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Further Result for Globally Asymptotic Stability of a Class of Memristor-Based Recurrent Neural Networks with Time-Varying Delays

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Abstract

This paper investigates the uniqueness and globally uniformly asymptotic stability for a class of memristor-based recurrent neural networks with time-varying delays. By employing a homeomorphism and suitable Lyapunov functional and differential condition, a sufficient conclusion for the uniqueness and globally uniformly asymptotic stability of a class of memristor-based recurrent neural networks is attained. Comparing with the previous corresponding results, we can derive that our results are new and improve the previous result reported on global uniform asymptotic stability. Two illustrative examples are given to demonstrate the applicability and advantages of our result.

Keywords*: Memristive neural network; Global uniform asymptotic stability;Time delay; Lyapunov functional; Differential inclusion*

1. Introduction

In 1971, Chua [1] found besides the resistor, capacitor and inductor, there should be the fourth circuit element from physical symmetry arguments. Then, he called it a memristor which has many valuable features. But no much attention for this element was paid to. In 2008, the scientists of HP Lab announced that they had built a type of the memristor [2]. Recently, the researchers have shown a lot of promising applications of memristive devices [3-7].

This memristor shares many properties of resistor and shares the same unit of measurement. The memristor is a nonlinear, whose value is not unique and it has the feature of pinched hysteresis. Because of this feature, the new models of networks based on memristor have been designed and analyzed, one of which is to apply this device to building a novel model of neural network to emulate the human brains.

It is known to us that the neural networks have found many important applications in the fields associated with memory. Assume that we make use of the memristors instead of the resistor, then we can build a novel model where the parameters change according to its state. The organization of the paper is as follows. Next, we will propose a mathematical model of memristors, and the existence and uniqueness for a class of memristor-based recurrent neural networks are analyzed. At last, we analyzed the globally uniformly asymptotic stability of the memristor-based recurrent neural networks model with time-varying delays.

2. Model Description and Preliminaries

According to the current-voltage characteristics and the feature of the memristor, we provide a model of the memristance:

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$$M(v(t)) = \begin{cases} M' & \dot{v}(t) > 0, \\ M'' & \dot{v}(t) < 0, \\ unchanged & \dot{v}(t) = 0. \end{cases} \quad \textbf{(1)}$$

where v is the voltage applied to the memristor, $\dot{v}(t)$ is the derivative of $v(t)$ with respect to t. M' and M'' are constants. "Unchanged " implies that the memristance holds the voltage value. We can build the connection links of the neural networks which are called a memristor-based neural networks by utilizing memristors to replace resistors in the circuit.

Next, we consider a model of the memristor-based recurrent neural networks with time-varying delays as follows:

$$\dot{z}_i(t) = -d_i z_i(t) + \sum_{j=1}^{n} a_{ij}(z_i - z_j)\hat{f}_j(z_j(t)) + \sum_{j=1}^{n} b_{ij}(z_i - z_j)\hat{g}_j(z_j(t-\tau(t))) + s_i, \quad \textbf{(2)}$$

where $z_i(t)$ is the state variable of the i th neuron; d_i is the i th feedback connection weight; $a_{ij}(z_i - z_j)$ and $b_{ij}(z_i - z_j)$ are memritor-based connection weights and associated with time delays, respectively; s_i is an external input to the i th neuron; $\hat{f}(\cdot)$ and $\hat{g}(\cdot)$ are the i th activation functions; n denotes the number of neurons in the indicated neural networks.

Throughout this paper, we will use the following assumptions:

(H) $\hat{f}(\cdot)$ and $\hat{g}(\cdot)$ are Lipschitz continuous, that is,

$$\left|\hat{f}_i(x) - \hat{f}_i(y)\right| \le \sigma_i |x-y|, \quad |\hat{g}_i(x) - \hat{g}_i(y)| \le \lambda_i |x-y|.$$

where $\forall x, y \in R$, $x \ne y$ and $\sigma_i > 0$, $\lambda_i > 0$ $(i = 1, 2, \cdots, n)$.

Because the connection weights are carried out by utilizing memristors, the value of $w_{ij}(z_i - z_j)$ is defined as

$$w_{ij}(z_i - z_j) = \begin{cases} w'_{ij} & \dot{z}_i - \dot{z}_j > 0, \\ w''_{ij} & \dot{z}_i - \dot{z}_j < 0, \\ unchanged & \dot{z}_i - \dot{z}_j = 0. \end{cases} \quad (3)$$

where w can be a or b, a_{ij} and b_{ij} are constants. For convenience, we define that $a'_{ii} = a''_{ii} = a_{ii}$ and $b'_{ii} = b''_{ii} = b_{ii}$.

Recently, there are a number of results about the global stability of memristor-based recurrent neural networks to have been obtained by many scholars [7-13]. For system (2), because $a_{ij}(\cdot)$ and $b_{ij}(\cdot)$ are discontinuous, a differential equation with a discontinuous right-hand side has same solution set as a certain differential inclusions based on A.F.Filippov [15]. Then, we will turn to studying the relevant differential inclusions to analyze the global stability of system (2).

Next, some useful definitions and lemmas are introduced.

Consider the following ordinary differential equations

$$\dot{x} = k(t, x), \quad (4)$$

where $k: R \times R^n \to R^n$ is discontinuous, which the Filippov solution of (4) is given as follows.

Definition 1 A function $x(\cdot)$ is called a solution of (4) on $[t_0, t_1]$, if $x(\cdot)$ is absolutely continuous on $[t_0, t_1]$ and for almost all $t \in [t_0, t_1]$

$$\dot{x} \in \tilde{k}(t, x). \tag{5}$$

where

$$\tilde{k}(t, x) \overset{\Delta}{=} \bigcap_{\sigma>0} \bigcap_{\mu(N)=0} \overline{co}\{k(t, B_\sigma(x) \setminus B)\}. \tag{6}$$

where $\overline{co}$ denotes the convex closure of a set and μ is the usual Lebesgue measure of R^n.

There is an equivalent definition when $k(t, x)$ is locally bounded (see [16]). For $t \geq 0$, there exists a set $N_0^t \subset R^n$ with $\mu(N_0^t) = 0$ such that

$$\tilde{k}(t, x) = co\{v : \exists\{x_i\} with\, x_i \to x \quad such that\, x_i \notin N_0^t \cup N and \quad v = \lim k(t, x_i)\} \tag{7}$$

for any $x \subset R^n$ and $N \subset R^n$ with $\mu(N) = 0$.

Therefore, the switching system

$$\dot{x} = \varphi_\sigma(t, x), \sigma \in \Omega, \tag{8}$$

has the same solution sets as the following differential inclusion

$$\dot{x} \in \{\varphi_\sigma(t, x)\}, \tag{9}$$

where $\sigma : [0, \infty) \to \Omega$ is an arbitrary switching signal and Ω is the index set. $\varphi(t, x)$ is locally bounded and $co\{\varphi_\sigma(x)\}$ denotes the convex hull of $\varphi_\sigma(x)$.

In the following, we will investigate the differential inclusion (9) in order to further study the stability of (8).

Lemma 1 Denote $\tilde{k}(t, x)$ be set-valued map defined in (6) or (7). If $\tilde{k}(t, x)$ is locally bounded and Lebesgue measurable with respect to $(t, x) \in [0, +\infty) \times R^n$, then $\tilde{k}(t, x)$ as a set-valued map of x satisfies:

(H_1): $\tilde{k}(t, x)$ is a nonempty, compact, convex subset of R^n for any $t \geq 0$ and $x \in R^n$;

(H_2): $\tilde{k}(t, x)$ is Lebesgue measurable for any $x \in R^n$;

(H_3): $\tilde{k}(t, x)$ is upper semicontinuous for any $t \geq 0$;

(H_4): $\tilde{k}(t, x)$ is locally bounded.

Lemma 2[14] Let $\tilde{k}(t, x)$ be a set-valued map. If $\tilde{k}(t, x)$ satisfies H_1, H_2, H_3 and H_4, the for any $(t, x) \in [0, +\infty) \times R^n$, there exists an interval I and a solution $x(t) : I \to R^n$ of (5) such that $t_0 \in I$ and $x(t_0) = x_0$.

Definition 2 The differential inclusion (5) is said to be globally uniform asymptotically stable, if the equilibrium point of (5) is uniformly stable, globally uniformly attractive and all the solutions are uniformly bounded. That is, there exist two functions $m:(0,+\infty)\to(0,+\infty)$ and $T:(0,+\infty)\times(0,+\infty)\to(0,+\infty)$ such that

1) for any $r>0$, $(t_0,x_0)\in(0,+\infty)\times R^n$ and any solution $x(\cdot)$

$\|x_0\|\le r\Rightarrow\|x(t)\|<m(r),\quad \forall t\ge t_0$;

2) $\lim\limits_{t\to 0^+} m(r)=0$;

3) for any $r>0$, $\varepsilon>0$ any $(t_0,x_0)\in(0,+\infty)\times R^n$,

$\|x_0\|\le r\Rightarrow\|x(t)\|<\varepsilon,\quad \forall t\ge t_0+T(r,\varepsilon)$.

Remark 1 According to Definition 2, it is easy to see that the global asymptotic stability is weaker than the global uniform asymptotic stability. In addition, if a differential inclusion is globally exponential stability, then it is also globally uniform attractive. Therefore, the global exponential stability is stronger than the global uniformly asymptotic stability.

Lemma 3[14] Let $\tilde{k}:[0,+\infty)\times R^n\to R^n$ be a set-valued map such that the existence of the solution of the differential inclusion (5) is insured. Suppose that there exists a Lyapunov functional $V=V(t,x)$, for some functions $\varphi_1,\varphi_2,\varphi\in\kappa_0^\infty$, such that

$$\varphi_1(\|x\|)\le V(t,x)\le\varphi_2(\|x\|),\quad \forall t\in[0,+\infty), x\in R^n; \qquad (10)$$

$$V(t_2,x(t_2))-V(t_1,x(t_1))\le-\int_{t_1}^{t_2}\varphi(\|x(\tau)\|)d\tau,\quad t_1\le t_2; \qquad (11)$$

for any pair of time instants (t_1,t_2) and any solution $x(\cdot):[t_1,t_2]\to R^n$ of differential inclusion (5), then the equilibrium point of the differential inclusion(5) is globally uniformly asymptotical stability.

Lemma 4 If $H(x)\in C^0$ satisfies the following conditions**:**

1) $H(x)$ is injective on R^n,

2) $\|H(x)\|\to\infty$ as $\|x\|\to\infty$.

then $\|H(x)\|$ is a homeomorphism of R^n.

Throughout this paper, the following notations will be used: let $v=(v_1,v_2,\cdots,v_n)^T$ be a n-dimensional vector and $Q=(q_{ij})$ be a real $n\times n$ matrix. Then $|v|$ will denote $|v|=(|v_1|,|v_2|,\cdots,|v_n|)^T$ and $|Q|=(|q_{ij}|)$.The following norms will also be used:

$$\|v\|_1=\sum_{i=1}^n|v_i|,\quad \|v\|_2=(\sum_{i=1}^n v_i^2)^{\frac{1}{2}},\quad \|v\|_\infty=\max_{1\le i\le n}|v_i|.$$

$$\|Q\|_1=\max_{1\le i\le n}\sum_{j=1}^n|q_{ji}|,\quad \|Q\|_2=(\lambda_{\max}(Q^TQ))^{\frac{1}{2}},\quad \|Q\|_\infty=\max_{1\le i\le n}\sum_{j=1}^n|q_{ij}|.$$

3. Main Results

The System (2) can be rewritten the vector form as follows:

$$\dot{z}(t)=P(z)=-Dz(t)+A(z)\hat{f}(z(t))+B(z)\hat{g}(z(t-\tau(t)))+S. \qquad (12)$$

Because the activation functions $\hat{f}_i(z)$ and $\hat{g}_i(z)$ are locally bounded and Lipschitz continuous, $P(z)$ is also locally bounded. Through the theories of differential inclusion and set-valued maps, the system (12) has the same solutions as the following differential inclusion:

$$\dot{z} \in co\{P(z)\} = -Dz(t) + A(z)\hat{f}(z(t)) + B(z)\hat{g}(z(t-\tau(t))) + S. \qquad (13)$$

where $D = diag(d_1, d_2, \cdots, d_n)$, $A(z) = (\xi_{ij}a'_{ij} + (1-\xi_{ij})a''_{ij})_{n\times n}$,

$B(z) = (\xi_{ij}b'_{ij} + (1-\xi_{ij})b''_{ij})_{n\times n}$, ξ_{ij} s are some constants and satisfies $0 \le \xi_{ij} \le 1$ and $\xi_{ij} + \xi_{ji} = 1$. $S = (s_1, s_2, \cdots, s_n)^T$, $\hat{f}(z(t)) = [\hat{f}_1(z_1(t)), \hat{f}_2(z_2(t)), \cdots, \hat{f}_n(z_n(t))]^T$,

$\hat{g}(z(t)) = [\hat{g}_1(z_1(t)), \hat{g}_2(z_2(t)), \cdots, \hat{g}_n(z_n(t))]^T$. Thus, according to Lemma1 and Lemma2, the existence of the solution for (13) is ensured since $P(z)$ is locally bounded.

The differential inclusion (13) implies that there exists some $\xi_{ij}(i = 1,2,\cdots,n)$ such that

$$\dot{z}_i = -d_i z_i + \sum_{j=1}^{n}[\xi_{ij}a'_{ij} + (1-\xi_{ij})a''_{ij}]\hat{f}_j(z_j(t)) + \sum_{j=1}^{n}[\xi_{ij}b'_{ij} + (1-\xi_{ij})b''_{ij}]\hat{g}_j(z_j(t-\tau_j)) + s_i. \qquad (14)$$

For any ξ_{ij}, assumption (H) certifies the existence of an equilibrium point z^* for (14). Then, the equilibrium point can be shifted to the origin by using the translation $x = z - z^*$, then system (14) can be rewritten as

$$\dot{x}_i = -d_i x_i + \sum_{j=1}^{n}[\xi_{ij}a'_{ij} + (1-\xi_{ij})a''_{ij}]\hat{f}_j(x_j(t)) + \sum_{j=1}^{n}[\xi_{ij}b'_{ij} + (1-\xi_{ij})b''_{ij}]\hat{g}_j(x_j(t-\tau_j)) + s_i \qquad (15)$$

The system (15) can be rewritten as the differential inclusion:

$$\dot{x} \in -Dx(t) + Af(x(t)) + Bg(x(t-\tau)), \qquad (16)$$

where $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t))]^T = \hat{f}(z) - \hat{f}(z^*)$ and $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \cdots, g_n(x_n(t))]^T = \hat{g}(z) - \hat{g}(z^*)$. It can be easy to see that the functions $f_i(\cdot)$ and $g_i(\cdot)$ satisfy the assumption (H) and meanwhile satisfy the following assumption:

(H') For any $x \in R$, there exist positive constants σ_i and λ_i such that

$$|f_i(x)| \le \sigma_i|x|, \quad |g_i(x)| \le \lambda_i|x|, \quad i = 1,2,\cdots,n.$$

Theorem 1 The memristor-based recurrent neural networks (12) has a unique equilibrium point for every input vector S and is globally uniformly asymptotical stability, if there exist constants $\alpha_i > 0(i = 1,2,\cdots,n)$ and $r_1 \in [0,1], r_2 \in [0,1]$, such that the following conditions hold:

$$\Theta = 2\alpha_i d_i - \sum_{j=1}^{n}\alpha_i(\sigma_j^{2r_1}|a_{ij}|_{\max} + \lambda_j^{2r_2}|b_{ij}|_{\max})$$

$$-\sum_{j=1}^{n}\alpha_j(\sigma_i^{2(1-r_1)}\left|a_{ji}\right|_{\max}+\lambda_i^{2(1-r_2)}\left|b_{ji}\right|_{\max})>0, \qquad i=1,2,\cdots,n. \qquad (17)$$

where $\left|a_{ij}\right|_{\max}=\max\{\left|a'_{ij}\right|,\left|a''_{ij}\right|\}$, $\left|b_{ij}\right|_{\max}=\max\{\left|b'_{ij}\right|,\left|b''_{ij}\right|\}$.

Proof: For every S, we will consider the mapping associated with system (12) in order to obtain the existence and uniqueness of the equilibrium point for system (12):

$$H(x)=-Dx+Af(x)+Bg(x)+S. \qquad (18)$$

Let x^* denote the equilibrium point of the model (12), and x^* must satisfy the following equation:

$$-Dx^*+Af(x^*)+Bg(x^*)+S=0.$$

It is obvious the solution of $H(x)=0$ is an equilibrium point of (12). By Lemma 4, for every input vector S, we can obtain that there exists a unique equilibrium point for the system (12) if $H(x)$ is homeomorphism of R^n. Next, we will demonstrate that the condition of Theorem 1 implies that $H(x)$ is a homeomorphism of R^n. Considering two vectors $x\in R^n$, $y\in R^n$ and $x\neq y$, for this case, $H(x)$ which defined by (18) satisfies the following equation:

$$H(x)-H(y)=-D(x-y)+A(f(x)-f(y)+B(g(x)-g(y)). \qquad (19)$$

By multiplying by $2(x-y)^T\Gamma$ both sides of (19), where $\Gamma=diag(\alpha_1,\alpha_2,\cdots,\alpha_n)$, we obtain

$$2(x-y)^T\Gamma(H(x)-H(y))$$

$$=-2\sum_{i=1}^{n}\alpha_i d_i(x_i-y_i)^2+2\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i(\xi_{ij}a'_{ij}+(1-\xi_{ij})a''_{ij})(x_i-y_i)(f_j(x_j)-f_j(y_j))$$

$$+2\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i(\xi_{ij}b'_{ij}+(1-\xi_{ij})b''_{ij})(x_i-y_i)(g_j(x_j)-g_j(y_j)). \qquad (20)$$

There have $2ab\le ka^2+\frac{1}{k}b^2$ for any real numbers a and b, where k is a positive constant number. Then, we can derive the following inequality,

$$2\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i(\xi_{ij}a'_{ij}+(1-\xi_{ij})a''_{ij})(x_i-y_i)(f_j(x_j)-f_j(y_j))$$

$$\le 2\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\left|\xi_{ij}a'_{ij}+(1-\xi_{ij})a''_{ij}\right|\left|x_i-y_i\right|\left|f_j(x_j)-f_j(y_j)\right|$$

$$\le\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\left|a_{ij}\right|_{\max}\sigma_j(\sigma_j^{2r_1-1}(x_i-y_i)^2+\sigma_j^{1-2r_1}(x_j-y_j)^2)$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_i\left|a_{ij}\right|_{\max}\sigma_j^{2r_1}+\alpha_j\left|a_{ji}\right|_{\max}\sigma_i^{2(1-r_1)})(x_i-y_i)^2;$$

$$2\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i(\xi_{ij}b'_{ij}+(1-\xi_{ij})b''_{ij})(x_i-y_i)(g_j(x_j)-g_j(y_j))$$

$$\leq 2\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\left|\xi_{ij}b'_{ij}+(1-\xi_{ij})b''_{ij}\right|\left|x_i-y_i\right|\left|g_j(x_j)-g_j(y_j)\right|$$

$$\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\left|b_{ij}\right|_{\max}\lambda_j(\lambda_j^{2r_2-1}(x_i-y_i)^2+\lambda_j^{1-2r_2}(x_j-y_j)^2)$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_i\left|b_{ij}\right|_{\max}\lambda_j^{2r_2}+\alpha_j\left|b_{ji}\right|_{\max}\lambda_i^{2(1-r_2)})(x_i-y_i)^2.$$

Therefore

$$2\sum_{i=1}^{n}\alpha_i(x_i-y_i)(h_i(x_i)-h_i(y_i))\leq-\sum_{i=1}^{n}\Theta(x_i-y_i)^2. \qquad (21)$$

It is noted that $\Theta>0$ that we can easily get $\sum_{i=1}^{n}\alpha_i(x_i-y_i)(h_i(x_i)-h_i(y_i))<0$. So, there exists at least one index i such that $h_i(x_i)\neq h_i(y_i)$ which follows that $H(x)\neq H(y)$ for all $x\neq y$. Therefore, $H(x)$ is an injective map.

In order to prove $\Theta>0$ which implies $\|H(x)\|\to\infty$ as $\|x\|\to\infty$, we will consider (21) for such case $y=0$ as the following:

$$2x^T\Gamma(H(x)-H(0))\leq-\Theta\|x\|_2^2.$$

By taking the absolute value of both sides of the above inequality, we can get

$$\left|2x^T\Gamma(H(x)-H(0))\right|\geq\Theta\|x\|_2^2.$$

Because

$$\left|2x^T\Gamma(H(x)-H(0))\right|\leq 2\alpha\|x\|_\infty\|H(x)-H(0)\|_1,$$

where $\alpha=\max(\alpha_i)$. Therefore, we can write

$$2\alpha\|x\|_\infty\|H(x)-H(0)\|_1\geq\Theta\|x\|_2^2.$$

By virtue of $\|x\|_\infty\leq\|x\|_2$ and $\|H(x)-H(0)\|_1\leq\|H(x)\|_1+\|H(0)\|_1$, it follows that $\|H(x)\|_1\geq\frac{\Theta}{2\alpha}\|x\|_2-\|H(0)\|_1$. Because $\|H(0)\|_1$ is finite, we can get $\|H(x)\|\to\infty$ as $\|x\|\to\infty$.Then, $H(x)$ is homeomorphism of R^n. This completes the proof about the existence and uniqueness of the equilibrium point for the neural networks defined by (12).

Next, in order to prove that $\Theta > 0$ is a sufficient condition of the global uniform asymptotic stability for system (12), we will prove that the origin of system (15) is globally uniformly asymptotical stability.

Next, the following Lyapunov functional is constructed

$$V(t) = \sum_{i=1}^{n} \alpha_i x_i^2(t) + \sum_{i=1}^{n} \int_{t-\tau_i}^{t} r_i x_i^2(s) ds .$$

where $r_i = \sum_{j=1}^{n} \alpha_j \lambda_i^{2(1-r_2)} \left| b_{ji} \right|_{\max}$.

Computing the upper Dini-derivative of $V(t)$ along the system (16), we can derive

$$D^+V(t)$$

$$= \sup\{2\sum_{i=1}^{n} \alpha_i x_i(t)[-d_i x_i(t) + \sum_{j=1}^{n} (\xi_{ij} a'_{ij} + (1-\xi_{ij}) a''_{ij}) f_j(x_j(t)))$$

$$+ \sum_{j=1}^{n} (\xi_{ij} b'_{ij} + (1-\xi_{ij}) b''_{ij}) g_j(x_j(t-\tau_j))] + \sum_{i=1}^{n} r_i [x_i^2(t) - x_i^2(t-\tau_i)]\}$$

$$\le \sup\{-2\sum_{i=1}^{n} \alpha_i d_i x_i^2(t) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \left| \xi_{ij} a'_{ij} + (1-\xi_{ij}) a''_{ij} \right| \left| x_i(t) \right| \sigma_j \left| x_j(t) \right|$$

$$+ 2\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \left| \xi_{ij} b'_{ij} + (1-\xi_{ij}) b''_{ij} \right| \left| x_i(t) \right| \lambda_j \left| x_j(t-\tau_j) \right| + \sum_{i=1}^{n} r_i [x_i^2(t) - x_i^2(t-\tau_i)]\}$$

$$\le -2\sum_{i=1}^{n} \alpha_i d_i x_i^2(t) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \sigma_j \left| a_{ij} \right|_{\max} \left| x_i(t) \right| \left| x_j(t) \right|$$

$$+ 2\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \lambda_j \left| b_{ij} \right|_{\max} \left| x_i(t) \right| \left| x_j(t-\tau_j) \right| + \sum_{i=1}^{n} r_i [x_i^2(t) - x_i^2(t-\tau_i)]\}$$

$$\le -2\sum_{i=1}^{n} \alpha_i d_i x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \left| a_{ij} \right|_{\max} \sigma_j [\sigma_j^{2r_1-1} x_i^2(t) + \sigma_j^{1-2r_1} x_j^2(t)]$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \left| b_{ij} \right|_{\max} \lambda_j [\lambda_j^{2r_2-1} x_i^2(t) + \lambda_j^{1-2r_2} x_j^2(t-\tau_j)^2 + \sum_{i=1}^{n} r_i [x_i^2(t) - x_i^2(t-\tau_i)]\}$$

$$= -2\sum_{i=1}^{n} \alpha_i d_i x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \left| a_{ij} \right|_{\max} \sigma_j^{2r_1} x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_j \left| a_{ji} \right|_{\max} \sigma_i^{2(1-r_1)} x_i^2(t)$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \left| b_{ij} \right|_{\max} \lambda_j^{2r_2} x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_j \left| b_{ji} \right| \lambda_i^{2(1-r_2)} x_i^2(t-\tau_i)^2$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\left|b_{ji}\right|_{\max}\lambda_i^{2(1-r_2)}x_i^2(t)-\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_j\left|b_{ji}\right|\lambda_i^{2(1-r_2)}x_i^2(t-\tau_i)^2$$

$$=-\sum_{i=1}^{n}[2\alpha_i d_i-\sum_{j=1}^{n}\alpha_i(\sigma_j^{2r_1}\left|a_{ij}\right|_{\max}+\lambda_j^{2r_2}\left|b_{ij}\right|_{\max})$$

$$-\sum_{j=1}^{n}\alpha_j(\sigma_i^{2(1-r_1)}\left|a_{ji}\right|_{\max}+\lambda_i^{2(1-r_2)}\left|b_{ji}\right|_{\max})]x_i^2(t)$$

$$\le-\Theta\sum_{i=1}^{n}x_i^2(t)\le-\Theta\left\|x(t)\right\|_2^2. \qquad (22)$$

From (17), it follows that $\Theta>0$. Then, by integrating (22) from t_1 to t_2, we get

$$V(x(t_1))-V(x(t_2))\le-\int_{t_1}^{t_2}\Theta\left\|x(\tau)\right\|_2^2dt,$$

which implies that (11) holds. Since $V(t)$ is positive definite, (10) is satisfied. Thus, differential inclusion (13) is globally uniformly asymptotically stable by Lemma3. However, (12) has the same solution set as (13) which implies (12) is also globally uniformly asymptotical stability.

4. Comparisons and Examples and Simulations

In this section, we make two examples and compare our result with the previous ones to show the effectiveness of ours. Firstly, we restate the result obtained in [8].

Theorem 2[8] The memristor-based recurrent neural network(12) is globally uniform asymptotically stable if there exist constants $\alpha_i(i=1,2,\cdots,n)$ such that

$$2d_i\alpha_i-\sum_{j=1}^{n}\alpha_i(\mu_j\left|a_{ij}\right|_{\max}+\lambda_j\left|b_{ij}\right|_{\max})-\sum_{j=1}^{n}\alpha_j(\mu_i\left|a_{ji}\right|_{\max}+\lambda_i\left|b_{ji}\right|_{\max})>0, \qquad (23)$$

where $\left|a_{ij}\right|_{\max}=\max\{\left|a'_{ij}\right|,\left|a''_{ij}\right|\}$, $\left|b_{ij}\right|_{\max}=\max\{\left|b'_{ij}\right|,\left|b''_{ij}\right|\}$.

Remark 2 It is easily to prove that the result given in [8] should be considered as the special case of our paper. Let $r_1=\frac{1}{2},r_2=\frac{1}{2}$, the result of Theorem 2 can be derived from the result of Theorem 1.

Now, consider the following examples:

Example 1 Consider the memristor-based recurrent neural network with time delays

$$\dot{x}(t) = -Dx(t) + A(x)f(x(t)) + B(x)g(x(t-\tau)) + S, \qquad (24)$$

Where

$$D = \begin{pmatrix} 19/2 & 0 \\ 0 & 9 \end{pmatrix},\ A(x) = \begin{pmatrix} 2 & a_{12} \\ a_{21} & -4 \end{pmatrix},\ B(x) = \begin{pmatrix} 2 & b_{12} \\ b_{21} & 4 \end{pmatrix},\ a_{12} = \begin{cases} -7 & \dot{x}_1 > \dot{x}_2 \\ 1 & \dot{x}_1 < \dot{x}_2 \end{cases},$$

$$a_{21} = \begin{cases} 1.5 & \dot{x}_1 > \dot{x}_2 \\ -2 & \dot{x}_1 < \dot{x}_2 \end{cases},\ b_{12} = \begin{cases} -1 & \dot{x}_1 > \dot{x}_2 \\ 0.5 & \dot{x}_1 < \dot{x}_2 \end{cases},\ b_{21} = \begin{cases} -2 & \dot{x}_1 > \dot{x}_2 \\ 6 & \dot{x}_1 < \dot{x}_2 \end{cases},$$

and

$$f_i(x) = g_i(x) = \frac{1-e^x}{1+e^x}, \qquad i = 1,2.$$

It is clear that f_i and g_i are Lipschitz continuous with the Lipschitz constants $\mu_i = \lambda_i = \frac{1}{2}$. Note that $|A|_{\max} = \begin{pmatrix} 2 & 7 \\ 2 & 4 \end{pmatrix}$, $|B|_{\max} = \begin{pmatrix} 2 & 1 \\ 6 & 2 \end{pmatrix}$. If let $r_1 = 1, r_2 = 0$, and positive constants $\alpha_1 = 2$ and $\alpha_2 = 1$, the condition (23) of Theorem 2 can not be satisfied (when $i = 1$ (23) is not satisfied). Therefore, Theorem 2 can not be used to ascertain the stability. But the condition (17) of Theorem 1 is satisfied, and then (24) is globally uniformly asymptotically stable.

Example 2 Consider the following memristor-based recurrent neural network with time delays

$$\dot{x}(t) = -Dx(t) + A(x)f(x(t)) + B(x)g(x(t-\tau)) + S,$$

Where

$$D = \begin{pmatrix} 10 & 0 \\ 0 & 15 \end{pmatrix},\ A(x) = \begin{pmatrix} 2 & a_{12} \\ a_{21} & -3 \end{pmatrix},\ B(x) = \begin{pmatrix} -5 & b_{12} \\ b_{21} & -4 \end{pmatrix},\ a_{12} = \begin{cases} -1 & \dot{x}_1 > \dot{x}_2 \\ 0.5 & \dot{x}_1 < \dot{x}_2 \end{cases},$$

$$a_{21} = \begin{cases} 1.5 & \dot{x}_1 > \dot{x}_2 \\ -2 & \dot{x}_1 < \dot{x}_2 \end{cases},\ b_{12} = \begin{cases} 2 & \dot{x}_1 > \dot{x}_2 \\ 3 & \dot{x}_1 < \dot{x}_2 \end{cases},\ b_{21} = \begin{cases} -2 & \dot{x}_1 > \dot{x}_2 \\ 1 & \dot{x}_1 < \dot{x}_2 \end{cases},$$

and

$$f_i(x) = g_i(x) = \frac{1}{2}(|x+1| - |x-1|), \qquad i = 1,2.$$

It is clear that f_i and g_i are Lipschitz continuous with the Lipschitz constants $\mu_i = \lambda_i = 1$.

Note that $|A|_{\max} = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$, $|B|_{\max} = \begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix}$. Using Theorem 1, we can suppose that there exist positive constants α_1 and α_2 such that (17) is satisfied. Substituting $d_1 = 10$,

$d_2 = 15$, $|a_{12}|_{\max} = 1$, $|a_{21}|_{\max} = 2$, $|b_{12}|_{\max} = 3$, $|a_{21}|_{\max} = 2$, $\mu_i = \lambda_i = 1$ into the (17), inequalities results in $2\alpha_2 < \alpha_1 < 3\alpha_2$. Thus, the condition of Theorem 1 is satisfied, and then (25) is globally uniform asymptotically stable.

Let $S = (0.1, -0.2)^T$. The following four cases are considered: Case1 the initial state $x_1(t) = 2$ for $t \in [-\tau_1, 0]$, and $x_2(t) = -3$, for $t \in [-\tau_2, 0]$, and the delay parameters $\tau_1 = 0.3$ and $\tau_2 = 0.4$; Case2 the initial state $x_1(t) = 5$ for $t \in [-\tau_1, 0]$, and $x_2(t) = 3$, for $t \in [-\tau_2, 0]$, and the same delay parameters as in Case1; Case3 the initial state $x(t) = (-4.5, 1)^T$ for $t \in [-0.3, 0]$, and the delay parameters $\tau_1 = \tau_2 = 0.3$; Case4 $x(t) = (-2, 6)^T$ for $t \in [-0.3, 0]$ and the same delay parameters as in Case3. Figure 1 shows the time responses of the state variables $x_1(t)$ and $x_2(t)$ for different time delays. This illustrates that the solutions of system (25) globally converge to the equilibrium point x^*.

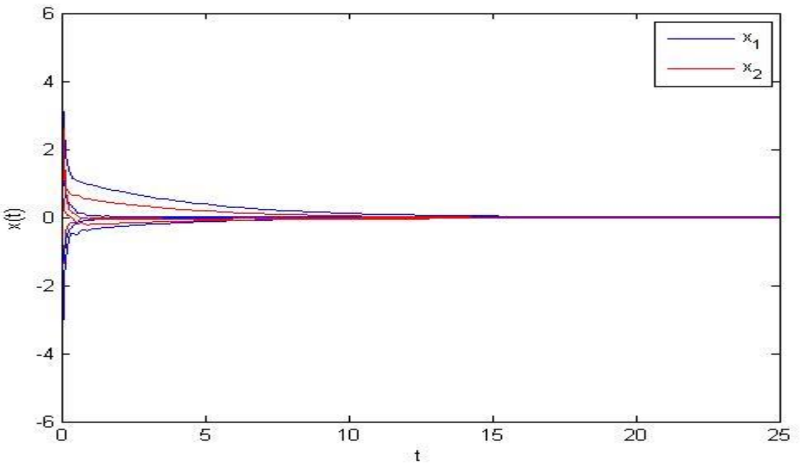

Figure 1. Transient Behavior of System (25) With Different Time Delays

5. Conclusion

In this paper, the uniqueness and global uniform asymptotic of the equilibrium point for the memristor-based recurrent neural networks with time delays are studied. By using the homeomorphism theory and Lyapunov functional and differential inclusion, we obtain a new result on the uniqueness and global uniform asymptotic stability for memristor-based recurrent neural networks with time delays. Two examples are given to illustrate effectiveness of the proposed result.

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