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Stochastic Model Analysis and Control Strategy Design on Preventive Maintenance Based on Condition

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Abstract

It is well known that condition based preventive maintenance (CBM) plays a key role in improving reliability of the devices and increasing security of the production process. But compared with traditional time based preventive maintenance (TBM), due to being an unplanned examining-maintenance activity beforehand, stationary operation of the devices is possibly interrupted so as to make whole production process become unstable, for example, connected-grid wind power generators, large-size nuclear power facilities, and railway transportation signals, and etc. Hence, in this paper the full life cycle model of the devices operation is firstly constructed with preventive maintenance (PM) involved, and then its reliability indexes and steady-state operation characteristics are analyzed with the model parameters being time-invariant corresponding to TBM. On the basis of it, the operation behavior of time-variant system is researched corresponding to CBM under some limitations, such as stability, accessibility, and controllability, and etc. The investigations indicate that TBM is a predictable and controllable process such that it can satisfy one's expectation by selecting suitable model parameters, whereas CBM is usually unpredictable such that it can not arrive at desired aim. Therefore, based on the relationships of the two, this paper proposes a CBM control scheme based on TBM, and investigates the dynamical behavior and real-time characteristics of control systems. The goal to do this makes the CBM exhibit some expected behaviors like the TBM such that the behaviors of the time-variant system can meet the desired requirements. The investigation results show that the proposed technology is quite effective, and is fit to control stationary operating of the systems under CBM scene. And so the proposed method possesses certain theory instruction significances for PM maintenance practices for those large-scale production processes.

Keywords*: Stochastic model, preventive maintenance, TBM, CBM, control theorem, reliability, stability, controllability*

1. Introduction

With the rapidly development of condition based preventive maintenance (CBM) technology, CBM has been broadly applied in modern large-size in industry enterprises to instruct their operation maintenance practices to ensure the safety of production processes, for example, wind power plants, nuclear power stations, and railway transportation facilities [1-3]. In particular, wind power plants, which locate in the relatively remote regions with large-area distribution of the generators and abominable climate environment, often require a fireheight-ascending maintenance for wind turbines. Thus, once failure occurs, the restoration of wind turbines often become difficult [4]. For this reason, modern wind turbine is installed a set of the monitoring cell to guard its status [5]. If one cell is in failure it can be detected out quickly and restored to the good state by a timely repairing [6]. Clearly, CBM plays a significant role during operation

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maintenance management of the devices. However, compared with traditional time based maintenance (TBM), CBM likely arouses some unexpected interruptions of the production process for an unplanned maintenance practice activities beforehand, which is aroused by the randomness of the CBM. And conversely, TBM is a planed preventive maintenance activity, and so can guarantee the stability of the production operation due to a large number of substitutes in advance, but possesses a higher cost [7]. Known from the above analysis, TBM is a maintenance activity arranged well in advance whose process is controllable and stationary, whereas CBM is a random one whose process is uncontrollable such that the stability of the production can be influenced, possibly. Hence, it is important to make its behavior become controllable like TBM while applying CBM. And thus we not only can fully play a role of CBM but also overcome its disadvantages, and simultaneously, the maintenance cost is also reduced as soon as possible. In [8], a preventive maintenance (PM) model on TBM is firstly proposed and analyzed, and then CBM is investigated further on the basis of it. But the model is simplified and the involving parameters are less during analysis process. In [9], a real-time control strategy is proposed based on reference model, and the stability is analyzed using Lyapunov method on CBM. Similarly, the model is simplified and the established theorem lakes ubiquity. In [10], a model reference adaptive control strategy is proposed to ensure the reliable completion of the task and reduce maintenance cost based on hyperstability theorem. But the constructed model is simple and the discussed parameters are inadequate. Hence, in this paper, we firstly construct a stochastic model of PM with time-invariant parameters, and discuss its reliability indexes and analyze its stability. And then time-variant model is investigated and its stability and controllability are analyzed under some given limitations. This paper still investigates accessibility, accessibility time distribution, and control convergence properties of time-variant system, and etc, and expects that the time-variant systems can possess some good behaviors like the time-invariant under some control conditions. The investigation results show that the proposed scheme is quite effective, and is a suitable control technology on PM with time-variant parameters, and therefore possesses certain theory instruction significances for maintenance practices of large-size enterprises.

2. TBM Model

2.1 TBM Model Description

To establish the full life cycle model of repairable devices under PM, the following assumptions require to be considered below.

Assumption1. The preventive maintenance and the corrective maintenance are separately expressed each using one state.

Assumption2. Whether at the operating or storage state, there are no devices the failures of which are no detectable.

Assumption3. All transfer rates between the states equal constants, follow exponential distribution. In particular, when the check and maintenance time are constants, we can define it as TBM model, a fixed time-interval PM method.

Assumption4. Preventive maintenance cannot change the natural failure rate of the equipments, the instantaneous failure rate after the equipment is repaired is same with one before repaired, and only the residual life is owned when it returns to work.

Assumption5. Preventive maintenance is perfect, if there are faults detected out during preventive maintenance, and then it would be able to get a timely repair.

Assumption6. Corrective maintenance is perfect, the equipment can restore to the original state after once corrective maintenance.

Based on the above assumptions, the life cycle model of the devices can be represented using the state transition diagram in Figure 1.

In Figure 1, the symbol Si expresses the state of the equipment, where the subscript i ∈[1, 2, 3, 4] is applied to respectively express the equipment storage, work, preventive maintenance, and corrective maintenance of four kinds of states; and □1 expresses the storage rate of a functioning equipment; and □2 expresses the using rate of a stored equipment; and v1 presents the failure rate of a stored equipment; and v2 presents the failure rate of a functioning equipment; and u1 presents the check rate of a stored equipment; and u2 presents the check rate of a functioning equipment; and □1 expresses the stored probability of the device after a preventive maintenance intervention; and 1-□1 expresses the used probability of the device after a preventive maintenance intervention; and □2 expresses the stored probability of the device after a corrective maintenance intervention; and 1-□2 expresses the used probability of the device after a corrective maintenance intervention; and □1 expresses the repair risk rate function of the preventive maintenance; □2 expresses the repair risk rate function of the corrective maintenance; □0 expresses the probability that the device being at PM state is transferred into corrective maintenance state. The physical meaning of the transfer rate is to ensure that it is the bounded and the nonnegative. Let S(t) be the situated state of the device at time t, and then S(t) is a continuous time Markov process, namely, at any time t, as the specific numerical value of S(t) is given, and the operation behavior after t of the process S(t) has nothing to do with the history before t.

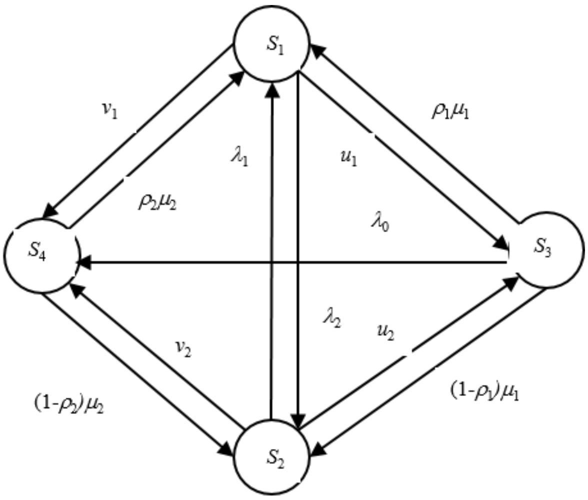

Figure 1. State Transition Diagram with Four-State

For ∀ $t \geq 0$, we use $x_i(t)$ to express the situated state of system $S(t)$ at time t, and then

$$x_i(t) = p\{S(t) = S_i\}, \quad i=1, 2, 3,4 . \tag{1}$$

Deinition1. Let $A_V(t)$ express availability degree of the equipment which can be defined as a probability that it can function normally at time t. According to Figure 1, at states S_1 and S_2 the equipments are normal, and so these states are available. For S_3, it can be thought to be unavailable if functioning equipments terminate working due to PM interference, which is equivalent to smaller repairing of PM. And conversely, if the state S_3 can be considered to be available unless the equipments working is not terminated due to PM interference, which is equivalent to smallest repairing of PM[11]. No matter what kinds of PM mentioned above, we here always assume that they follow the Assumption 4, and mainly focus on the former case. In S_4, where corrective maintenance is implemented and the equipment is thought shun down, thus it is unavailable, According to the definition on availability, we then have

$$A_V(t) = x_1(t) + x_2(t) = 1^T \boldsymbol{x}(t) \tag{2}$$

where $\mathbf{1}^T$ =[1 1], and $\boldsymbol{x}(t)$=[$x_1(t)$ $x_2(t)$]^T.

Deinition2. The reliability degree of the equipment is defined as the probability that it can work normally within the scope of the time from zero to t. It does not allow failure and any maintenance. According to the definition, the reliability degree $R(t)$ should have nothing to do with traversing what states before the device becomes the failure for the first time. In this system, we always ignore the natural failure or obsolescence due to ageing. Similarly, based on assumption 4, we think although the preventive state S_3 could arouse machines halt, but not influence the equipments reliability. And so does S_4. But when computing MTTF, the time that equipment stays at these two states should be deducted before it is possibly given an eventual substitution. Hence, according to [12], we then have

$$\begin{aligned} R(t) &= P_r(T > t) = 1 - P_r(T \le t) = 1 - F(t) = 1 - \int_0^t f(t)\mathrm{d}t \\ &= \int_t^{\infty} f(t)\mathrm{d}t = F(\infty) - F(t) = R(t) - R(\infty) \\ &= x_1(t) + x_2(t) - x_1(\infty) - x_2(\infty) \end{aligned} \tag{3}$$

where $F(t)$ is the failure time distribution function, and whose probability density function(*pdf*) is $f(t)$ for $\forall t \ge 0$.

And then based on (3), MTTF can be defined by

$$\mathrm{MTTF} = \int_0^{\infty} R(t)\mathrm{d}t = 1^T \int_0^{\infty} [\boldsymbol{x}(t) - \boldsymbol{x}(\infty)]\mathrm{d}t \tag{4}$$

where the definition on $\mathbf{1}^T$ and $\boldsymbol{x}(t)$ is same with definition 1, and $\boldsymbol{x}(\infty)$=[$x_1(\infty)$ $x_2(\infty)$]^T.

According to (4), the physical meaning of MTTF can be here understood as mean time that the devices arrive at stable-state $\boldsymbol{x}(\infty)$ from any non-zero initial state starting. On the scope of time interval, it is impossible for it to be given an eventually substitution though PM exists. In other words, the devices can be substituted only when it is in stable-state.

2.2 TBM Model Analysis

According to Assumption 3, the average time of the system to stay at each state follows the exponential distribution, such that after linearization and ignoring high-order items the transition rates between each state are constants, namely, a kind of fixed time interval PM, and is defined as TBM, where the parameters of the model are constants.

According to Figure 1 and reliability mathematics theory [13, 14] and we then have the following Kolmogorov equation-group.

$$\begin{cases} \mathrm{d}x_1(t)/\mathrm{d}t = -(\lambda_2+u_1+v_1)\, x_1(t)+\lambda_1 x_2(t)+\rho_1\mu_1 x_3(t)+\rho_2\mu_2 x_4(t) \\ \mathrm{d}x_2(t)/\mathrm{d}t = \lambda_2\, x_1(t)-(\lambda_1+u_2+v_2)\, x_2(t)+(1-\rho_1)\mu_1\, x_3(t)+(1-\rho_2)\mu_2\, x_4(t) \\ \mathrm{d}x_3(t)/\mathrm{d}t = u_1 x_1(t)+u_2\, x_2(t)-(\mu_1+\lambda_0)x_3(t) \\ \mathrm{d}x_4(t)/\mathrm{d}t = v_1 x_1(t)+v_2\, x_2(t)+\lambda_0 x_3(t)-\mu_2 x_4(t) \end{cases} \tag{5}$$

The physical bounds of the transition rates satisfy

$0 \le \lambda_1 \le \lambda_{1\max}$, $0 \le \lambda_2 \le \lambda_{2\max}$, $0 \le v_1 \le v_{1\max}$, $0 \le v_2 \le v_{2\max}$, $0 \le u_1 \le u_{1\max}$, $0 \le u_2 \le u_{2\max}$,
$0 \le \mu_1 \le \mu_{1\max}$, $0 \le \mu_2 \le \mu_{1\max}$, $0 \le \lambda_0 \le \lambda_{1\max}$, $0 \le \rho_1 \le 1$, $0 \le \rho_2 \le 1$, $\forall t \ge 0$.

The system states are mutually exclusive, and so the following equation is bound to hold.

$$x_1(t)+ x_2(t)+ x_3(t)+ x_4(t)=1, \quad \forall t \ge 0 \qquad (6)$$

where $0 \le x_1(t) \le 1$, $0 \le x_2(t) \le 1$, $0 \le x_3(t) \le 1$, $0 \le x_4(t) \le 1$, $\forall t \ge 0$.
From (6), we obtain

$$x_4(t)=1-[x_1(t)+x_2(t)+x_3(t)] \qquad (7)$$

Substituting (4) into the former three in (5), and we then have

$$\begin{cases} \mathrm{d}x_1(t)/\mathrm{d}t = -(\lambda_2+\lambda_3+\rho_2\mu_2)\, x_1(t)+(\lambda_1-\rho_2\mu_2)x_2(t)+(\rho_1\mu_1-\rho_2\mu_2)x_3(t)+\rho_2\mu_2 \\ dx_2(t)/\mathrm{d}t = [\lambda_2-(1-\rho_2)\mu_2\,)]x_1(t)-[(\lambda_1+\lambda_4+(1-\rho_2)\mu_2]\, x_2(t)+[(1-\rho_1)\mu_1-(1-\rho_2)\mu_2]x_3(t) +(1-\rho_2)\mu_2 \\ dx_3(t)/\mathrm{d}t = u_1x_1(t)+u_2x_2(t)-(\mu_1+\lambda_0)x_3(t)\; x_1(t)+x_2(t)+x_3(t) \le 1, \quad \forall t \ge 0 \end{cases} \qquad (8)$$

where $\lambda_3=u_1+v_1$ and $\lambda_4= u_2+v_2$.
Writting (5) into matrix form, and we obtain

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{\mu} \qquad (9)$$

where

$$\boldsymbol{A} = \begin{bmatrix} -d_1 & a_1 & a_2 \\ b_1 & -d_2 & b_2 \\ u_1 & u_2 & -d_3 \end{bmatrix} \quad \boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad \boldsymbol{\mu} = \begin{bmatrix} \rho_2\mu_2 \\ (1-\rho_2)\mu_2 \\ 0 \end{bmatrix}$$

$d_1=\lambda_2+\lambda_3+\rho_2\mu_2$, $a_1=\lambda_1-\rho_2\mu_2$, $a_2=\rho_1\mu_1-\rho_2\mu_2$, $b_1=\lambda_2-(1-\rho_2)\mu_2$, $d_2=\lambda_1+\lambda_4+(1-\rho_2)\mu_2$, $b_2=(1-\rho_1)\mu_1-(1-\rho_2)\mu_2$, $d_3=\mu_1+\lambda_0$.

where d_1 means the probability transiting out from state S_1 per unit time Δt, and b_1 and u_1 respectively are the probabilities transiting into S_2 and S_3 and leaving from S_1 at the same time, and a_1 and a_2 respectively express the probabilities leaving from S_2 and S_3 and entering into S_1 at the same time. Clearly, according to physical significance of the parameters, the conditions $d_1 \ge b_1+u_1$, and $d_1+a_1+a_2>0$ must be satisfied. Likewise, the following bound conditions also must be satisfied: $d_2 \ge a_1+u_2$, and $d_2+b_1+b_2>0$, and as well as $d_3 \ge a_2+b_2$, and $d_3+u_1+u_2>0$. In addition, we still have $a_1+\rho_2\mu_2>0$, and $b_1+(1-\rho_2)\mu_2>0$. Thus $\boldsymbol{A}$ is a diagonally dominant matrix.

Theorem1. The system described by (9) is a stable, and such that

$$\boldsymbol{x}(t) = e^{\boldsymbol{A}t}[\boldsymbol{x}(0)+\boldsymbol{A}^{-1}\boldsymbol{\mu}]-\boldsymbol{A}^{-1}\boldsymbol{\mu} \qquad (10)$$

Further,

$$x(\infty) = \lim_{t\to\infty}\boldsymbol{x}(t) = \lim_{t\to\infty}[e^{\boldsymbol{A}t}[\boldsymbol{x}(0)+\boldsymbol{A}^{-1}\boldsymbol{\mu}]-\boldsymbol{A}^{-1}\boldsymbol{\mu}] = -\boldsymbol{A}^{-1}\boldsymbol{\mu} \qquad (11)$$

Proof. According to (9), we easily obtain det$\boldsymbol{A}$<0 except for some trivial cases, and thus $\boldsymbol{A}$ is invertible. Similarly, since $\boldsymbol{A}$ and $\boldsymbol{\mu}$ are continuous in $[0, +\infty)$, and integral. And so we take the integration at both sides of (9) from zero to t, and $t\in[0, +\infty)$, and then we can obtain (10), immediately, wherein $\boldsymbol{x}(0)$ is the initial state at t=0. Below we prove the characteristic roots of $\boldsymbol{A}$ possess the negative real parts.

Let s_1, s_2, and s_3 be the characteristic roots of $\boldsymbol{A}$, respectively, from $\boldsymbol{A}$, we have

$$\begin{cases} s_1 \times s_2 \times s_3 = \det \boldsymbol{A} < 0 \\ s_1 + s_2 + s_3 = -(d_1 + d_2 + d_3) \\ \rho(\boldsymbol{A}) \le \|\boldsymbol{A}\|_m < (d_1 + d_2 + d_3) \end{cases} \quad (12)$$

where $\rho(\boldsymbol{A})$ is spectral radius and $\| \|_m$ is one operator norm of $\boldsymbol{A}$.

Known from the first formula in (12), the $\boldsymbol{A}$ either possesses three characteristic values with real parts being negative, or at least one characteristic value with negative real number. On the other hand, seen from the second in (12), the absolute value of this sole negative real root should be the maximal. And then the last formula in (12) shows that absolute value of the sole negative real root does not exceed the sum of all the diagonally dominant elements, which predicts that another two roots either are the negative real roots or a pair of conjugate complex roots with real parts being negative. In a word, all characteristic values of $\boldsymbol{A}$ possess negative real parts. And thus, based on (10) and (11), we have

$$\boldsymbol{x}(t) - \boldsymbol{x}(\infty) = e^{\boldsymbol{A}t}[\boldsymbol{x}(0) - \boldsymbol{x}(\infty)]$$

Hence, we have

$$\begin{aligned} \| \boldsymbol{x}(t) - \boldsymbol{x}(\infty) \| &= \| e^{\boldsymbol{A}t}[\boldsymbol{x}(0) - \boldsymbol{x}(\infty)] \| \le \| e^{\boldsymbol{A}t} \| \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| = \| \boldsymbol{P} e^{\boldsymbol{\Lambda}t} \boldsymbol{P}^{-1} \| \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| \\ &\le \text{cond}(\boldsymbol{A}) \| e^{\boldsymbol{\Lambda}t} \| \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| \le \text{cond}(\boldsymbol{A}) \| e^{Re\boldsymbol{\Lambda}t} \| \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| \end{aligned}$$

where $\| \;\; \|$ is matrix operator norm, and $\boldsymbol{P}$ is the matrix formed by characteristic vectors of the characteristic values of $\boldsymbol{A}$, and $\text{cond}(\boldsymbol{A}) = \| \boldsymbol{P} \| \| \boldsymbol{P}^{-1} \|$ is condition number of $\boldsymbol{A}$, and $\boldsymbol{\Lambda}$ is the diagonal matrix formed by characteristic values of $\boldsymbol{A}$, and the symbol Re means taking real part. Clearly,

$$\lim_{t\to\infty} \| \boldsymbol{x}(t) - \boldsymbol{x}(\infty) \| \le \lim_{t\to\infty} \text{cond}(\boldsymbol{A}) \| e^{Re\boldsymbol{\Lambda}t} \| \| \boldsymbol{x}(0) - \boldsymbol{x}(\infty) \| = 0$$

End.

In fact, let $(\boldsymbol{X}, \| \|)$ is a Banach space with a basis $\{x_i\}$, i=1,2,3, and then projections $\boldsymbol{A}$: $\boldsymbol{X} \to \boldsymbol{X}$, defined by (9), are tense bound linear operator in interval [0, $+\infty$), and so $\|e^{\boldsymbol{A}t}\|$ is convergent[15].

Proposition1. System instantaneous availability degree can be expressed by

$$\boldsymbol{A}_{\mathrm{V}}(t) = 1^{\mathrm{T}} \boldsymbol{x}(t) = 1^{\mathrm{T}} \{ e^{\boldsymbol{A}t}[\boldsymbol{x}(0) + \boldsymbol{A}^{-1}\boldsymbol{\mu}] - \boldsymbol{A}^{-1}\boldsymbol{\mu} \} \quad (13)$$

Further, the steady-state availability exists and has nothing to do with the initial states.

$$\boldsymbol{A}_{\mathrm{V}} = -1^{\mathrm{T}} \boldsymbol{A}^{-1} \boldsymbol{\mu} \quad (14)$$

where $\mathbf{1}^{\mathrm{T}}$=[1 1 0], $\boldsymbol{x}(t)=[x_1(t) \;\; x_2(t) \;\; x_3(t)\,]^{\mathrm{T}}$.

Proof. According to (2) in Definition 1 and (10), the formula (13) is evident. To prove (14), we have

$$A_{\mathrm{V}} = \lim_{t\to\infty} A_{\mathrm{V}}(t) = 1^{\mathrm{T}} \lim_{t\to\infty} \boldsymbol{x}(t) = 1^{\mathrm{T}} \lim_{t\to\infty} \{ e^{\boldsymbol{A}t}[\boldsymbol{x}(0) + \boldsymbol{A}^{-1}\boldsymbol{\mu}] - \boldsymbol{A}^{-1}\boldsymbol{\mu} \} = -1^{\mathrm{T}} \boldsymbol{A}^{-1}\boldsymbol{\mu}$$

End.

Proposition2. System instantaneous reliability degree can be expressed by

$$R(t) = 1^{\mathrm{T}} e^{\boldsymbol{A}t}[\boldsymbol{x}(0) + \boldsymbol{A}^{-1}\boldsymbol{\mu}] \quad (15)$$

Further, system MTTF can be expressed by

$$\text{MTTF} = -1^{\mathrm{T}} \boldsymbol{A}^{-1}[\boldsymbol{x}(0) - \boldsymbol{x}(\infty)] \quad (16)$$

where $\mathbf{1}^{\mathrm{T}}=[1\ \ 1\ 0]$, $\boldsymbol{x}(t)=[x_1(t)\ \ x_2(t)\ \ x_3(t)\ ]^{\mathrm{T}}$.

Proof. According to (10) and (11), we have

$$\boldsymbol{x}(t)+\boldsymbol{A}^{-1}\boldsymbol{\mu}=e^{\boldsymbol{A}t}[\boldsymbol{x}(0)+\boldsymbol{A}^{-1}\boldsymbol{\mu}]\rightarrow\boldsymbol{x}(t)-\boldsymbol{x}(\infty)=e^{\boldsymbol{A}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]$$

And then according to (3) in Definition 2, and we have

$$R(t)=1^{\mathrm{T}}[\boldsymbol{x}(t)-\boldsymbol{x}(\infty)]=1^{\mathrm{T}}e^{\boldsymbol{A}t}[\boldsymbol{x}(0)+\boldsymbol{A}^{-1}\boldsymbol{\mu}]$$

From (4), we obtain

$$\mathrm{MTTF}=\int_0^\infty R(t)\mathrm{d}t=1^{\mathrm{T}}\int_0^\infty e^{\boldsymbol{A}t}(\boldsymbol{x}(t)-\boldsymbol{x}(\infty))\mathrm{d}t=-1^{\mathrm{T}}\boldsymbol{A}^{-1}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]$$

End.

3. CBM Model

3.1 CBM Model Description

Compared with traditional TBM, CBM is a kind of proactive maintenance mode and implements PM maintenance based on real state of the devices. Theoretically, it is an optimal maintenance model. The typical characteristic of CBM lies in that the parameters of the model used to describe the system are time-variant, and not constants. Taking (9) as an example, its corresponding time-variant system can be written by

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=\boldsymbol{A}(t)\boldsymbol{x}(t)+\boldsymbol{\mu}(t) \tag{17}$$

where physical meaning of the parameters are the same with (9).

The system (17) may be thought as a generalized model on CBM, different from TBM, where $\boldsymbol{A}(t)$ is time-variant. Below we firstly consider its convergence by imposing some restrictions on $\boldsymbol{A}$(t).

Theorem2. Let$\boldsymbol{A}(\boldsymbol{t})$ be the measurable functions in $[0, t]$ in L_2, namely

$$\int_0^t |A(s)|^2\,\mathrm{d}s<\infty$$

Then, the system (17) converges.

Proof. Clearly, the system (17) is linear, and its solution can be obtained by respectively solving the following two equations:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=\boldsymbol{A}(t)\boldsymbol{x}(t) \tag{18}$$

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=\boldsymbol{\mu}(t) \tag{19}$$

The former is called as zero-input response, and the latter is zero-state response.

From (18), we have

$$\boldsymbol{x}(t)=\int_0^t \boldsymbol{A}(s)\boldsymbol{x}(s)\,\mathrm{d}s+\boldsymbol{x}(0)\qquad s\in[0,t] \tag{20}$$

where $\boldsymbol{A}(s)$ and $\boldsymbol{x}(0)$ are known functions in $L_2[0, t]$.

Let us now consider operator $\bar{\boldsymbol{A}}$ dominated by $\psi(t)=\bar{\boldsymbol{A}}x(t)$ and compare with (20), then

$$\psi(t)=\int_0^t \boldsymbol{A}(s)\boldsymbol{x}(s)\,\mathrm{d}s \tag{21}$$

The operator as shown in (21) is called as Hilbert-Smith operator, wherein $\boldsymbol{A}(s)$ is also named as Hilbert-Smith core, which defines a tense linear operator $\bar{\boldsymbol{A}}$ in $L_2[0, t]$ [16], and satisfies

$$\|\bar{\boldsymbol{A}}\|=\sqrt{\int_0^t |A(s)|^2\,\mathrm{d}s} \tag{22}$$

Since $\bar{\boldsymbol{A}}$ is tense linear operator in $L_2[0, t]$, the system shown in (18) clearly possesses solitary solution.

As for (19), since $\boldsymbol{A}(\boldsymbol{t})$ is square convergence, as the element of $\boldsymbol{A}(\boldsymbol{t})$, and so does $\boldsymbol{\mu}(t)$, which also determinates $\boldsymbol{\mu}(t)$ is average convergent, namely

$$\boldsymbol{x}(t)=\int_0^t \mu(s)\mathrm{d}s+\boldsymbol{x}(0)\le\int_0^t|\mu(s)|\,\mathrm{d}s+\|\boldsymbol{x}(0)\|=\|\mu\|+\|\boldsymbol{x}(0)\| \quad s\in[0,t] \tag{23}$$

Evidently, $\boldsymbol{x}(t)$ is dominated by $\boldsymbol{\mu}(t)$ alone and bounded, and so convergent. Since the two systems deminated by (18) and (19) are convergent, and then Theorem4 holds.

End.

Theorem3 Let $\boldsymbol{A}(\boldsymbol{t})$ be state transition density matrix of the inhomogeneous Markov process as shown in (17), and $\boldsymbol{A}$ be one of the homogeneous Markov process as shown in (9), under the condition of Theorem4, if $\boldsymbol{A}(\boldsymbol{t})$ converges to $\boldsymbol{A}$ in L_2, and then $\boldsymbol{x}(t)$ converges to $\boldsymbol{x}(\boldsymbol{t})$ in L_2, and such that the system (17) possesses same properties with (9).

Proof. Since $\boldsymbol{A}(\boldsymbol{t})$ converges to $\boldsymbol{A}$ in L_2, and $\boldsymbol{A}(\boldsymbol{t})$ also can converge to $\boldsymbol{A}$ in distribution or probability. Let $\boldsymbol{G}(t)$ be the distribution function of $\boldsymbol{A}(\boldsymbol{t})$, and satisfy $\boldsymbol{G'}(t)=\boldsymbol{A}(\boldsymbol{t})$, and then from (17) we easily obtain

$$\boldsymbol{x}(t)=\int_0^t e^{G(t)-G(\tau)}\mu(\tau)\,\mathrm{d}\tau+\boldsymbol{x}(0)\,e^{G(t)-G(0)} \quad \tau\in[0,t] \tag{24}$$

Namely,

$$\boldsymbol{x}(t)=\int_0^t e^{\boldsymbol{A}(\xi_1)(t-\tau)}\mu(\tau)\mathrm{d}\tau+\boldsymbol{x}(0)\,\mathrm{e}^{A(\xi_2)t} \quad \tau\in[0,t],\ \xi_1\in[\tau,t],\ \xi_2\in[0,t] \tag{25}$$

From (9), similarly, we can obtain

$$\boldsymbol{x}(t)=\int_0^t e^{\boldsymbol{A}(t-\tau)}\mu\,\mathrm{d}\tau+\boldsymbol{x}(0)\,\mathrm{e}^{\boldsymbol{A}t} \quad \tau\in[0,t]$$

Then

$$\begin{aligned}
\|\boldsymbol{x}(t)-x(t)\| &= \|\int_0^t e^{\boldsymbol{A}(\xi_1)(t-\tau)}\mu(\tau)\mathrm{d}\tau+\mathrm{e}^{A(\xi_2)t}\,\boldsymbol{x}(0)-\int_0^t e^{\boldsymbol{A}(t-\tau)}\mu\,\mathrm{d}\tau+\mathrm{e}^{\boldsymbol{A}t}\boldsymbol{x}(0)\| \\
&\le \int_0^t \|e^{\boldsymbol{A}(\xi_1)(t-\tau)}\mu(\tau)-e^{\boldsymbol{A}(t-\tau)}\mu\|\,\mathrm{d}\tau+\|\mathrm{e}^{A(\xi_2)t}\,\boldsymbol{x}(0)-\mathrm{e}^{\boldsymbol{A}t}\boldsymbol{x}(0)\| \\
&\le \int_0^t \|e^{\boldsymbol{A}(\xi_1)(t-\tau)}-e^{\boldsymbol{A}(t-\tau)}\|\|\mu(\tau)\|\mathrm{d}\tau+\int_0^t\|e^{\boldsymbol{A}(t-\tau)}\|\|\mu(\tau)-\mu\|\mathrm{d}\tau+ \\
&\quad +\|e^{A(\xi_2)t}-e^{At}\|\|\boldsymbol{x}(0)\|+\|e^{\boldsymbol{A}t}\|\|\boldsymbol{x}(0)-\boldsymbol{x}(0)\| \\
&= \int_0^t \|e^{\boldsymbol{A}(\xi_1)(t-\tau)}-e^{\boldsymbol{A}(t-\tau)}\|\|\mu(\tau)\|\mathrm{d}\tau+\int_0^t\|e^{\boldsymbol{A}(t-\tau)}\|\|\mu(\tau)-\mu\|\mathrm{d}\tau+ \\
&\quad +\|e^{A(\xi_2)t}-e^{At}\|\|\boldsymbol{x}(0)\|+\|e^{\boldsymbol{A}t}\|\|\int_0^\infty[\boldsymbol{x}(t)-\boldsymbol{x}(t)]\delta(t)\mathrm{d}t\|
\end{aligned}$$

where and $\delta(t)$ is a generalized function.

According to Dyson's expansion formula [17], we have

$$\|e^{\boldsymbol{A}(\xi_i)} - e^{\boldsymbol{A}}\| = \|e^{\boldsymbol{A}+(\boldsymbol{A}(\xi_i)-\boldsymbol{A})} - e^{\boldsymbol{A}}\| = \|\int_0^1 e^{(1-\tau)\boldsymbol{A}(\xi_i)}[(\boldsymbol{A}(\xi_i)-\boldsymbol{A})e^{\tau\boldsymbol{A}(\xi_i)}\mathrm{d}\tau\|$$

$$\le \int_0^1 \|e^{(1-\tau)\boldsymbol{A}(\xi_i)}\|\|\boldsymbol{A}(\xi_i)-\boldsymbol{A}\|\|e^{\tau\boldsymbol{A}(\xi_i)}\|\mathrm{d}\tau, \quad i=1,2$$

Clearly, there exist one integer N to make $n>N$, as $\boldsymbol{A}(\xi)$ converges to $\boldsymbol{A}$ such that $\boldsymbol{x}(t)$ converges to $\boldsymbol{x}(t)$ almost everywhere excluding some zero-measurable sets, which implies system (17) is equivalent with (9) in L_2, and possesses same Markov property, such as recurrence, ergodicity, and reducibility, and so on. End.

Theorem3 provides a general control scheme for time-variant system (17) by selecting a desirable time-invariant system (9), but this is difficult for n large enough specified. In practice, we may reduce the requirement of square convergence to $\boldsymbol{A}$ of $\boldsymbol{A}(t)$, $\boldsymbol{A}(t)$ is any square integral functions [16].

Proposition3 Let the system (17) starts from same initial state with the (9) and possesses same inputs, namely, $\boldsymbol{\mu}(t)=\boldsymbol{\mu}$, and $\boldsymbol{A}(t)$ be a continuous with $\boldsymbol{A}(0)=\boldsymbol{A}$, then there exists one time T to make the error $\|\boldsymbol{x}(t) - \boldsymbol{x}(t)\|$ arrive at the maximum in $[0, t]$, and T satisfies

$$T_1 = \frac{\ln \boldsymbol{A}^{-1}\boldsymbol{\beta}}{\boldsymbol{\beta}-\boldsymbol{A}} \tag{26}$$

Proof. According to Theorem2 and the conditions in Proposition3, we have

$$\|\Delta\boldsymbol{x}(t)\| = \|\boldsymbol{x}(t) - x(t)\| = \|[\int_0^t e^{\boldsymbol{A}(\xi_1)(t-\tau)}\boldsymbol{\mu}(\tau)\mathrm{d}\tau - \int_0^t e^{\boldsymbol{A}(t-\tau)}\boldsymbol{\mu}\mathrm{d}\tau] + \mathrm{e}^{A(\xi_2)t}\boldsymbol{x}(0) - \mathrm{e}^{\boldsymbol{A}t}\boldsymbol{x}(0)\|$$

$$\le \|\int_0^t [e^{\boldsymbol{A}(\xi_1)(t-\tau)} - \int_0^t e^{\boldsymbol{A}(t-\tau)}]\boldsymbol{\mu}\mathrm{d}\tau\| + \|\mathrm{e}^{A(\xi_2)t}\boldsymbol{x}(0) - \mathrm{e}^{\boldsymbol{A}t}\boldsymbol{x}(0)\|$$

$$\le \|\mathrm{e}^{\beta t}\|\|\int_0^t [e^{-\boldsymbol{\beta}\tau} - e^{-\boldsymbol{A}\tau}]\boldsymbol{\mu}\mathrm{d}\tau\| + \|\mathrm{e}^{\beta t} - \mathrm{e}^{\boldsymbol{A}t}\|\|\boldsymbol{x}(0)\|$$

$$= \|\mathrm{e}^{\beta t}\|\int_0^t \|e^{-\boldsymbol{\beta}\tau} - e^{-\boldsymbol{A}\tau}\|\mathrm{d}\tau\|\boldsymbol{\mu}\| + \|\mathrm{e}^{\boldsymbol{\beta}t} - \mathrm{e}^{\boldsymbol{A}t}\|\|\boldsymbol{x}(0)\|$$

where $\boldsymbol{\beta}$ is a suitable scalar matrix and satisfies

$$e^{\boldsymbol{\beta}} = \sup\{e^{\boldsymbol{A}(\xi)}, \xi\in[0, t]\}$$

Let

$$f(\tau) = \exp(-\boldsymbol{\beta}\tau) - \exp(-\boldsymbol{A}\tau)$$

And then

$$f'(\tau) = -\boldsymbol{\beta}\exp(-\boldsymbol{\beta}\tau) + \boldsymbol{A}\exp(-\boldsymbol{A}\tau)$$

Let $f'(\tau)=0$, and then we obtain

$$\frac{\boldsymbol{\beta}}{\boldsymbol{A}} = \frac{\exp(\boldsymbol{\beta}\tau)}{\exp(\boldsymbol{A}\tau)}$$

For n large enough, the $\exp(\boldsymbol{\beta}t)$ and $\exp(\boldsymbol{A}t)$ can be approximated by

$$\exp(\boldsymbol{\beta}t) \approx (\boldsymbol{I} + \boldsymbol{\beta}\frac{t}{n})^n, \quad \exp(\boldsymbol{A}t) \approx (\boldsymbol{I} + \boldsymbol{A}\frac{t}{n})^n$$

By increasing n-times power, and then the above two matrixes will be non-negative [18]. And thus, we have

$$\ln\boldsymbol{A}^{-1}\boldsymbol{\beta} = \ln e^{\boldsymbol{\beta}\tau} - \ln e^{\boldsymbol{A}\tau}, \quad \tau = \frac{\ln \boldsymbol{A}^{-1}\boldsymbol{\beta}}{\boldsymbol{\beta} - \boldsymbol{A}}$$

Then, we take

$$T_1 = \tau \approx m_{11}\Delta t$$

where Δt is a small but infinite time interval for an effective implementation, and m_{11} is a suitable integer.

End.

The control strategy is to apply suitable approaches for time interval larger than $m_{12} \times T_1$ to control the course of $\| \boldsymbol{x}(t) - \boldsymbol{x}(t) \|$, where m_{12} is a suitable integer. If the difference can not remain bounded, then the system (17) must be reinitialized with

$$\boldsymbol{x}(m_1\Delta t) = \boldsymbol{x}(m_1\Delta t)$$

where $m_1 = m_{12} \times m_{11}$.

4. TBM Based CBM Control Strategy Design

Known from the from discussion, a significant difference between TBM and CBM lies in that the former parameters matrix is time-invariant, and so controllable, whereas the latter is time-variant, and uncontrollable, typically such as checking rate $\boldsymbol{u}(t)$, and etc. It is necessary to implement the suitable control mode on CBM in order to obtain a desired steady-state property similar to TBM. It is easy to see from (9) and (11), the steady-state probability $x_3(\infty)$ of the state S_3 is a function of $x_1(\infty)$ and $x_2(\infty)$ not having nothing to do with $x_4(\infty)$ only since its corresponding input is zero, which implies $x_3(t)$ is dispensable as the system being at steady-state. In addition, from the third equation of (8), during transient process $\boldsymbol{x}_3(t)$ possesses more quick decay speed since its damping coefficient μ_1 is generally far larger than u_1 and u_2 in practice. And thus the influence of $x_3(t)$ may be ignored. This kind of analysis problem approach is often used in other subject areas, such as electromechanical transient analysis [19] and preventive maintenance model [20]. And thus we acquire the following simplified TBM model.

$$\frac{d\boldsymbol{x}_T(t)}{dt} = \boldsymbol{A}_T\boldsymbol{x}_T(t) + \boldsymbol{\mu}_T \tag{27}$$

Where

$$\boldsymbol{A}_T = \begin{bmatrix} -d_1 & a_1 \\ b_1 & -d_2 \end{bmatrix} \quad \boldsymbol{x}_T(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \boldsymbol{\mu}_T = \begin{bmatrix} \rho_2\mu_2 \\ (1-\rho_2)\mu_2 \end{bmatrix}$$

$d_1=\lambda_2+\lambda_3+\rho_2\mu_2$, $a_1=\lambda_1-\rho_2\mu_2$, $b_1=\lambda_2-(1-\rho_2)\mu_2$, $d_2=\lambda_1+\lambda_4+(1-\rho_2)\mu_2$, $\lambda_3=u_1+v_1$, $\lambda_4=u_2+v_2$.

It is easy to prove this system (27) possesses sole steady-state operation point [21].

Consider now corresponding CBM model as follows

$$\frac{\mathrm{d}\boldsymbol{x}_{\mathrm{c}}(t)}{\mathrm{d}t}=\boldsymbol{A}_{\mathrm{c}}(t)\boldsymbol{x}_{\mathrm{c}}(t)+\boldsymbol{\mu}_{\mathrm{c}}(t) \quad (28)$$

The parameter matrix possesses same physical meaning with (27) but time-variant. From (28) we can obtain

From (27) and (28), we obtain

$$\boldsymbol{x}_{\mathrm{T}}(t)=\int_0^t \boldsymbol{A}_{\mathrm{T}}\boldsymbol{x}_{\mathrm{T}}(t)\mathrm{d}t+\int_0^t \boldsymbol{\mu}_{\mathrm{T}}\mathrm{d}t+\boldsymbol{x}_{\mathrm{T}}(0) \quad (29)$$

$$\boldsymbol{x}_{\mathrm{c}}(t)=\int_0^t \boldsymbol{A}_{\mathrm{c}}(s)\boldsymbol{x}_{\mathrm{c}}(s)\mathrm{d}t+\int_0^t \boldsymbol{\mu}(s)\mathrm{d}t+\boldsymbol{x}_{\mathrm{c}}(0) \quad (30)$$

And then

$$\boldsymbol{x}_{\mathrm{c}}(t)\text{-}\boldsymbol{x}_{\mathrm{T}}(t)=\int_0^t [\boldsymbol{A}_{\mathrm{c}}(s)\boldsymbol{x}_{\mathrm{c}}(s)\text{-}\boldsymbol{A}_{\mathrm{T}}\boldsymbol{x}_{\mathrm{T}}(s)]\mathrm{d}s+\int_0^t (\boldsymbol{\mu}(s)\text{-}\boldsymbol{\mu}_{\mathrm{T}})\mathrm{d}s+\boldsymbol{x}_{\mathrm{c}}(0)\text{-}\boldsymbol{x}_{\mathrm{T}}(0) \quad (31)$$

In according with coollary1, if $\boldsymbol{A}_{\mathrm{c}}(t)$ converges to $\boldsymbol{A}_{\mathrm{T}}$, and $\boldsymbol{\mu}(t)$ converges to $\boldsymbol{\mu}_{\mathrm{T}}$ in L_2, and then $\boldsymbol{x}_{\mathrm{c}}(t)$ converges to $\boldsymbol{x}_{\mathrm{T}}(t)$ almost everywhere. But under most of conditions, $\boldsymbol{A}_{\mathrm{c}}(t)$ does not possibly converges to $\boldsymbol{A}_{\mathrm{T}}$, which leads to a undesired steady-state behavior. For CBM, one more pay attention to the change of the checking rate $\boldsymbol{u}(t)$ aroused by condition monitoring than any other parameters. Hence, we may select $\boldsymbol{u}(t)$ as control variable to make the system (28) obtain the desired properties similar to (27).

Theorem4 Assume that the system (28) start from same initial state with (27), and system parameter $\boldsymbol{A}_{\mathrm{c}}(t)$ is square integral in L_2. To obtain desired steady-state behavior similar to system (27), $\boldsymbol{u}(t)$ must satisfies

$$\boldsymbol{u}(t)=\boldsymbol{X}(t)^{-1}[(\boldsymbol{A}_{\mathrm{c1}}(t)-\boldsymbol{A}_{\mathrm{T}})\boldsymbol{x}(t)+\boldsymbol{\mu}_{\mathrm{c}}(t)-\boldsymbol{\mu}_{\mathrm{T}}] \quad (32)$$

Proof. Isolating out $\boldsymbol{u}(t)$ from $\boldsymbol{A}_{\mathrm{c}}(t)$ in (28) we obtain

$$\frac{\mathrm{d}\boldsymbol{x}_{\mathrm{c}}(t)}{\mathrm{d}t}=\boldsymbol{A}_{\mathrm{c1}}(t)\boldsymbol{x}_{\mathrm{c}}(t)\text{-}\boldsymbol{U}(t)\boldsymbol{x}_{\mathrm{c}}(t)+\boldsymbol{\mu}_{\mathrm{c}}(t)$$

Namely,

$$\frac{\mathrm{d}\boldsymbol{x}_{\mathrm{c}}(t)}{\mathrm{d}t}=\boldsymbol{A}_{\mathrm{c1}}(t)\boldsymbol{x}_{\mathrm{c}}(t)\text{-}\boldsymbol{X}_{\mathrm{c}}(t)\boldsymbol{u}(t)+\boldsymbol{\mu}_{\mathrm{c}}(t) \quad (33)$$

where

$$\boldsymbol{U}(t)=\begin{vmatrix} u_1(t) & 0 \\ 0 & u_2(t) \end{vmatrix}, \quad \boldsymbol{u}(t)=\begin{vmatrix} u_1(t) \\ u_2(t) \end{vmatrix}, \quad x_c(t)=\begin{vmatrix} x_{c1}(t) \\ x_{c2}(t) \end{vmatrix}, \quad \boldsymbol{X}(t)=\begin{vmatrix} x_{c1}(t) & 0 \\ 0 & x_{c2}(t) \end{vmatrix}$$

$$\boldsymbol{A}_c(t)=\boldsymbol{A}_{c1}(t)+\boldsymbol{U}(t)$$

From (27) and (31), we obtain

$$\frac{\mathrm{d}\boldsymbol{x}_c(t)}{\mathrm{d}t}-\frac{\mathrm{d}\boldsymbol{x}_T(t)}{\mathrm{d}t}=\boldsymbol{A}_{c1}(t)\boldsymbol{x}_c(t)\text{-}\boldsymbol{A}_T\boldsymbol{x}_T(t)\text{-}\boldsymbol{X}_c(t)\boldsymbol{u}(t)+\boldsymbol{\mu}_c(t)-\boldsymbol{\mu}_T$$

Let the left side equal zero and $\boldsymbol{x}_c(t)$=$\boldsymbol{x}_T(t)$= $\boldsymbol{x}(t)$, we then have

$$\boldsymbol{u}(t)=\boldsymbol{X}(t)^{-1}[(\boldsymbol{A}_{c1}(t)-\boldsymbol{A}_T)\boldsymbol{x}(t)+\boldsymbol{\mu}_c(t)-\boldsymbol{\mu}_T]$$

Clearly, the right parameter matrixes and states are detectable under CBM, and so $\boldsymbol{u}(t)$ is controllable. End.

Theorem5 The motion of the system (28) tends to the desired steady-state determined by the system (27) as $t\rightarrow\infty$, under the role of control law (32) starting from arbitrary initial location. Moreover, $\boldsymbol{x}(t)$ approximates $\boldsymbol{x}(\infty)$ in exponential rate.

Proof. Substituting (32) into (28), we will obtain the expected system (27) below.

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}=\boldsymbol{A}_\mathrm{T}\boldsymbol{x}(t)+\boldsymbol{\mu}_\mathrm{T}$$

From Theorem1, we have

$$x(\infty)=-\boldsymbol{A}_\mathrm{T}^{-1}\boldsymbol{\mu}_\mathrm{T}$$

In addition, according to (10) and (11), we obtain

$$\boldsymbol{x}(t)-\boldsymbol{x}(\infty)=e^{\boldsymbol{A}_\mathrm{T}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]$$

Then

$$\begin{aligned}\left\|\boldsymbol{x}(t)-\boldsymbol{x}(\infty)\right\|&=\left\|e^{\boldsymbol{A}_\mathrm{T}t}[\boldsymbol{x}(0)-\boldsymbol{x}(\infty)]\right\|\le\| e^{\boldsymbol{A}_\mathrm{T}t}\|\|\boldsymbol{x}(0)-\boldsymbol{x}(\infty)\|\\ &\le\| e^{\mathrm{Re}\boldsymbol{A}_\mathrm{T}t}\|\|\boldsymbol{x}(0)-\boldsymbol{x}(\infty)\|=\| e^{\frac{\boldsymbol{A}_\mathrm{T}+\boldsymbol{A}_\mathrm{T}^*}{2}t}\|\|\boldsymbol{x}(0)-\boldsymbol{x}(\infty)\|\\ &=\|\boldsymbol{P}e^{\boldsymbol{\Lambda}t}\boldsymbol{P}^{-1}\|\|\boldsymbol{x}(0)-\boldsymbol{x}(\infty)\|\le\|\boldsymbol{P}\|\|e^{\boldsymbol{\Lambda}t}\|\|\boldsymbol{P}^{-1}\|\|\boldsymbol{x}(0)-\boldsymbol{x}(\infty)\|\\ &=\|e^{\boldsymbol{\Lambda}t}\|\|\boldsymbol{x}(0)-\boldsymbol{x}(\infty)\|\le\|\boldsymbol{x}(0)-\boldsymbol{x}(\infty)\|\end{aligned}$$

where for any matrix $\boldsymbol{A}$, $\| e^{\boldsymbol{A}}\|\le\| e^{\mathrm{Re}\boldsymbol{A}}\|$ can be found in [17], and $\boldsymbol{A}^*$ is the conjugate transpose matrix of any matrix $\boldsymbol{A}$, and $\boldsymbol{\Lambda}$ is a diagonal matrix composed of the eignvalues of Re $\boldsymbol{A}$, and $\boldsymbol{P}$ is a matrix composed of each characteristic vector belonging to each eignvalue of Re $\boldsymbol{A}$. Clearly, Re $\boldsymbol{A}=(\boldsymbol{A}+\boldsymbol{A}^*)/2$ is a Hermitian operator.

Corollary1. Under the role of control rule (32), the MTTF of system (28) starting from any regions can be calculated by

$$\mathrm{MTTF}[x_1(0),x_2(0)]=\int_0^\infty R(t)dt=1^\mathrm{T}\boldsymbol{E}^{-1}[x(0)-\bar{x}]=T[x_1(0),x_2(0)] \quad (34)$$

and

$$T(\xi,\eta)=\frac{[(e_{22}-e_{21})(\xi-x_1(\infty)]+[(e_{11}-e_{12})(\eta-x_2(\infty)]}{\det\boldsymbol{E}} \quad (35)$$

where $\boldsymbol{E}$ is an expected scalar matrix, and e_{ij} are its entries.

Proof. According to Definition2 and Theorem5, it is not difficult to obtain (34) and (35).

5. State Estimation and Error Control

Although the system (28) can obtain desired properties under the role of control rule (32), theoretically, it is difficult to completely follow it in process of concretely implementation due to diverse reasons in practice, for instance, a certain amount of time cost is required in acquisition, disposing, and discussion making, and as well as reaction of information, and possibly, there is error existing in the collected information disturbed, and etc. This makes the error appear between the observed states and real states such that the desired system behavior can not be ensured. For that we require to further investigate and estmate the error behavior such that suitable measures are taken to restrict it.

Definition3. Let $x(t)$ be the observed vector determined by (28), and $x(t)$ be the statistical vector determined by

$$\tilde{x}_i(t) = n_i(t) / n(t) \text{, i=1, 2.} \quad (36)$$

where $n_i(t)$ represents the number of the device being in S_i at time t, and $n(t)$ is the total number of the same device at the same time.

Clearly, the error between them be

$$\Delta \boldsymbol{x}(t) = \boldsymbol{x}(t) - \boldsymbol{x}(t) \quad (37)$$

Clearly, $E[x(t)] = E[x(t)] = \boldsymbol{x}(t)$, and $\boldsymbol{x}(t)$ is real vector. Hence, $E[\Delta \boldsymbol{x}(t)]=0$, and $\Delta \boldsymbol{x}(t)$ follows $N(0, \boldsymbol{\sigma}^2(t))$, where $\boldsymbol{\sigma}^2(t)$ is a bounded variance function. Clearly, $\Delta \boldsymbol{x}(t)$ is a normal process.

Theorem6. Stochastic process $\Delta x_i(t)$, i=1, 2, can arrive at any value c_i in probability one in finite time, namely,

$$P_0(T_{ci} \geq \infty) = 0 \quad (38)$$

where T_{ci} is a time that $\Delta x_i(t)$ arrives at c_i, and P_0 represents $\Delta x_i(t)$ starts from zero.

Further, $\Delta x_i(T_{ci})$ follows uniform distribution in vertical line and

$$E(T_{ci}) = +\infty \quad (39)$$

Proof. Clearly, if at time $t=n$ that the orbit $\Delta x_i(t)$ still stays at interior of c_i, and then the absolute values of all the i.i.d increments of $\Delta x_i(1)$-$\Delta x_i(0)$, $\Delta x_i(2)$-$\Delta x_i(1)$, …, $\Delta x_i(n)$-$\Delta x_i(n-1)$ in line interval must be smaller than $2c_i$. Thus we have

$$P_0(T_{ci} \geq \infty) \leq [P\{| \Delta x_i(1) - \Delta x_i(0) | < 2c_i\}]^n = \varepsilon_i^n$$

wherein ε_i is bound to be smaller than one. Therefore for any n, we have $P_0\{T_{ci}=\infty\} \leq \varepsilon_i^n$, and such that $P_0\{T_{ci}=\infty\}=0$.

Clearly, $\Delta x_i(T_{ci})$ situates at points $\pm c_i$. Known from symmetry of normal distribution, the probability that $\Delta x_i(t)$ arrives at $\pm c_i$ should be equal.

Known from the one-dimensional symmetric random wandering[22], the formula (39) can be obtained at once. End.

Theorem7. Let X be a stochastic variable with density function $f(x)$ and moment-generating function $M(t)$, and X_t possess density function f_t denoted by

$$f_t(x) = \frac{e^{tx} f(x)}{M(t)} \quad (40)$$

show that for any t>0, and we have

$$P\{X > a\} \le M(t)e^{-ta}P\{X_t > a\} \tag{41}$$

In addition, show that if $E(X_{t^*})=a$, and then

$$\min_t M(t)e^{-ta} = M(t^*)e^{-t^*a} \tag{42}$$

Proof. We clearly have

$$P\{X > a\} = \int_a^\infty f(x)\mathrm{d}x = \int_a^\infty \frac{f_t(x)M(t)}{e^{tx}}\mathrm{d}x = M(t)\int_a^\infty e^{-tx}f_t(x)\mathrm{d}x$$
$$\le M(t)e^{-ta}P\{X_t > a\}$$

In addition, from (40) we obtain

$$E(X_t) = \int_{-\infty}^{+\infty} xf_t(x)\mathrm{d}x = \int_{-\infty}^{+\infty} x\frac{e^{tx}f(x)}{M(t)}\mathrm{d}x = \frac{M^{'}(t)}{M(t)}$$

And then from (41), according to Markov inequality we have

$$P\{X > a\} \le M(t)e^{-ta}P\{X_t > a\} \le M(t)e^{-ta}\frac{E(X_t)}{a} = \frac{1}{a}M^{'}(t)e^{-ta}$$

Then we have

$$M^{'}(t) = M(t)E(X_t)$$

And

$$M(t) = M(0)e^{\int_0^t E(X_s)\mathrm{d}s}$$

Then

$$M(t)e^{-ta} = M(0)e^{\int_0^t [E(X_s)-a]\mathrm{d}s}$$

Let $\psi(t)=M(t)e^{-ta}$ and $\psi'(t)=0$, then

$$\psi^{'}(t) = [E(X_t)-a]M(0)e^{\int_0^t [E(X_s)-a]\mathrm{d}s} = 0$$

Thus we obtain $E(X_t)=a$. Since $E(X_t)$ is a function of t, let t^* be specified by $E(X_t)=a$. And then (42) is proven immediately. End.

Theorem8. Let ζ_n be a i.i.d sequence with zero mean value with nonzero finite variance σ^2, and η_n be a nonnegative integer random variable sequence, and satisfy

$$\frac{\eta_n}{n} \xrightarrow{P} \alpha > 0 \tag{43}$$

Show that

$$\frac{\zeta_1 + \zeta_2 + \cdots + \zeta_{\eta_n}}{\sqrt{\eta_n}} \xrightarrow{d} N(0, \sigma^2) \tag{44}$$

Further, let X_n be a normal-recurrent irreducible Markov chain with stationary distribution π, and f be a function defined on state space. Let

$$\tau_j^0 = 0,\ \tau_j^n = \inf\{k > \tau_j^{(n-1)} : X_k = j\},\quad \mu = E_\pi f(X_0) < \infty.$$
$$Y_m = f(X_{\tau_{j+1}^m} - \mu) + \cdots + f(X_{\tau_j^{m+1}} - \mu),\quad \sigma_0^2 \square E_j Y_0^2 < \infty. \tag{45}$$

And then show that under P_j as $n \to \infty$

$$\frac{\sum_{m=0}^{n} (f(X_m) - \mu)}{\sqrt{n}} \xrightarrow{d} N(0, \pi_j \sigma_0^2) \tag{46}$$

Proof. Put $S_n = \zeta_1 + \zeta_2 + \cdots + \zeta_n$. Clearly, S_n is a martingale with mean zero, and $E(S_n^2)$ is finite. Since $E(|S_n|) \le [E(S_n^2)]^{1/2}$, and then $E(|S_n|)$ is bounded. According to Martingale convergence theorem, $\{S_n, n \ge 1\}$ can converge to one limitation as $n \to \infty$. Let $[n\alpha]$ is the largest integer smaller than $n\alpha$, such that $S_{\eta n}$ and $S_{[n\alpha]}$ are two Cauchy subsequences of S_n, and so they are also convergent. At the same time, η_n and $[n\alpha]$ are also two stopping times of S_n. According to Doob stopping time theorem[23], we have

$$E[S_{\eta_n} \mid F_{[n\alpha]}] = S_{\eta_n \wedge [n\alpha]},\quad \text{a.s.}$$

And so,

$$E[S_{\eta_n} - S_{[n\alpha]} \mid F_{[n\alpha]}] = 0$$

Then

$$\frac{\zeta_1 + \zeta_2 + \cdots + \zeta_{\eta_n}}{\sqrt{\eta_n}} = \frac{S_{\eta_n}}{\sqrt{\eta_n}} \xrightarrow{P} \frac{S_{[n\alpha]}}{\sqrt{[n\alpha]}} \xrightarrow{d} N(0, \sigma^2)$$

And then (44) is proved.

Known from (45), $\{\tau_j^n; n \ge 1\}$ is a stopping time sequence, and then $\{Y_m; m \ge 1\}$ is i.i.d. According to strong ergodicity theorem [24], we have

$$\sigma_0^2 \square E_j Y_0^2 \to EY_0^2 = \pi_j \sigma_0^2$$

$$EY_0 = E_\pi f(X_0) = \mu \to EY_0 = \mu$$

And then we can obtain (46), immediately. End.

Known from Theorem 6, stochastic variable $\Delta x_i(t)$ can arrive at any value c_i in probability one in finite time, which also means it can leave one collection in same time in probability one. Assume that such collection be G, and the exit time be τ, and then exit probability be $P_x(\tau_i < \infty)$, $x \in G$. Hence, it is possible to define the random variables below.

$$\varepsilon_i(t) = \max_{\tau < t}\{|\Delta x_i(\tau)|\} = \max_{\tau < t}\{|x_i(\tau) - x_i(\tau)|\} \tag{47}$$

Clearly, τ is a stopping time. And then we have the following theorem.

Theorem9. For $n(t)$ sufficient large, existing $c_i(t)$ and satisfies

$$P(\varepsilon_i(t) \le c_i(t)) = \gamma_i(t) \tag{48}$$

where

$$c_i(t) = \sqrt{-\frac{1}{2n(t)} \ln \frac{1-\gamma_i(t)}{2}} \qquad (49)$$

Proof. According to (37) we have

$$|\Delta x_i(t)| = |x_i(t) - x_i(t)| = |x_i(t) - \frac{n_i(t)}{n(t)}|$$

Based on (36), let $p_i(t)$= $x_i(t)$ =$n_i(t)/n(t)$, and such that

$$x_i(t) \sim B(n(t), p_i(t))$$

where $B(n(t), p_i(t))$ represents the binomial distribution.

Clearly, $S_{n(t)}=\sum x_i(t) - n(t)p_i(t)$is a martingale of zero mean value.
Since

$$-p_i(t) \le S_{n(t)} - S_{[n(t)-1]} \le 1 - p_i(t)$$

According to Azuma inequality[25], we have

$$P(S_{n(t)} - n(t)p_i(t)) \ge n(t)c_i(t)) \le e^{-2n(t)c_i^2(t)}$$

which is equivalent to

$$P(\frac{S_{n(t)} - n(t)p_i(t)}{\sqrt{n(t)p_i(t)q_i(t)}} \ge c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}}) \le e^{-2n(t)c_i^2(t)} \qquad (50)$$

where $q_i(t)=1-p_i(t)$.

On the other hand, according to theorem12, and (47), and (48), τ is a stopping time such that $\Delta x_i(t)$ possesses same distribution with $\varepsilon_i(t)$. The left-side probability of the above formula tends to be standard distribution normal $N(0, 1)$ as $n(t)\to\infty$. And thus we have

$$P(\frac{S_{n(t)} - n(t)p_i(t)}{\sqrt{n(t)p_i(t)q_i(t)}} > c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}}) = \frac{1-\gamma_i(t)}{2} \qquad (51)$$

Comparing (50) with (51), we easily have

$$e^{-2c_i^2(t)n(t)} = \frac{1-\gamma_i(t)}{2}$$

Taking logarithmic in both sides of the above equation, and then we can obtain (49), immediately. End.

According to (47), we clearly have

$$\tilde{x}_i(t) \in [\hat{x}_i(t) - c_i(t), \hat{x}_i(t) + c_i(t)]$$

If $n(t)$ and $\gamma_i(t)$ do not change with time, and then $c_i(t)$ becomes a constant c. After the time interval $m_2\Delta t$, if one has $\varepsilon_i(t)>c$, the following formula should then be reinitialized.

$$\boldsymbol{x}(m_2\Delta t) = \boldsymbol{x}(m_2\Delta t)$$

where $m_2=m_{22}\times m_{21}$, and m_{21}, m_{22} are suitable integer.

As a matter of fact, applying the conclusion in theorem11 to left side item of (50) we easily obtain

$$t^* = c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}} \quad (52)$$

where t^* is $\Delta x_i(t)$ the shortest time to arrive at $c_i(t)$ starting from zero. And then

$$P\left(\frac{S_{n(t)} - n(t)p_i(t)}{\sqrt{n(t)p_i(t)q_i(t)}} > c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}}\right) \le e^{-\frac{t^{*2}}{2}} = e^{\frac{-[c_i(t)\sqrt{\frac{n(t)}{p_i(t)q_i(t)}}]^2}{2}} \le e^{-2c_i^2(t)n^2(t)}$$

In the above formula we applies $p_i(t)q_i(t) \le 1/4$ for $0 < p_i(t), q_i(t) < 1$, and then let

$$\frac{1-\gamma_i(t)}{2} = e^{-2c_i^2(t)n^2(t)}, \text{ while } T_2 = t^* = 2c_i(t)\sqrt{n(t)}.$$

And thus we still can obtain (49). This shows that the conclusions of the two methods are fully consistent, but the one in Theorem7 are more profound.

Known from temporal spatial homogeneity and strong ergodicity of Markov process, and as well as stopping-time theorem, after stopping time Markov process will reproduce the former process independent of it, which means Markov process will repeat the pass as long as the conditions of stopping time are same. Practically, under the conditions of theorem13, stationary Markov process forms a basic group who starts from the origin and again returns to the origin, constantly, whereas the shortest time interval is T_2 represented by (52).

Therefore, comprehensively consider (26) and (52), we should try to select $T=\inf\{T_1,T_2\}$ as reinitialization time if the error can not keep boundary.

5. Conclusion

This paper firstly establishes the life cycle model on preventive maintenance, and then discusses its operating behavior and controllability respectfully under TBM and CBM scenes. Based on it, the paper points out that TBM is a predictable and controllable process such that it can satisfy one’s expectation by selecting suitable parameters. In another hand, CBM is usually unpredictable process so as not to arrive at desired aim. Hence, according to the relationship between the two, this paper proposes a CBM control scheme based on TBM, and investigates the dynamical behavior and real-time of control systems. The aim to do this is to provide operation maintenance and discussion making supports for the devices under CBM, and better instructs production, operating, and maintenance practical activities of the enterprises, and possesses broad application prosperity.

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