

Here, ε/D_h is the relative roughness of the pipe and N_{Re} is the Reynold's number which can be calculated using dynamic viscosity (μ) of the fluid as,

$$N_{\mathrm{Re}} = \frac{\rho v D_h}{\mu} \tag{13}$$

However, Equation 12 is an implicit function of f_d and it has to be solved iteratively (increases computation time). Instead, an approximation to Equation 12 using Haaland equation can be used to calculate f_d as,

$$\frac{1}{\sqrt{f_d}} = -1.8\log_{10}\left(\frac{6.9}{N_{\mathrm{Re}}} + \left(\frac{\varepsilon / D_h}{3.7}\right)^{1.11}\right) \tag{14}$$

In the annulus, the drill fluid will be mixed with the fluid produced from the reservoir with a certain value of water cut. The impact of water cut on the viscosity of the fluid can be expressed using Brinkman formula [11] as,

$$\mu_r = (1 - W_c)^{-2.5} \tag{15}$$

Here, μ_r is the relative viscosity. If μ_d is the dynamic viscosity of the drill mud in the drill string, the dynamic viscosity of the mixture of fluids in the annulus can be calculated as,

$$\mu_a = \mu_r \mu_d \tag{16}$$

If $q_{res} = 0$, then $W_c = 0$. In such conditions, $\mu_a = \mu_d$.

The values of the parameters and variables used in the simulations are listed in Table 1.

Table 1: Values for the Parameters and Variables

Parameters /Variables	Value	Unit	Description
ρ_l	1150	kg / m^3	Drill mud density
W_c	0.1		Water cut of the reservoir fluid
A_d	0.0067	m^2	Cross sectional area of drill string
A_a	0.278	m^2	Cross sectional area of annulus
D_d	0.0925	m	Hydraulic diameter of drill string
D_a	0.211	m	Hydraulic diameter of annulus
L	1600	m	Vertical depth of the well
P_I	1.6667×10^{-4}	m^5 / Ns	Productivity Index value
P_{res}	250	bar	Reservoir pressure
P_{frac}	270	bar	Fracture pressure
P_{coll}	220	bar	Collapse pressure
P_0	4	bar	Pressure downstream the choke valve
ε / D_d	10^{-5}		Relative roughness of pipe in drill string
ε / D_a	10^{-4}		Relative roughness of pipe in the annulus
μ_d	0.015	kg / ms	Dynamic viscosity of the drill fluid
β_d	3×10^8	N / m^2	Bulk modulus in the drill string
β_a	2.4×10^8	N / m^2	Bulk modulus in the annulus
q_{mp}^{nom}	1500	l / min	Nominal flow rate of the drill fluid
u_c^{nom}	70	%	Nominal choke valve opening

3. Pipe Connection during Drilling

During the pipe connection procedure, the rotation of the drill bit is stopped and the pumping of the drill fluid into the drill string is stopped. A new pipe segment is added after which the mud pump is re-started. The drill bits are rotated again and the drilling continues. The connection procedure is repeated when another new pipe segment is added. The pipe connection procedure severely fluctuates the bottom hole pressure of the annulus.

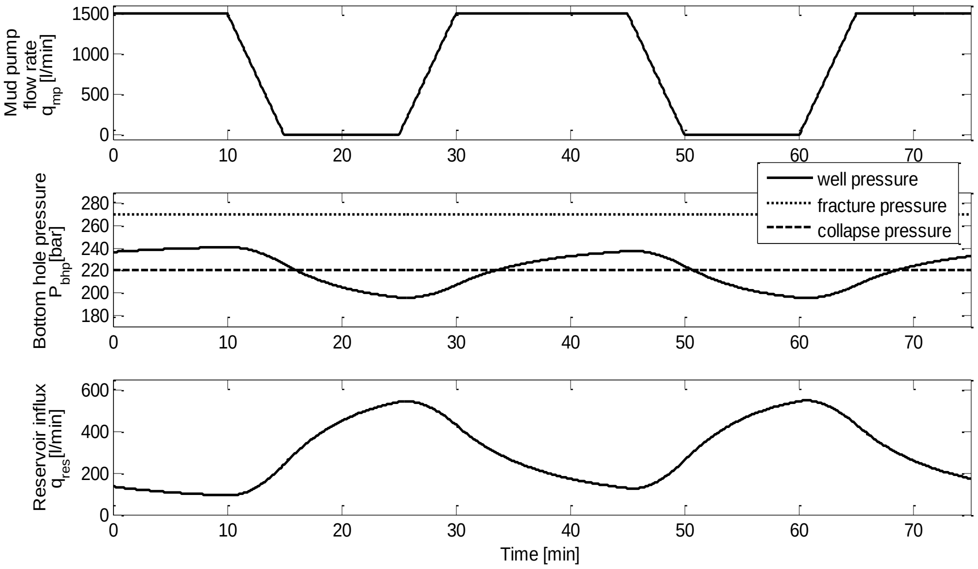

Figure 2. Bottom Hole Pressure Fluctuation during Pipe Connection

This is illustrated in Figure 2. Two pipe connections are considered, first connection at *t = 10 minutes* and second connection at *t = 45 minutes.* Steps changes in the flow rate of the drilling mud pump is avoided since it is seldom done in practice. To shut down the mud pump, the flow rate of the drilling mud pump is ramped down linearly from 1500 l/min to 0 l/min in a period of 5 minutes. Similarly, to start up the mud pump, the flow rate is ramped up linearly from 0 l/min to 1500 l/min in a period of 5 minutes. The valve opening of the choke valve at the well head is maintained at 70%.

Clearly from Figure 2, it can be seen that during the pipe connection procedure, the bottom hole pressure of the annulus falls below the collapse pressure (the lower limit of the drilling window). This should be avoided because the walls of the well bore may collapse, the drill string may be stuck and become unrecoverable. A new well should be drilled again, which will add more expenses to the drilling operation.

Assuming that the drilling is being performed near/at the reservoir zone, during the pipe connection period, the flow of fluid from the reservoir into the annulus (reservoir influx) is increased to about 600 l/min (see Figure 2). If this is not controlled, blow out of the well might occur leading to environmental damage and possible loss of lives.

The openloop simulation results in Figure 2 clearly indicates the necessity of a control system to maintain and manage the bottom hole pressure at the annulus during drilling.

4. Estimation of Bottom Hole Pressure

Unscented Kalman filter (UKF) is a nonlinear estimator widely used in process industries. The theory behind the development of UKF can be found in various literature as in [12] [13] and [14]. In this paper, the general steps that are followed while implementing an UKF are described.

Let $x=\left[P_{mp}, q_{bhp}, P_{chk}\right]^T \in \Re^{n\times 1}$ denote the states of the system, $y=\left[P_{mp}, P_{bhp}, P_{chk}\right]^T \in \Re^{m\times 1}$ denote the measurements taken from the system and $u=\left[u_c, q_{bkp}\right]^T \in \Re^{r\times 1}$ denote the control input variables that can be manipulated to keep track of the bottom hole pressure. Let the nonlinear model of the drilling operation described in section 2 be written in a general discrete form as,

$$\begin{aligned} x_{k+1} &= f(x_k, u_k, \theta_k, t_k) + p_k \\ y_k &= g(x_k, u_k, \theta_k, t_k) + m_k \end{aligned} \tag{17}$$

Here, k denotes the discrete time and θ_k are the parameters of the process. p_k is the process noise and m_k is the measurement noise which are modeled as randomly distributed white Gaussian noises such that,

$$\begin{aligned} E(p_k) = 0, \quad & E\left(p_k {p_k}^T\right) = P_{noise} \in \Re^{n\times n} \\ E(m_k) = 0, \quad & E\left(m_k {m_k}^T\right) = M_{noise} \in \Re^{m\times m} \end{aligned} \tag{18}$$

Here, P_{noise} is the process noise covariance matrix and M_{noise} is the measurement noise covariance matrix. The various steps involved in designing the UKF are listed below:

- Set the known initial mean $\hat{x}_k^+ = \hat{x}_0^+$ for $k=0$ and known initial covariance $P_k^+ = P_0^+$ for $k=0$ of the system states.
- Choose $2n$ sigma points $\hat{x}_k^{(i)}$ as,

$$\begin{aligned} \tilde{x}^{(i)} &= \left(\sqrt{nP_k^+}\right)_i^T \quad i = 1,\ldots,n \\ \tilde{x}^{(n+i)} &= -\left(\sqrt{nP_k^+}\right)_i^T \quad i = 1,\ldots,n \\ \hat{x}_k^{(i)} &= \hat{x}_k^+ + \tilde{x}^{(i)} \quad i = 1,\ldots,2n \end{aligned} \tag{19}$$

Here, $\left(\sqrt{nP_k^+}\right)_i$ is the i^{th} row of $\left(\sqrt{nP_k^+}\right)$.

- Perform the unscented transformation of the sigma points to find the transformed vectors $\hat{x}_{k+1}^{(i)}$ using the nonlinear model $f(\cdot)$ as,

$$\hat{x}_{k+1}^{(i)} = f(\hat{x}_k^{(i)}, u_k, \theta_k, t_k) \tag{20}$$

- Find the mean of the transformed vectors $\hat{x}_{k+1}^{(i)}$ to obtain *apriori* state estimate.

$$\hat{x}_{k+1}^- = \frac{1}{2n}\sum_{i=1}^{2n} \hat{x}_{k+1}^{(i)} \tag{21}$$

- Estimate the *apriori* error covariance and add the process noise covariance matrix.

$$P_{k+1}^- = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_{k+1}^{(i)} - \hat{x}_{k+1}^-\right)\left(\hat{x}_{k+1}^{(i)} - \hat{x}_{k+1}^-\right)^T + P_{noise} \tag{22}$$

- Using the *apriori* state estimate $\hat{x}_{k+1}^{-}$ and *apriori* error covariance P_{k+1}^{-}, choose $2n$ sigma points.

$$\begin{aligned}
\tilde{x}^{(i)} &= \left(\sqrt{nP_{k+1}^{-}}\right)_i^T \quad i = 1,\ldots,n \\
\tilde{x}^{(n+i)} &= -\left(\sqrt{nP_{k+1}^{-}}\right)_i^T \quad i = 1,\ldots,n \\
\hat{x}_{k+1}^{(i)} &= \hat{x}_{k+1}^{-} + \tilde{x}^{(i)} \quad i = 1,\ldots,2n
\end{aligned} \tag{23}$$

- Transform the sigma points $\hat{x}_{k+1}^{(i)}$ into predicted measurement vector $\hat{y}_k^{(i)}$ using the nonlinear measurement equation $g(\cdot)$ as,

$$\hat{y}_k^{(i)} = g(\hat{x}_{k+1}^{(i)}, u_k, \theta_k, t_k) \tag{24}$$

- Find the mean of the predicted measurement vector to obtain the predicted measurement at time *k.*

$$\hat{y}_k = \frac{1}{2n}\sum_{i=1}^{2n} \hat{y}_k^{(i)} \tag{25}$$

- Obtain the covariance of the predicted measurement and add the measurement noise covariance matrix.

$$P_y = \frac{1}{2n}\sum_{i=1}^{2n} \left(\hat{y}_k^{(i)} - \hat{y}_k\right)\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T + M_{noise} \tag{26}$$

- Obtain the cross covariance matrix between *apriori* state estimates $\hat{x}_{k+1}^{-}$ and measurement estimates $\hat{y}_k$.

$$P_{xy} = \frac{1}{2n}\sum_{i=1}^{2n} \left(\hat{x}_{k+1}^{(i)} - \hat{x}_{k+1}^{-}\right)\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T + M_{noise} \tag{27}$$

- Find the Kalman gain and update the *aposteriori* states and covariance estimates.

$$\begin{aligned}
K_k &= P_{xy}P_y^{-1} \\
\hat{x}_{k+1}^{+} &= \hat{x}_{k+1}^{-} + K_k\left(y_k - \hat{y}_k\right) \\
P_{k+1}^{(i)} &= P_{k+1}^{-} - K_k P_y K_k^T
\end{aligned} \tag{28}$$

- Repeate the above steps for *k = 1,2* until the end of the simulation time.

Figure 3 shows the simulation results of the application of the UKF during a pipe connection procedure. This is an openloop simulation results i.e. feedback control systems are not yet implemented. It is also assumed that the measurements of the bottom hole pressure are available even during the pipe connection period.

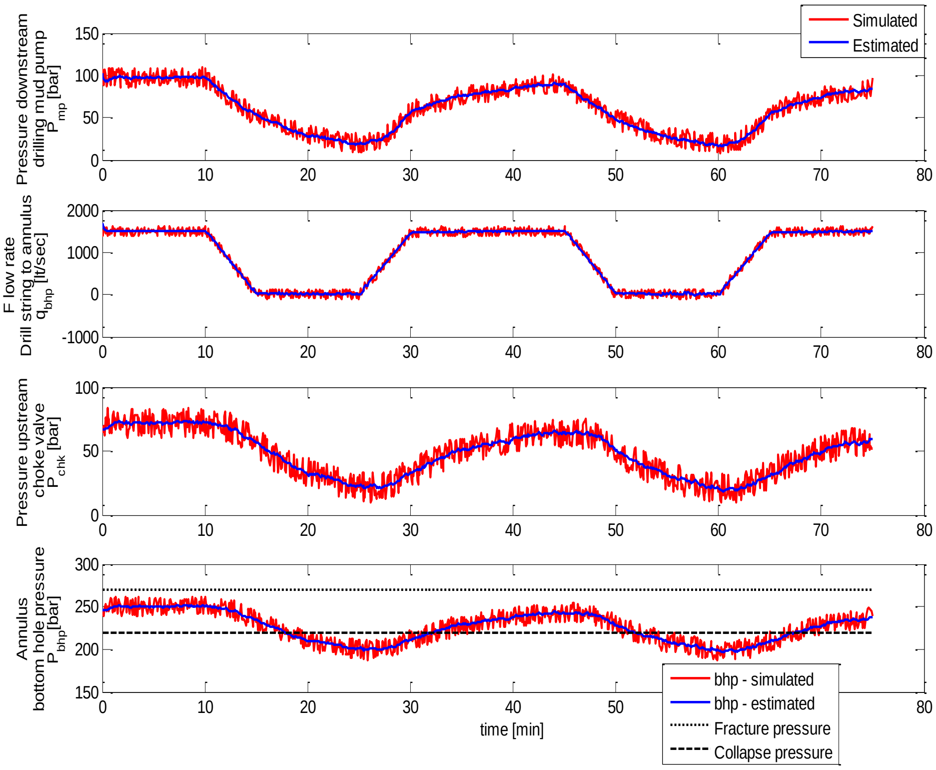

Figure 3. Nonlinear Estimation using UKF

The first pipe connection is started at the simulation time of *t = 10 minutes* and completed at *t = 30 minutes*. The second pipe connection starts at *t = 45 minutes* and ends at *t = 65 minutes*. The three states of the process which are the pressure downstream the drilling mud pump, flow rate of fluid from the annulus into the drill string at the bottom hole and the pressure upstream the choke valve are estimated properly using the UKF. Similarly, the measurement which is the bottom hole pressure at the annulus is also properly estimated by using the nonlinear estimator as can be seen in Figure 3.

4.1. Estimation in the Absence of Measurement Data

As also previously described, the measurement data of the bottom hole pressure is not always available. During the pipe connection period, the mud flow telemetry system is not able to provide a reliable measurement of the bottom hole pressure at the annulus. In the absence of such measurement data, UKF can still be used to estimate the states of the process and the measurements. Two different ways are generally used in such a situation.

- The last known value of the bottom hole pressure measurement denoted by P_{bhp}^{last} can be used to replace y_k in equation (28).

$$\begin{aligned} y_k &= P_{bhp}^{last} \\ \hat{x}_{k+1}^{+} &= \hat{x}_{k+1}^{-} + K_k\left(y_k - \hat{y}_k\right) \end{aligned} \tag{29}$$

- The *apriori* state estimate $\hat{x}_{k+1}^{-}$ can be used to calculate the value of y_k by utilizing the nonlinear measurement equation $g(\cdot)$. This value of y_k can then be used in equation (28).

$$\begin{aligned} y_k &= g(\hat{x}^{-}_{k+1}, u_k, \theta_k, t_k) \\ \hat{x}^{+}_{k+1} &= \hat{x}^{-}_{k+1} + K_k\left(y_k - \hat{y}_k\right) \end{aligned} \tag{30}$$

The simulation results comparing the above two ways of handling the absence of measurement data are shown in Figure 4.

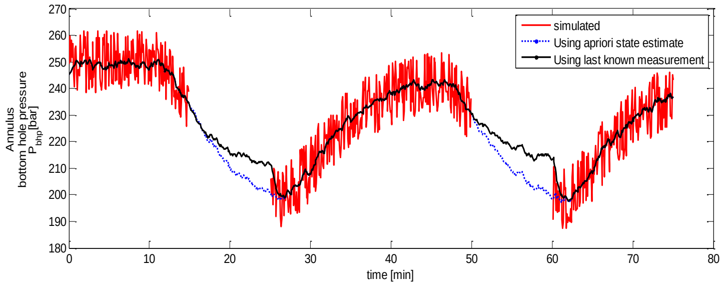

Figure 4. Bottom Hole Pressure Estimation in the Absence of Measurement Data

The measurement of the bottom hole pressure are not available in between the time period from *t = 15 minutes* to *t = 25 minutes* during the first pipe connection. It is not available during the second pipe connection between the periods from *t = 50 minutes* to *t = 60 minutes*. Clearly from Figure 4, the estimation of bottom hole pressure using the *apriori* state estimates is better in comparison to the estimates obtained using the last known measurement value. In the latter case, the estimates tends to diverge from actual value. However, as soon as the measurements are available again, the faulty estimates quickly converge to the actual value.

5. Control of Bottom Hole Pressure

Since the pipe connection procedure causes the bottom hole pressure to fluctuate, it should be controlled so that it lies within the drilling window. A nonlinear model predictive controller (NMPC) is implemented. NMPC utilizes the estimates of the states and the measurements provided by the UKF to generate control actions. In this paper, in addition to the choke valve opening (u_c), the flow rate of the back pressure pump (q_{bkp}) is also taken into account as control inputs.

5.1. Formulation of Optimal Control Problem

NMPC solves a nonlinear optimal control problem at each time step. The optimal control problem is formed by looking into the future horizon for which the nonlinear model of the process is used. Details about the theory regarding model-based control can be found in dedicated literature as in [15] and [16] and is therefore not described in this paper.

The goal is to control the pressure at the bottom hole of the annulus within the drilling window. For the control system, let us define a setpoint for the bottom hole pressure (P^{ref}_{bhp}) to be equal to the reservoir pressure i.e. $P^{ref}_{bhp} = P_{res}$. Then a nonlinear objective function for the optimal control problem can be defined as,

$$\min_{\Delta u} \; f(\Delta u) = \sum_{k=1}^{N_p} \lambda_{e,k}\left(P^{ref}_{bhp} - P_{bhp,k}\right)^T Q \left(P^{ref}_{bhp} - P_{bhp,k}\right) + \sum_{k=1}^{N_c} \lambda_{\Delta u_k}\left(\Delta u_k\right)^T R \left(\Delta u_k\right) \tag{31}$$

Here, N_p is the prediction horizon length and N_c is the control horizon length. For simplicity, $N_p = N_c = N$ is considered. $\lambda_{e,k}$ is the weighting factor for the set point error and $\lambda_{\Delta u_k}$ is the weighting factor the control deviation. To make the formulation and computation simpler, same value for the weighting factor for the entire horizon length is considered and denoted as λ_e and $\lambda_{\Delta u}$ .Here, the rate of change of control inputs is denoted by Δu_k . The control signal at time k can be calculated as $u_k = \Delta u_k + u_{k-1}$. The two control inputs are denoted by the vector $u_k = \left[u_{c,k},\ q_{bkp,k}\right]^T$.

The annulus bottom hole pressure should be greater than the well collapse pressure (P_{coll}) and lower than the fracture pressure (P_{frac}), i.e. the constraint in the output is,

$$P_{coll} \le P_{bhp,k}(\Delta u_k) \le P_{frac} \tag{32}$$

The choke valve opening should be between 0 and 100, i.e. the constraint in the control input is,

$$0 \le u_{c,k} \le 100 \tag{33}$$

In practice, the choke valves are opened in smaller steps and larger abrupt changes in its opening is usually avoided. The choke valve is assumed to be opened or closed by only 0.5% per second i.e.

$$-0.5 \le \Delta u_{c,k} \le 0.5 \tag{34}$$

Similarly, it is considered that the flow rate of the back pressure pump cannot be increased or decreased by more than 6 l/sec at each time step i.e.

$$-6 \le \Delta q_{bkp,k} \le 6 \tag{35}$$

The maximum flow rate capacity of the back pressure pump is assumed to be 400 l/sec and when it is not in use, it is 0 l/sec.

$$0 \le q_{bkp,k} \le 400 \tag{36}$$

Equations 32 - 36 form the constraints for the optimization problem.

5.2. Simulation Results

The tool used for simulation is MATLAB. However, to solve the nonlinear optimization problem, *Opti-Toolbox* [17], which is an open source toolbox for mathematical optimization, has been used. The nonlinear solver (open source) that is used to solve the nonlinear optimal control problem at each time step of the simulation is *IPOPT* [18]. The values of the parameter used for the simulation are listed in Table 1. The simulation time step is taken to be 15 seconds and $N = 25$ time steps is considered as the prediction horizon length.

To implement the nonlinear control system, two consecutive pipe connection procedure are considered. To shut down the drilling mud pump, the flow rate of the pump is ramped down linearly from 1500 l/min to 0 l/min in a period of 5 minutes. Similarly, to start up the pump, the flow rate is ramped up linearly. The first pipe connection is started at *t = 10 minutes* and second connection at *t = 45 minutes.* The simulation results are shown in Figure 5.

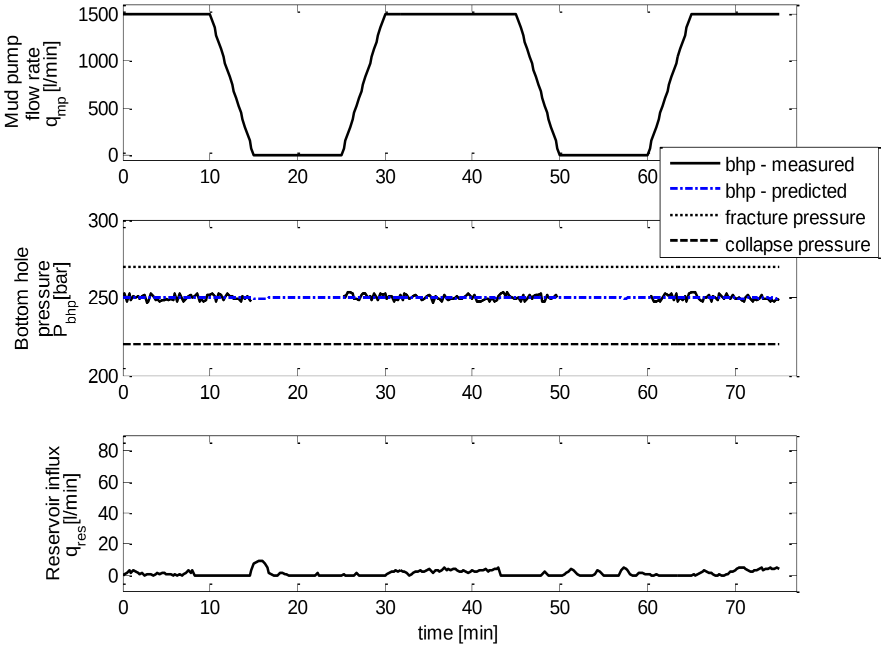

Figure 5. Nonlinear Control of Bottom Hole Pressure of the Annulus using NMPC

As can be clearly seen in Figure 5, the nonlinear control system is able to keep the bottom hole pressure within the drilling window even during the two consecutive pipe connection periods, where the fluctuation in the bottom hole pressure would otherwise have been severe. Furthermore, the controller is able to keep track of the setpoint that is equal to the reservoir pressure (pore pressure). Since there is no reliable measurement of the bottom hole pressure during the pipe connection, the predicted measurement has to be analyzed. From Figure 5, it can be seen that the predicted bottom hole pressure is well controlled by the nonlinear controller when used in conjunction to a nonlinear estimator.

The reservoir influx is also properly controlled. There is almost no influx from the reservoir. This means that there will be no sudden flow of reservoir fluid into the annulus and blow out of the well is prevented. Probably the occurrence of kick will also be avoided if the reservoir influx can be kept at such a low value, which is almost equal to zero throughout the drilling operation as seen in Figure 5.

At the same time, the controller keeps the bottom hole pressure to remain above the collapse pressure. This will ensure that the well will not collapse while drilling and hence improves the continuity of the drilling operation without any losses of well or connecting pipes. The bottom hole pressure is controlled to remain far below the fracture pressure. It prevents the cracking/fracture of the walls of the drilled well. The leakage/loss of the drilling fluid through the cracks is avoided. At the same time, this also prevents the drilling mud particles from being trapped within the cracks. If the opening of the pores of the reservoir are blocked by such particles due to the formation of cracks/fractures, the reservoir fluid cannot seep into the annulus sufficiently i.e. productivity of the well will be significantly reduced when it is set into production. However, this can be totally prevented by using a nonlinear model predictive controller.

Both control inputs are utilized to achieve a good control of the bottom hole pressure as can be seen in Figure 6. As the drilling mud pump is shut down (to

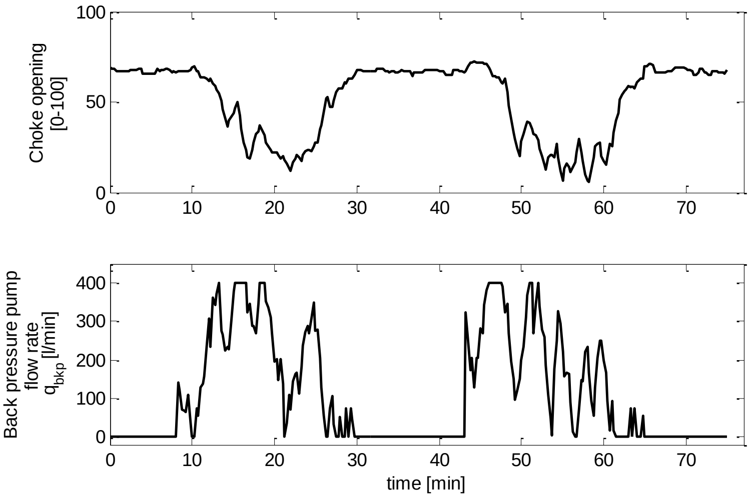

Figure 6. Manipulation of the Control Inputs: Choke Valve Opening and Flowrate of the Back Pressure Pump

initiate the first pipe connection procedure) at *t = 10 minutes*, the opening of the choke valve is lowered and at the same time, the back pressure pump kicks into action by increasing its flowrate. The combined action of these two control inputs ensures that there is enough accumulation of mass in the annulus for providing sufficient hydrostatic pressure for maintaining the bottom hole pressure. After the first pipe connection is completed at *t = 25 minutes*, the mud pump is restarted (its flow rate starts to ramp up towards its nominal value). The controller then increases the opening of the choke valve and at the same time decreases the flow rate of the back pressure pump towards zero. Similar control actions and their consequences are also seen during the second pipe connection at *t = 45 minutes.*

It is interesting to note that the back pressure pump is used only during the pipe connection period. During the normal drilling operation, it is not used. The choke valve takes the sole responsibility of maintaining the bottom hole pressure during the normal drilling operation. From an economic point of view, this is the right and a normal thing to do to save the electrical power utilized by the back pressure pump. If the drilling is being performed offshore, it is important and necessary to save the electrical power (which is generally produced using diesel generator), as there will not be any connection of electricity grid lines to such offshore platforms.

5.3. Comment on Computational Time

Nonlinear MPC is in general a computationally demanding algorithm/controller. During the simulation, at each time step (15 seconds period), there are 50 variables to be optimized and 250 inequality constraints to be satisfied. With a laptop having 2.50 GHz processor and 4 GB memory, for each time step, it takes roughly only 10 seconds to solve the nonlinear optimization problem once. In other words, to obtain a control action at each time step during drilling operation, it takes about 10 seconds to calculate the value of the control inputs. This computational time required to solve the nonlinear control problem is less than the sampling time. This suggests that the algorithm has a good potential to be used for real time application. Moreover, with better processing power (multi-core and multi-CPUs) and with parallelization, the algorithm can be easily adapted and used for real time control of drilling operation. Other tricks like grouping of control inputs for reducing the number of variables to optimize can also be applied to decrease the computational time required by the nonlinear MPC

6. Conclusion

Given that the mathematical model reflects the actual dynamics of the system, this research article shows the possibility of using a nonlinear model predictive controller along with a nonlinear estimation algorithm to maintain the bottom hole pressure of the annulus in a normal drilling operation and during the pipe connections. The algorithms used in this paper have a potential to be implemented for real time application in terms of computational requirement. Nonlinear MPC along with UKF offer a good combination to control and estimate the bottom hole pressure as they are comparatively easier to implement together. The changes in the density of the fluid in the annulus due to the reservoir influx has been modeled in the paper. However, the uncertainty in the calculation of density and frictional losses in the annulus due to drill cuttings, and the uncertainty in the choke valve operation have not been considered, and should be analyzed further in more detail.

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Authors

Roshan Sharma, He received his bachelor degree in Electronics and Communication Engineering from Tribhuvan University, his master degree in Systems and Control Engineering and his PhD degree from Telemark University College (TUC), Norway. Currently, he is associated at TUC at the department of Electrical Engineering, IT and Cybernetics. His area of research are within the field of advanced process control and optimization, modeling and simulation of dynamic systems and industrial automation.