

## The Design of Overload Synchronizing Controller Based on the Model of Supersonic Missile's Feature Points

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### Abstract

*Since the model of hypersonic missile cannot be built accurately and it can change unpredictably, traditional design method is not very effective. A synchronizing control method is introduced in this paper. A synchronizing system that has the similar structures with the controlled system is constructed, then a control law is design to make the synchronizing system trace the controlled system. So the controlled system can be controlled by controlling the synchronizing system. Finally, detailed numerical simulation was done and it showed that the synchronous control method is proved to be more effective than PID control method.*

**Keywords:** *hypersonic missile, sliding mode control, synchronizing control*

### 1. Introduction

Supersonic sonic missiles are well researched by many countries for its military application. And as its speed increase, the control of supersonic missiles become a very difficult problem for engineers and scientists. The main difficulty is caused by the modeling of missile movement [1-8]. Because of the flying environment in the experiments is not the same it's in the sky, so the model of supersonic missiles discussed in all papers are not accurate and the uncertainties [9-14] and errors always exist and unknown.

The PID control method is effective for certain and accurate system and that is why the missile model is reduced to a second order system according to the experiment information in engineering. But with the increase of uncertainty, the PID method will lost its effective and it will be more and more difficult for PID method to control an uncertain system. So it is meaningful to assume that how can we do if the system is totally unknown. And any theory or control method should be more effective if the system model is more accurate. For system with uncertainties, most papers tried to use adaptive methods or robust control method to solve it, but we have different strategy in this paper [15-23].

We constructed a totally accurate system and also the original system is assumed to be totally unknown. Then we make the two system is synchronized with each other, then we only have to control the accurate system, it will make the original system also be

controlled. That is the main concept of synchronous control. In this paper, a kind of synchronous method is tried to introduce to missile control system. It is a new missile control method which is mostly used in the secure communication in the past. What is worthy pointing out is that the missile model is only used a linear model and the output is overload which is very easy to be measured in engineering. But it will make the design more complex compared with the situation that the attack angle is chosen as an output.

## 2. Model Description

According to traditional aircraft design's feature points solidifying and linearization idea, the model of hypersonic missile's feature points can be got, the model is a second order system as follows:

$$\dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z \quad (1)$$

$$\dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z \quad (2)$$

$$n_y = \frac{v}{g}a_{34}\alpha + \frac{v}{g}a_{35}\delta_z \quad (3)$$

Where  $a_{ij}$  is aerodynamic the goal of control is to design a control law such that the system state  $n_y$  can trace the expected value  $n_y^d$ , without loss of generality, assume the expected value  $n_y^d = 1$ .

Model deforms:

$$\alpha = \frac{g}{va_{34}}n_y - \frac{a_{35}}{a_{34}}\delta_z \quad (4)$$

$$\dot{n}_y = [\omega_z - a_{34}(\frac{g}{va_{34}}n_y - \frac{a_{35}}{a_{34}}\delta_z) - a_{35}\delta_z + \frac{a_{35}}{a_{34}}\dot{\delta}_z] / \left(\frac{g}{va_{34}}\right) \quad (5)$$

$$\dot{\omega}_z = a_{24}(\frac{g}{va_{34}}n_y - \frac{a_{35}}{a_{34}}\delta_z) + a_{22}\omega_z + a_{25}\delta_z \quad (6)$$

It is got by transferring:

$$\dot{n}_y = b_{11}n_y + b_{12}\omega_z + b_{13}\delta_z + b_{14}\dot{\delta}_z \quad (7)$$

$$\dot{\omega}_z = b_{21}n_y + b_{22}\omega_z + b_{23}\delta_z \quad (8)$$

## 3. Design Synchronizing Control Law

A synchronizing system is designed for the control the controlled system. When the controlled system is the simplified model of missile, the synchronizing can be designed as follows:

$$\dot{\hat{n}}_y = \text{sign}(b_{12})\hat{\omega}_z + b_{13}\delta_z + b_{14}\dot{\delta}_z + v_1 \quad (9)$$

$$\dot{\hat{\omega}}_z = b_{23}\delta_z + v_2 \quad (10)$$

Where  $b_{13}, b_{14}, b_{23}$  is known.

According to the simplified model and synchronizing system, define a error variable as follows:

$$e_1 = n_y - \hat{n}_y \quad (11)$$

$$e_2 = \omega_z - \hat{\omega}_z \quad (12)$$

Then the error system can be got by subtracting the above two systems:

$$\dot{e}_1 = b_{11}n_y + b_{12}\omega_z - \text{sign}(b_{12})\hat{\omega}_z - v_1 \quad (13)$$

$$\dot{e}_2 = b_{21}n_y + b_{22}\omega_z - v_2 \quad (14)$$

Design synchronizing control law as follows:

$$v_1 = k_{n1}e_1 + k_{n2} \int e_1 dt \quad (15)$$

$$v_2 = k_{\omega1}e_2 + k_{\omega2} \int e_2 dt \quad (16)$$

So  $e_1$  and  $e_2$  can approach zero when control parameters  $k_{n1}, k_{n2}, k_{\omega1}, k_{\omega2}$  are large enough.

Define:

$$z = n_y - n_y^d \quad (17)$$

then

$$\dot{z} = \dot{n}_y = \text{sign}(b_{12})\hat{\omega}_z + b_{13}\delta_z + b_{14}\dot{\delta}_z + v_1 \quad (18)$$

The sliding surface of the system is made of error and error intergral and error differential, the sliding can be written as:

$$s = \dot{z} + k_{z1}z + k_{z2} \int z dt \quad (19)$$

Then:

$$\dot{s} = \ddot{z} + k_{z1}\dot{z} + k_{z2}z \quad (20)$$

Further deform:

$$\dot{s} = \text{sign}(b_{12})\hat{\omega}_z + b_{13}\dot{\delta}_z + b_{14}\ddot{\delta}_z + \dot{v}_1 + k_{z1}\dot{z} + k_{z2}z \quad (21)$$

Then:

$$\dot{s} = \text{sign}(b_{12})\hat{\omega}_z + b_{13}\dot{\delta}_z + b_{14}\ddot{\delta}_z + \dot{v}_1 + k_{z1}\text{sign}(b_{12})\hat{\omega}_z + k_{z1}b_{13}\dot{\delta}_z + k_{z1}b_{14}\dot{\delta}_z + k_{z1}v_1 + k_{z2}z \quad (22)$$

Put the derivative of angular speed into the above formula

$$\begin{aligned} \dot{s} = & \text{sign}(b_{12})b_{23}\delta_z + \text{sign}(b_{12})v_2 + b_{13}\dot{\delta}_z + b_{14}\ddot{\delta}_z + \dot{v}_1 \\ & + k_{z1}\text{sign}(b_{12})\hat{\omega}_z + k_{z1}b_{13}\dot{\delta}_z + k_{z1}b_{14}\dot{\delta}_z + k_{z1}v_1 + k_{z2}z \end{aligned} \quad (23)$$

Define:

$$T = (\text{sign}(b_{12})b_{23} + k_{z1}b_{13})\delta_z + (b_{13} + k_{z1}b_{14})\dot{\delta}_z + b_{14}\ddot{\delta}_z \quad \dot{s} = c\dot{e} + de + \ddot{e} \quad (24)$$

Design:

$$T = -k_{z2}z - \text{sign}(b_{12})v_2 - k_{z1}\text{sign}(b_{12})\hat{\omega}_z - \dot{v}_1 - k_{z1}v_1 - k_1s - k_2 \frac{s}{|s| + \varepsilon} - k_3 \frac{1 - e^{-\tau_1 s}}{e^{-\tau_1 s} + 1} \quad (25)$$

Then:

$$\dot{s} = -k_1s - k_2 \frac{s}{|s| + \varepsilon} - k_3 \frac{1 - e^{-\tau_1 s}}{e^{-\tau_1 s} + 1} \quad (26)$$

Choose a Lyapunov function as:

$$V = \frac{1}{2}s^2 \quad (27)$$

By derivative, it is easy to get:

$$\dot{V} = -k_1s^2 - k_2 \frac{s^2}{|s| + \varepsilon} - k_3s \frac{1 - e^{-\tau_1 s}}{e^{-\tau_1 s} + 1} \leq 0 \quad (28)$$

According to the principle of Lyapunov, the system is stable by now. based on the transfer function, the inverse solution of Control u is got.define:

$$\begin{aligned} T_1 &= \text{sign}(b_{12})b_{23} + k_{z1}b_{13} \\ T_2 &= b_{13} + k_{z1}b_{14} \\ T_3 &= b_{14} \end{aligned} \quad (29)$$

Then:

$$T_1\delta_z + T_2\dot{\delta}_z + T_3\ddot{\delta}_z = T \quad (30)$$

The inverse solution of control law is got:

$$\delta_z = \frac{T}{T_3S^2 + T_2S + T_1} \quad (31)$$

Where  $S$  is differential operator. In order to ensure the system being stable, there must be  $T_3T_2T_1 > 0$ .

#### 4. Numerical Simulation

The system is proved to be stable by theoretical derivation as above, in order to test the stability of the system, this section uses SIMULINK tool case in MATLAB to the simulation.

In this section, the parameters of missile are chosen as follows:

$$\begin{aligned} a_{25} &= -167.87; \quad a_{35} = 0.243; \quad a_{22} = -2.876; \quad a_{24} = -193.65; \quad a_{34} = 1.584; \\ g &= 9.764; \quad v = 804.4 \end{aligned}$$

Because the parameters are got from experimental data by wind tunnel, there exists gap between the measured value and the real value. So the data near the feature points will be change 50% even 90% to test the robustness of controller.

#### 4.1 Constant Perturbation

Assume that the value of constant perturbation is  $k$ , then the parameters can be written as:

$$A_{22} = a_{22} \cdot (1 + k); \quad A_{24} = a_{24} \cdot (1 + k); \quad A_{25} = a_{25} \cdot (1 + k);$$

$$A_{34} = a_{34} \cdot (1 + k); \quad A_{35} = a_{35} \cdot (1 + k)$$

(1) PID simulation

Choose PID control parameters as follows:

$$k_p = 1, k_i = 2, k_d = 2$$

Assume that the value expected is one, simulation results are as follows:

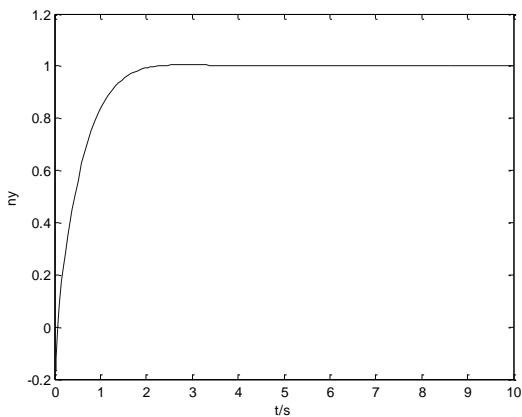


Figure 1. K=0 Curve Of Simulation

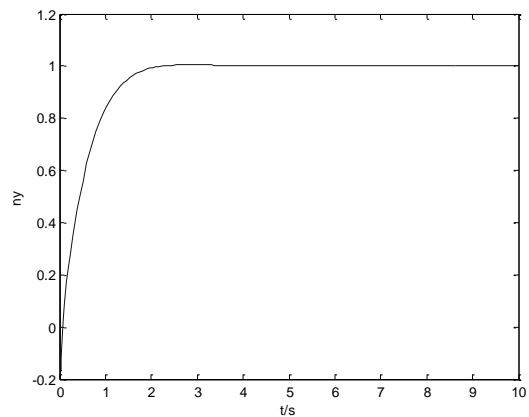


Figure 2. K=-5% Curve Of Simulation

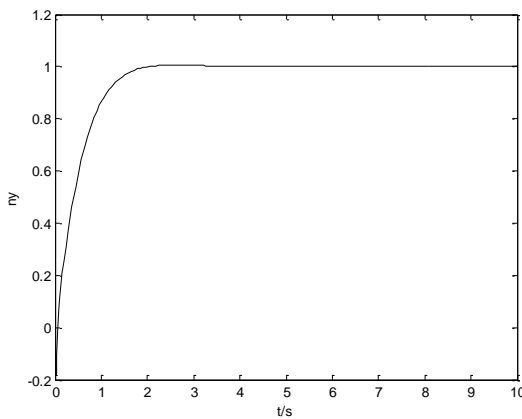


Figure 3. K=5% Curve Of Simulation

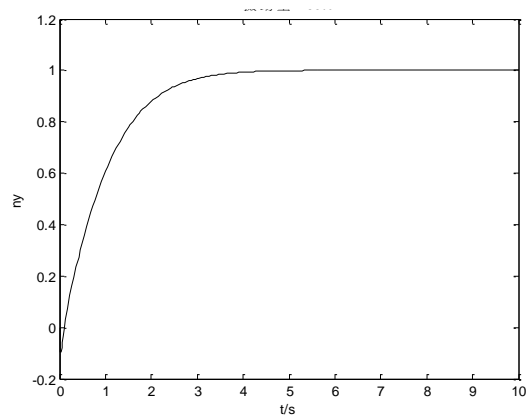
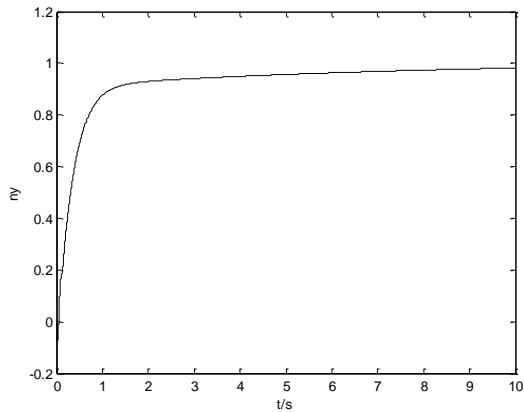
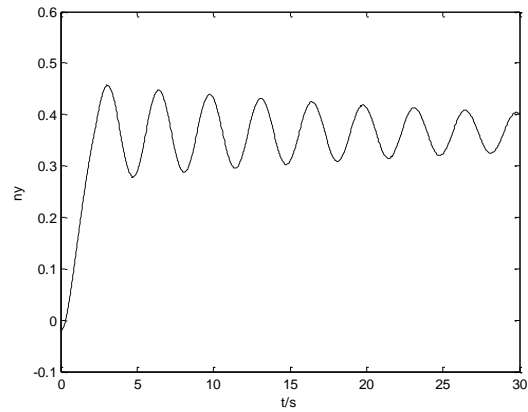


Figure 4. K=-50% Curve Of Simulation



**Figure 6. K=50% Curve Of Simulation**



**Figure 6. K=90% Curve Of Simulation**

May see from the simulation results, the system that is controlled by PID is stable when the perturbation is small, but the more perturbation is, the more system is unstable. This shows that PID control method has its range of application, this should be noticed in engineering practice.

(2) Overload sliding mode synchronizing control simulation

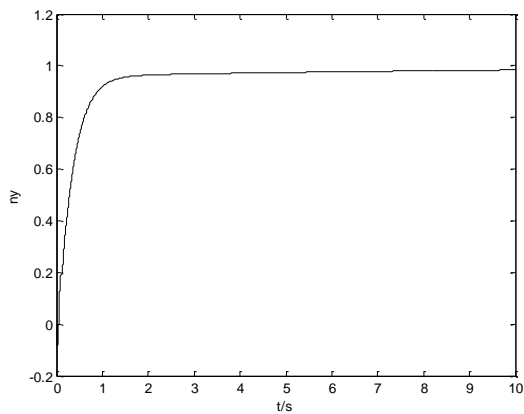
According to the above theory, choose control parameters as follows:

$$k_{\alpha 1}=30, k_{\alpha 2}=0.1, k_{\omega 1}=30, k_{\omega 2}=0.1, k_{s1}=7, k_{s2}=12,$$

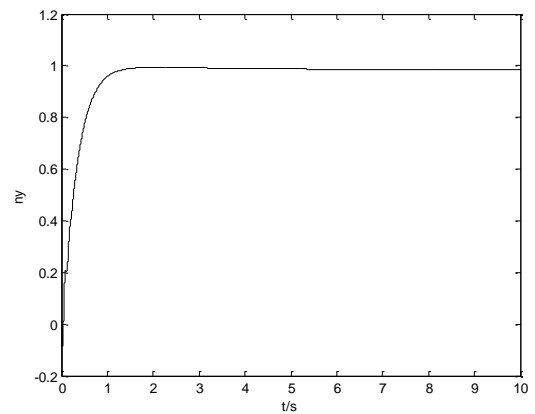
$$k_{z1}=50, k_{z2}=5, k_{n1}=20, k_{\omega 1}=20,$$

$$k_{n2}=2, k_{\omega 2}=2, k_{s3}=2, k_{s4}=5/57.3, e=0.5, b_{ij} (i=1,2,3 j=1,2,3)。$$

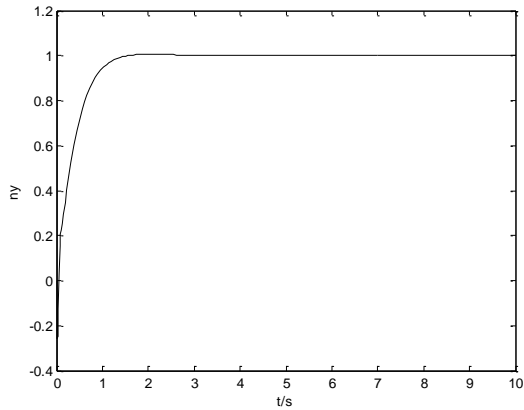
Assume that the value expected is one, simulation results are as follows:



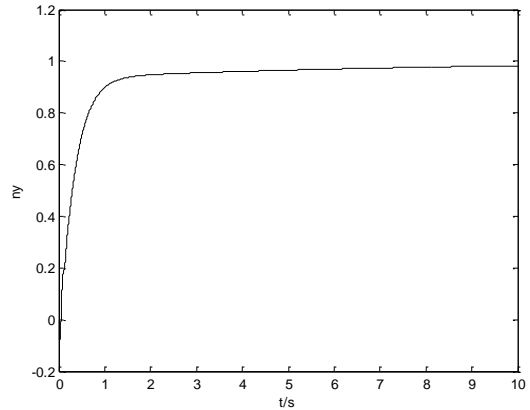
**Figure 7. K=0 Curve of Simulation**



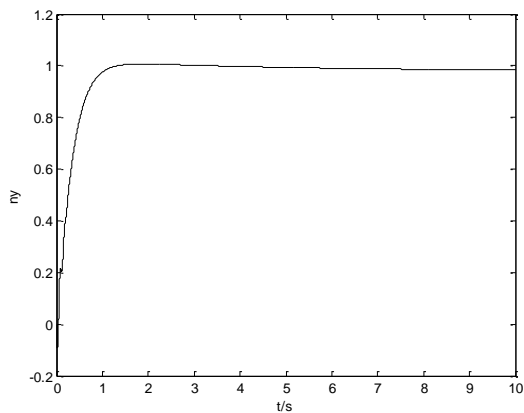
**Figure 8. K=-5% Curve of Simulation**



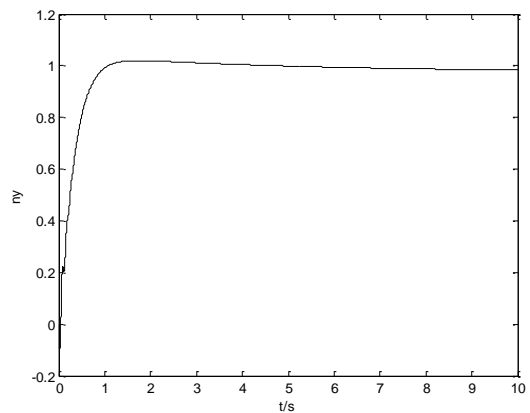
**Figure 9. K=5% Curve of Simulation**



**Figure 10. K=-50% Curve of Simulation**



**Figure 11. K=50% Curve of Simulation**



**Figure 12. K=90% Curve of Simulation**

May see from the simulation results, the controlled system is always stable with different perturbation, this shows that overload sliding mode synchronizing control method has better control effect and strongly practice applicability.

#### 4.2 Random perturbation

Assume that the value of random perturbation is  $k_s$ , then the value of random perturbation can be written in matlab as:

$$k_s = 2K * (rand(5, 1) - 0.5)$$

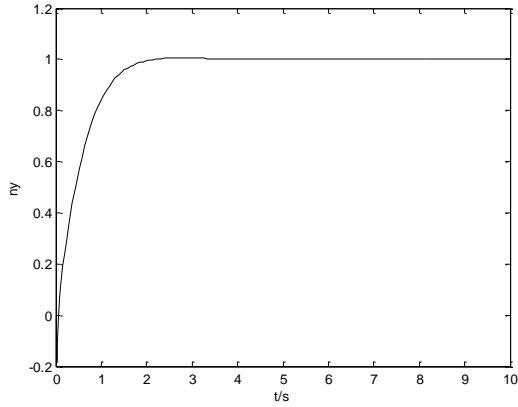
So five value can be got:  $k_{s1}$ ,  $k_{s2}$ ,  $k_{s3}$ ,  $k_{s4}$ ,  $k_{s5}$   
then the parameters can be written as:

$$A_{22} = a_{22} \cdot (1 + k_{s1}); A_{24} = a_{24} \cdot (1 + k_{s2}); A_{25} = a_{25} \cdot (1 + k_{s3});$$

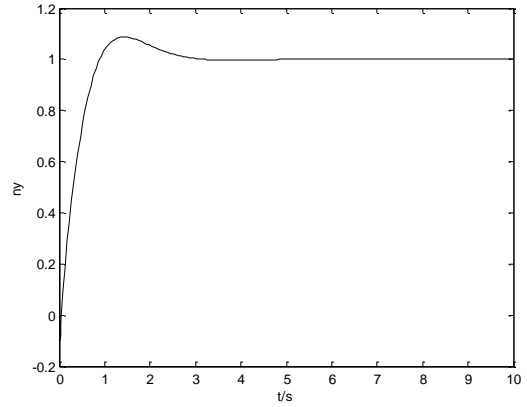
$$A_{34} = a_{34} \cdot (1 + k_{s4}); A_{35} = a_{35} \cdot (1 + k_{s5})$$

#### (1) PID simulation

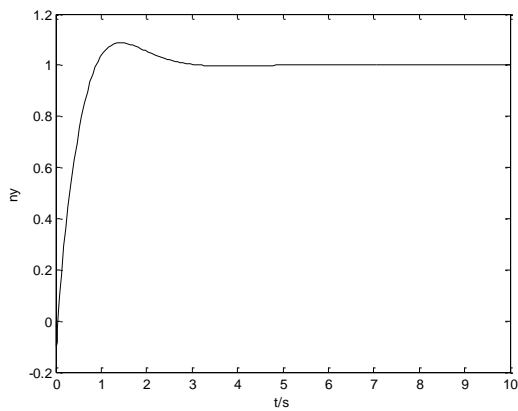
Assume that the value expected is one, simulation results are as follows:



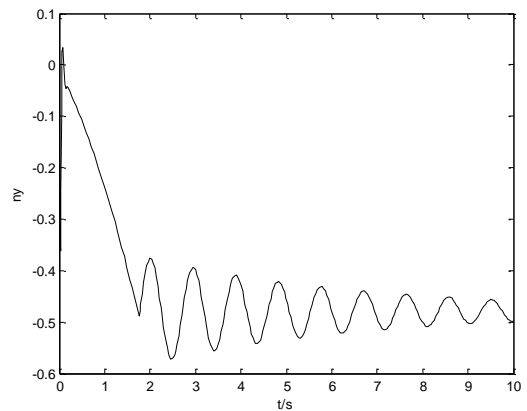
**Figure 13.**  $k_s \in (-5\%, 5\%)$  Curve of Simulation



**Figure 14.**  $k_s \in (-10\%, 10\%)$  Curve of Simulation



**Figure 15.**  $k_s \in (-50\%, 50\%)$  Curve of Simulation

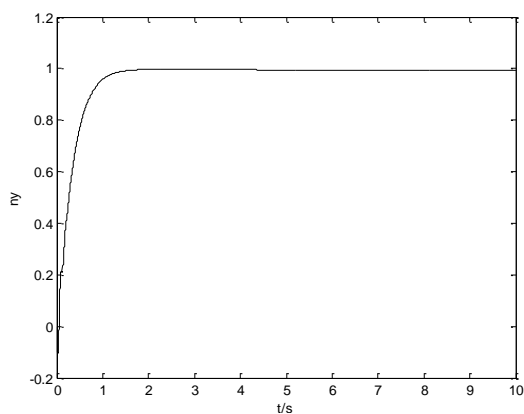


**Figure 16.**  $k_s \in (-90\%, 90\%)$  Curve of Simulation

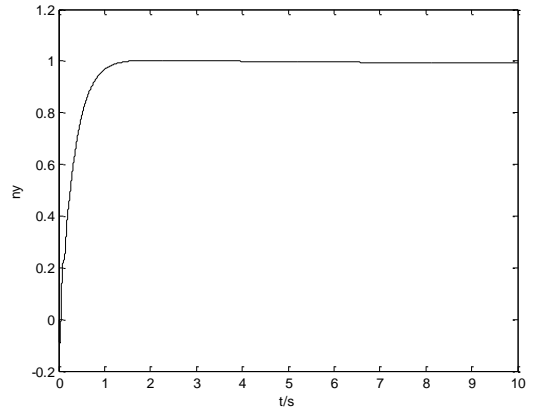
We can see from the simulation result that the output become unstable when the system meet larger perturbation, this show that the robustness of PID control method is not strong.

(2) overload sliding mode synchronizing control simulation

Use the above control parameters and assume that the value expected is one, simulation results are as follows:

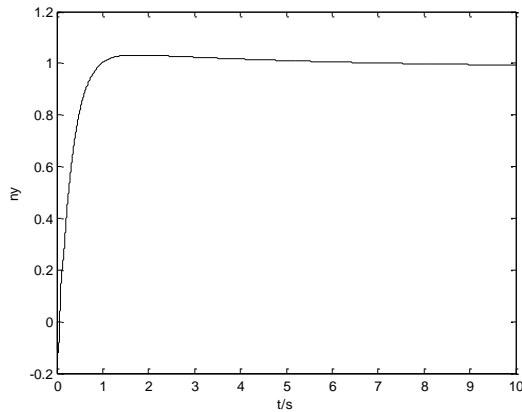


**Figure 17.**  $k_s \in (-5\%, 5\%)$  Curve of Simulation

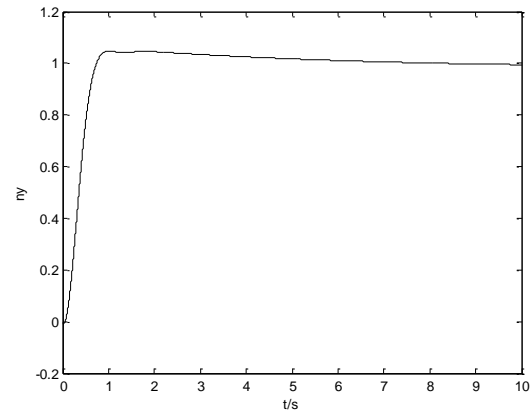


**Figure 18.**  $k_s \in (-10\%, 10\%)$  Curve of Simulation





**Figure 19.**  $k_s \in (-50\%, 50\%)$  Curve of Simulation



**Figure 20.**  $k_s \in (-90\%, 90\%)$  Curve of Simulation

All the simulation are stable, this shows that the system that is controlled by overload sliding mode synchronizing control method has robustness and can keep stable when parameters change.

By comparing the two control method, it shows that the system can be stable when the perturbation is small, but there are some difference between PID method and overload sliding mode synchronizing control method, especially when perturbation is large. So overload sliding mode synchronizing control method has better robustness than PID method.

## 5. Conclusion

This paper introduces overload sliding mode synchronizing control method and compares the control effect of PID control method and overload sliding mode synchronizing control method in situation of perturbation. The control effect of overload sliding mode synchronizing control method is better in the situation of the large variation of parameters. Because the modern anti-ship missile need be able to supersonic cruise, large aerospace maneuvering and long-range strike, the parameters of missile also have time-varying, nonlinear and uncertain characteristic. So based on the simulation results, the overload sliding mode synchronizing control method can be applied in military practice.

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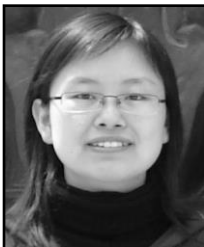
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