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Hybrid Strategy for Nonlinear Control of (6 DOF) Underactuated Trirotor UAV

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Abstract

This paper, proposed a hybrid strategy for nonlinear control of six degree of freedom (6 DOF) underactuated trirotor Unmanned Aerial Vehicle (UAV). The hybrid control strategy consists of Nonlinear Disturbance Observer (NDO) along with Proportional, Integral and Derivative (PID) controller to control the model of nonlinear (6 DOF) underactuated trirotor UAV. The control model of the UAV is divided in to two sub-models, altitude control and the attitude control such that, the PD controller is used to control the altitude of the UAV and (NDO)with the PID controller is used to control the attitude of UAV. However, the stability of the aircraft is proved by using Lyapunov stability criteria. The robustness of the proposed control strategy is compared with the nonlinear observer design with backstepping control of UAV. It shows that the proposed hybrid strategy have better response less steady state error and good robustness in the presence of continuous disturbance in the model of UAV.

Keywords: *Unmanned aerial vehicle, nonlinear disturbance observer, hybrid controller*

1. Introduction

Unmanned Aerial Vehicle (UAV) exists many important effects like, aerodynamic forces, aerodynamic moments, inertial torques and gravitational forces, *etc*., the inclusion of these effects in a body of UAV, it is difficult to design the controller for the underactuated, highly nonlinear and multivariable trirotor UAV [1].

A trirotor UAV has six degree of freedom (6 DOF) with three actuators the aerodynamic forces and its moments substitute in a body of UAV which is produce by the rotors [2]. In quadrotor UAV there are four rotors which is fixed in a frame of aircraft, in which two rotor rotates in the opposite direction of the remaining two rotors to stabilize the torque of UAV. But in trirotor UAV one rotor rotates in the opposite direction to stabilize the torques of UAV [3].

As compared to quadrotor, trirotor UAV have three rotors in which "R_1" is the rotor 1, "R_2" is the rotor 2 and "R_3" is rotor 3, to achieve the altitude and attitude tracking control, it has three different conditions. (1) *Altitude control:* The speed of all three rotors are same $(R_1 = R_2 = R_3)$;(2) *Roll control:* To control the roll of the UAV, it has two conditions first in anticlockwise in which the speed of rotors are $(R_3 > R_1 > R_2)$, the second one is clockwise in that case the speed of rotors are $(R_2 > R_1 > R_3)$;(3) *Pitch Control:* To control the pitch of the UAV, it has also two conditions for nose-up $(R_2 = R_3 > R_1)$ and for nose down $(R_1 > R_2 = R_3)$;(4) Yaw control: For yaw control the speed of all rotors are same $(R_1 = R_2 = R_3)$ along with tilt angle $(\alpha = u_4 > 0)$ [4]. The term "u_4" is the yaw control input which is defined in the control section.

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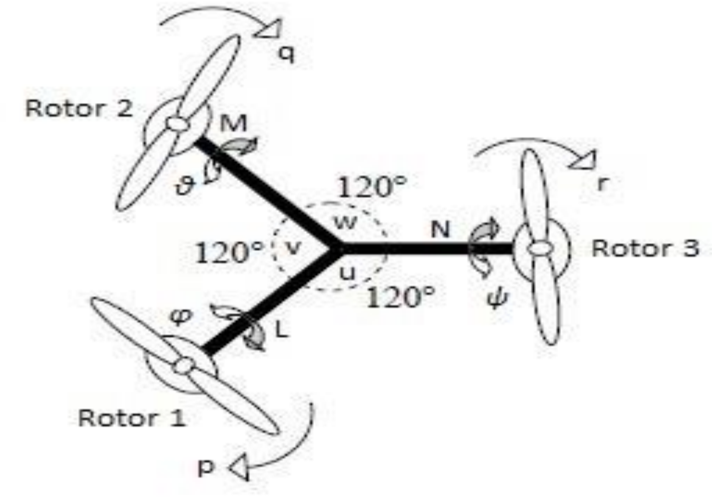

Figure 1. Aerodynamic Forces and Moments Acting on trirotor UAV

Figure 1, shows the aerodynamic forces and moments which is exerted in a body [1]. In which, Euler angles of UAV are (φ, θ, ψ), the linear and angular velocity can be written as (u, v, w) and (p, q, r). The fixed body frame of a trirotor is $B_F = \{x, y, z\}$ and for the Earth frame is $E_F = \{X, Y, Z\}$ respectively [5].

Previously, many control algorithms were already proposed to control the dynamics and stability of the UAV. Recently, the adaptive hybrid controller to stabilize the attitude and altitude of an underactuated (6 DOF) trirotor UAV was discussed in [6]. In this paper, a hybrid control strategy is proposed for controlling the motion of an underactuated trirotor UAV. In the proposed scheme, robustness against continuous disturbance is presented by a nonlinear disturbance observer along with the uncertainties in the UAV model. The integral action in the pole-placement technique is designed to linearize the system.

In this paper, the hybrid control strategy is based on nonlinear disturbance observer with the adaptive pole-placement as constraints of the PID controller. To control the attitude, the nonlinear observer disturbance with the PID controller is used and the desired position of UAV is controlled by using proportional and derivative controller. Moreover, the simulated results of our proposed hybrid controller is compared with the dual controller approach which consists of Nonlinear Disturbance Observer with the Backstepping controller called as (NDO-BKS) of [7].

The core contributions of this article are as follows: (1) hybrid strategy to control the multivariable, underactuated and highly nonlinear (6 DOF) UAV system; (2) nonlinear disturbance observer with adaptive tuning gains of the PID controller; (3) the designed control strategy uses linear and angular velocity components in the system, which shows to be real in practical point of view;(4) in spite of uncertainties in the system model, the close loop system error converges to zero, which is deal by hybrid controller and the stability of the system is ensured by Lyapunov stability criteria.

The breakup of this article is organized as follows. Section 2 defines the Preliminaries of UAV including proposed system model which is followed by the section 3 describing control strategy of trirotor UAV. In Section 4 simulation results and discussions are discussed. Lastly, the article is concluded in Section 5.

2. Preliminaries of UAV

The Euler angle (φ, θ, ψ) is used to derive the attitude of the aircraft around $\{x, y, z\}$ axis the fix body frame of the UAV respectively [8]. The rotational matrix can be written as,

$$\begin{cases} R_{\varphi}^{x} R_{\theta}^{y} R_{\psi}^{z} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & sin\varphi \\ 0 & -sin\varphi & cos\varphi \end{bmatrix} \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ R_{\varphi}^{x} R_{\theta}^{y} R_{\psi}^{z} = \begin{bmatrix} cos\theta cos\psi & cos\theta sin\psi & -sin\theta \\ sin\varphi sin\theta cos\psi - cos\psi sin\psi & sin\varphi sin\theta sin\psi - cos\varphi cos\psi & sin\varphi cos\theta \\ cos\varphi cos\theta cos\psi + sin\varphi sin\psi & cos\varphi sin\theta sin\psi - sin\theta cos\psi & cos\psi cos\theta \end{bmatrix} \end{cases} \quad (01)$$

The input and output forces and torque of the aircraft is written as,

$$\begin{cases} f_i = K_t\, u_i^2 \rightarrow kt\, u_i|u_i| \\ \tau_i = K_\tau\, u_i^2 \rightarrow k\tau\, u_i|u_i| \end{cases} \quad (02)$$

Where $i = 1, 2, 3$ for all three rotors of the aerial vehicle, f_i the thrust force from the actuator, τ_i the induced torque by the actuator, K_τ is the torque constant, K_t thrust constant. The external forces and mass which is exerted on the triangular frame of the UAV can be written as,

$$\begin{cases} F_{ext.} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} 0 \\ K_t\,(u_1|u_1|sinu_4) \\ -K_t\,(u_1|u_1|cosu_4 + u_2|u_2| + u_3|u_3|) \end{pmatrix} \\ M_{F_{ext.}} == \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} (\sqrt{3}/2)l * K_t(u_2|u_2| - u_3|u_3|) \\ 0.5 * l * K_t(u_2|u_2| + u_3|u_3|) - l * K_t(u_1|u_1|cosu_4) + K_\tau(u_1|u_1|sinu_4) \\ -l * K_t(u_1|u_1|sinu_4) - K_\tau(u_1|u_1|cosu_4) + u_2|u_2| + u_3|u_3| \end{pmatrix} \end{cases} \quad (03)$$

Translational dynamics: To neglect all the effects that is exerted on the body frame "B_F" of UAV. However, Earth frame "E_F" is used to define the translational velocity components for governing the aircraft which is written as,

$$\ddot{X} = (u_1/m)(\cos\varphi \cos\psi \sin\theta + \sin\psi \sin\varphi) \quad (04)$$
$$\ddot{Y} = (u_1/m)(\sin\theta \sin\psi \cos\varphi - \sin\varphi \cos\psi) \quad (05)$$
$$\ddot{Z} = -g + (u_1/m)(\cos\theta \cos\varphi) \quad (06)$$

Rotational dynamics: The rotational velocity components of the UAV depend on the fix body frame which is "B_F". In which the position of the aircraft is considered from the center of the mass of the body which is written in terms of the inertial frame that is (I_x, I_y, I_z). Now rotational dynamics is written as,

$$\ddot{\varphi} = qr(I_y - I_z/I_x) + (l/I_x)u_2 \quad (07)$$
$$\ddot{\theta} = pr(I_z - I_x/I_y) + (l/I_y)u_3 \quad (08)$$
$$\ddot{\psi} = pq(I_x - I_y/I_z) + (l/I_z)u_4 \quad (09)$$

3. Control Strategy

In this part of the article, the complete control strategy of our proposed hybrid controller is defined. Now the input vector "$U(t)$" can be written as,

$$U(t) = [u_1, u_2, u_3, u_4] \quad (10)$$

In which,

$$\begin{cases} u_1 = lK_t(\delta_1^2 - \delta_2^2) \\ u_2 = lK_t(\delta_1^2 - \delta_2^2) - lK_t\delta_3^2 \, cosu_4 \\ u_3 = lK_\tau(-\delta_1^2 - \delta_2^2) + lK_\tau\delta_3^2 + lK_\tau\delta_3^2 sinu_4 \\ u_4 = K_t(\delta_1^2 + \delta_2^2) + K_t\delta_3^2 \, cosu_4 \end{cases} \quad (11)$$

Where, the collective input force or vertical input is"u_1". The roll, pitch and yaw forces are denoted by"u_2, u_3, u_4", these forces are generated by the rotors of the aerial vehicle [9-11]. The reference or desired trajectory tracking scenarios to neglect the higher order dynamics of UAV. Let us consider,

$$\begin{cases} \ddot{X} = -g\varphi \\ \ddot{Y} = g\theta \\ \ddot{Z} = U'(t)/m \end{cases} \quad (12)$$

Now, by adding external disturbances in the model of UAV which can be described as,

$$\begin{cases} \ddot{x}_1 = \dot{x}_2 \\ \ddot{x}_2 = g(x) + h(x)U(t) + d'(t) \\ g(x) = g_0(x) \\ h(x) = h_0(x) \end{cases} \quad (13)$$

Where $x = [x_1, x_2]$ the state variables, x_1 shows the location and x_2 shows the linear velocity component of UAV. However, the control command vector is $U(t)$ which defines the current position of the UAV, $g(x)$ and $h(x)$ are model-able nonlinear functions, $g_0(x)$ and $h_0(x)$ are nominal models, $d'(t)$ is the external disturbance which is also bounded. The complete control strategy of hybrid control scheme which is shown in figure 2.

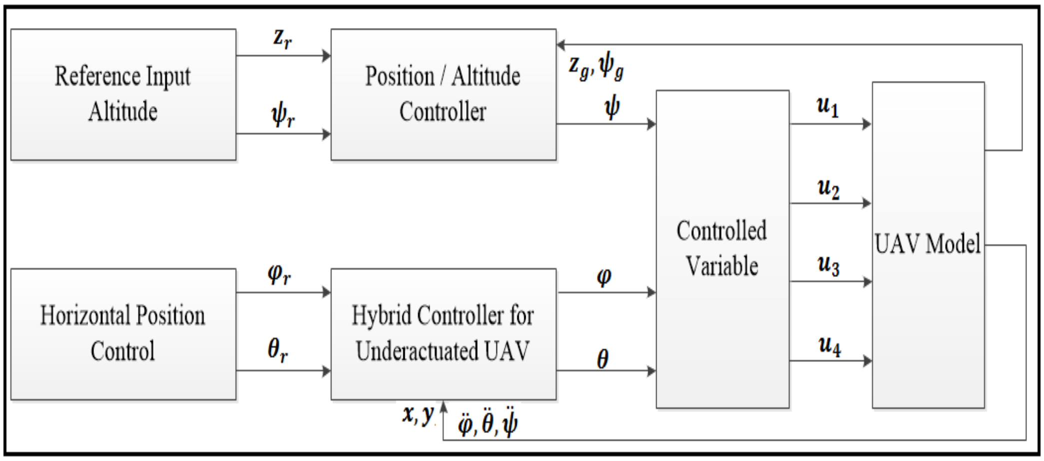

Figure 2. Control System Block Diagram of UAV.

A nonlinear observer is designed to control the attitude of UAV which is written as,

$$\begin{bmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} (I_y - I_z/I_x)qr \\ (I_z - I_x/I_x)pr \\ (I_x - I_y/I_z)pq \end{bmatrix} + \begin{bmatrix} l/I_x & 0 & 0 \\ 0 & l/I_z & 0 \\ 0 & 0 & l/I_z \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} \quad (14)$$

The attitude of UAV which can be written in the state space vector that is, $x = [x_1, x_2]$, and their states are,

$$\begin{cases} x_1 = [\dot{\varphi}, \dot{\theta}, \dot{\psi}] \\ x_2 = [\ddot{\varphi}, \ddot{\theta}, \ddot{\psi}] \end{cases} \quad (15)$$

The nonlinear functions $g_0(x)$ and $h_0(x)$ that can be written as,

$$g_0(x) = \left[(I_y - I_z/I_x)qr, (I_z - I_x/I_x)pr, (I_x - I_y/I_z)pq\right]^T \quad (16)$$

$$h_0(x) = \begin{bmatrix} l/I_x & 0 & 0 \\ 0 & l/I_z & 0 \\ 0 & 0 & l/I_z \end{bmatrix} \quad (17)$$

The angular movement vector for the input control of UAV can be written as,

$$U'(t) = [u_2, u_3, u_4] \quad (18)$$

Now the angular error signal in terms of adaptive control can be written as,

$$e_{att.} = x_a - x_r \quad (19)$$

In which x_a and x_r are the actual and reference states of the UAV. $x_a = [\varphi_a, \theta_a, \psi_a]$ and $x_r = [\varphi_r, \theta_r, \psi_r]$. The attitude error e_{att} along with $e_{att} = [\varphi_a - \varphi_r, \theta_a - \theta_r, \psi_a - \psi_r]$. The controlled input reference vector $CI_r = [CI_\varphi, CI_\theta, CI_\psi]$. Which stabilize the state x_1 that is based on PID algorithm.

$$\begin{cases} CI_r = G_P e_{att} + G_I \int_0^t e_{att}\, dt + G_D (e_{att} d/dt)x_2 \\ CI_z = k_R(z_g - z) + k_S \int_0^t (z_g - z)dt + k_T(z_g - z)/dt \end{cases} \quad (20)$$

Where G_P, G_I, G_D are the adaptive gains of the PID controller for tuning the attitude of UAV, CI_z the control input reference altitude of UAV. The external disturbances "α"can be written as,

$$\begin{cases} \alpha = d'(t) + U(t) \\ \alpha = \ddot{x}_1 - g_0(x)h_0(x)U(t) \end{cases} \quad (21)$$

The external disturbances are compensated by the signals of available velocity which is redefined by using the Euler angles [12].

$$\hat{\alpha} = V(x) + \overline{w} \quad (22)$$

In which $\overline{w} \in R^2$the variable vector "$V(x)$" nonlinear function for the designing of vector "x". The nonlinear observer error can be written as $\tilde{\alpha} = \alpha - \hat{\alpha}$.The derivative of error must be $\dot{\alpha} = 0$. Initially, there is no information regarding the derivative of disturbance.

The equation for the nonlinear observer and the error of the observer is written as,

$$\begin{cases} \tilde{\dot{\alpha}} = j\big(w + V(x) + g_0(x) - h_0(x)U(t)\big) - j\ddot{x}_1 \\ \tilde{\dot{\alpha}} + j\hat{\alpha} = 0 \end{cases} \quad (23)$$

Now the Lyapunov candidate function is used to proof the stability criteria of nonlinear observer that is,

$$V(\hat{\alpha}) = (0.5)\tilde{\alpha}^2 > 0 \quad (24)$$

and its derivative is,

$$\dot{V}(\hat{\alpha}) = \hat{\alpha}\dot{\hat{\alpha}} \quad (25)$$

If the condition $\hat{\alpha} > |\alpha|$ are satisfied, the term with an indefinite signal is dominated and then,

$$\dot{V}(\hat{\alpha}) \le -\tilde{\alpha}^2 < 0 \quad (26)$$

Which assurances that $\hat{\alpha} = 0$ is a globally asymptotically stable (GAS) at equilibrium point, which is used for the trim conditions of UAV, because the equation (26) is a positive definite function.

$$d'(t) = j^{-1}diag(A, A) \tag{27}$$

The nonlinear function $V(x)$is to be designed as,

$$V(x) = j^{-1}Ax \tag{28}$$

The nonlinear disturbance observer is $\hat{\alpha} = \bar{w} + V(x)$,

$$\dot{\bar{w}} = -j(\bar{w} + V(x) + g_0(x) + h_0(x)U(t)) \tag{29}$$

$$\hat{U}(t) = U(t) - \hat{\alpha}/h_0(x) \tag{30}$$

To check the robustness of UAV, the reference path is achieved by requiting the disturbance. Then, referred path relates to the external disturbance that will reduce the tracking stability of the controller. In which, altitude or position control is developed using PD controller to control the reference attitude of the UAV at the time of to follow the reference path. For the hovering case the reference altitude and yaw of the UAV is same as the given which can be written as,

$$\begin{cases} z_r = z_g \\ \psi_r = \psi_g \end{cases} \tag{31}$$

The linear and angular both velocities are used to control the state variable of the system which is,

$$\left[V_{(x,y)r}\right] = \begin{bmatrix} e_{xr} - e_{xa} \\ e_{yr} - e_{ya} \end{bmatrix} \begin{bmatrix} Cos\psi & -Sin\psi \\ Sin\psi & Cos\psi \end{bmatrix} \tag{32}$$

Where $[e_x e_y]$ is the error vector to control the position of the aircraft in an inertial frame.

$$\begin{cases} \theta_r = G_P(V_{yd} - V_y) + G_D dV_y/dt \\ \varphi_r = G_P(V_{xd} - V_x) + G_D dV_x/dt \end{cases} \tag{33}$$

G_P and G_D are the gains of proportional and derivative controller, $V_{xr}$and V_{yr} are the reference velocities in x and y direction respectively, φ_r, θ_r, z_r and ψ_r, the reference roll, pitch, altitude and yaw, z_g and ψ_g are the given altitude and given yaw. The disturbance which is added to the model of UAV is expressed as,

$$d'(t) = \begin{bmatrix} 0.15Sin(\pi t) + 0.15Sin(\pi t/10) \\ 0.15Sin(\pi t) + 0.15Sin(\pi t/10) \\ 0.15Sin(\pi t) + 0.15Sin(\pi t/10) \end{bmatrix} N \tag{34}$$

4. Simulation Results and Discussions

In this section of the article, the validity of our designed hybrid control strategy is compared by the Nonlinear Disturbance Observer with the Backstpping controller called as (NDO-BKS) of [7]. In which the parameters of the controller is described in the Table 1.

Table 1. Parameters of Trirotor UAV

Parameters	Values	S.I. Units
(Mass) "m"	*0.81*	*kg*
l	*0.2995*	*m*
g	*9.8*	m/s^2
K_t	*0.0156*	Nms^2/rad^2
K_τ	*0.007*	Nms^2/rad^2
I_x	*0.012*	$Kg.m^2$
I_Y	*0.012*	$Kg.m^2$
I_z	*0.027*	$Kg.m^2$

To simulate the nonlinear close loop control strategy in which the initial conditions of the attitude of UAV are $\varphi = -0.01$, $\theta = 0.4$ and $\psi = 0.01$rad after passing five seconds the attitude of the UAV converges to zero in figure 3 along with the reference altitude, it will initiate from $z_r = 0$m, and finally reached at the referenced altitude is about $z_r =$ 1m, which is shown in figure 4 respectively. To compare the results of our proposed hybrid controller with (NDO-BKS) it shows that the hybrid controller reaches at the reference value with very low oscillations as compared to the (NDO-BKS).

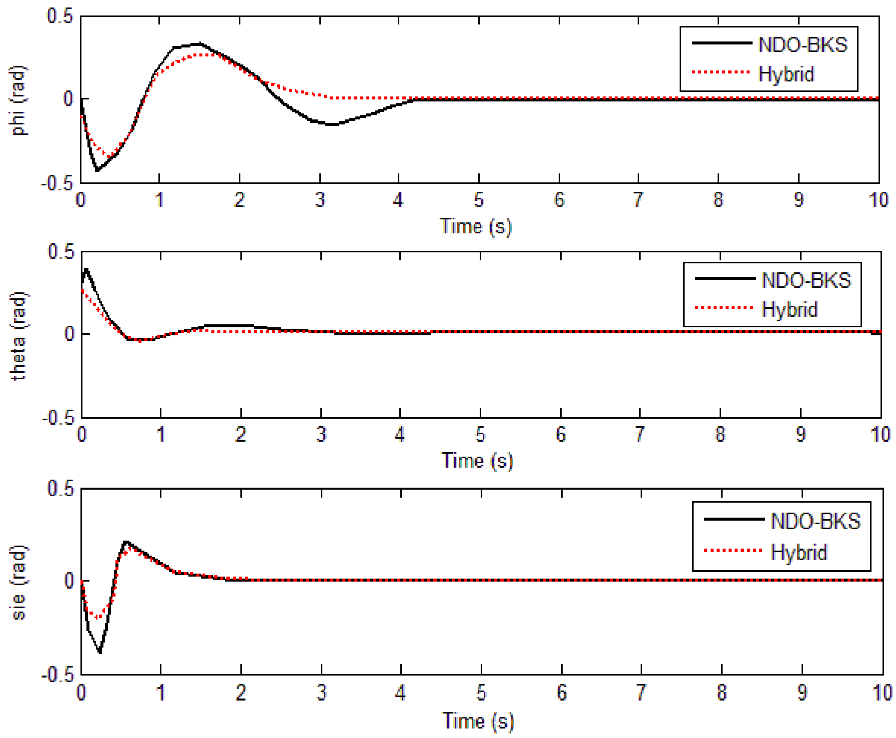

Figure 3. Attitude Angle Responses

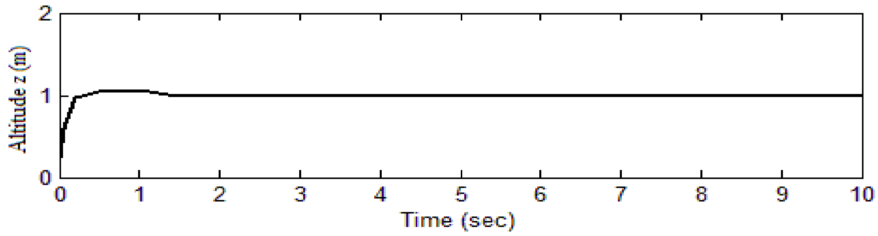

Figure 4. The Reference Altitude of Trirotor UAV

Figure 4, shows the referred altitude or height achieve by the UAV, such that the proposed controller is capable to stabilize the Euler angles and enables the trirotor aerial robot to hover at a referred point. Figure 5, shows the input control commands of the UAV. The two out of three rotors of the trirotor aerial robot, rotates in the same direction while the third one rotates in the opposite direction to make the UAV in equilibrium state. The produce enough lift that is produce by the rotors of the aircraft to stunned the load of the UAV and enable it to hover at refer point.

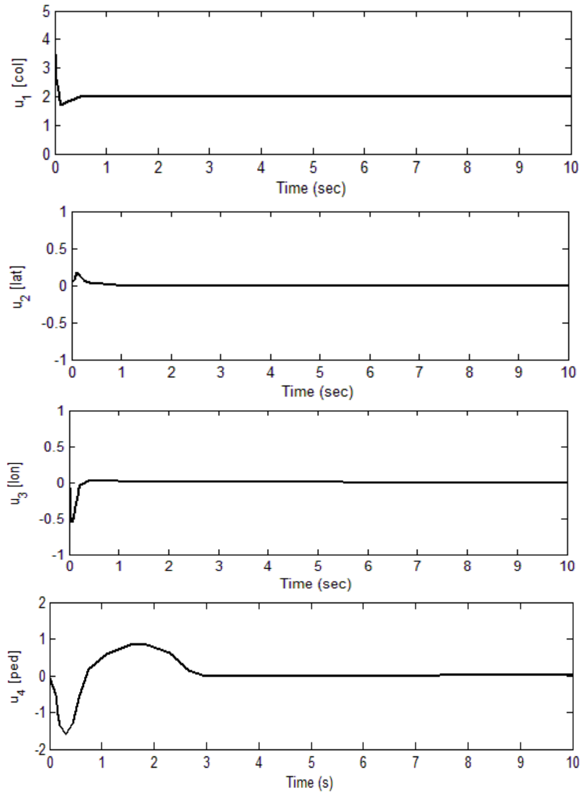

Figure 5. Input Control Commands of UAV

Figure 5, shows the input control commands of the UAV that is (u_1, u_2, u_3, u_4) the given input of the proposed controller that converges to the referred $(2, 0, 0, 0)$ at a very short interval of time. Moreover,$u_1 \neq 0$ that shows that the input is time invariant in a limited period of time. Lastly, the starting values from u_1 to u_4 have no high oscillations.

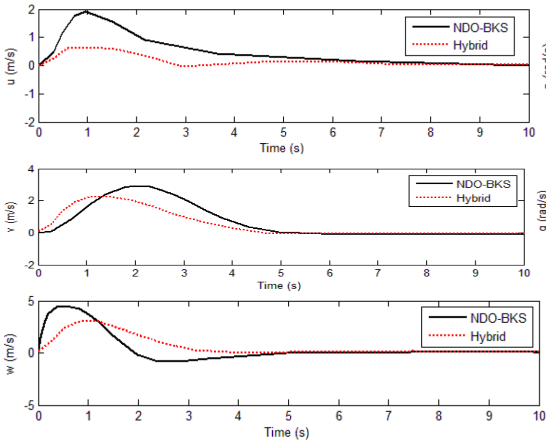

Figure 6. Linear Velocity Components

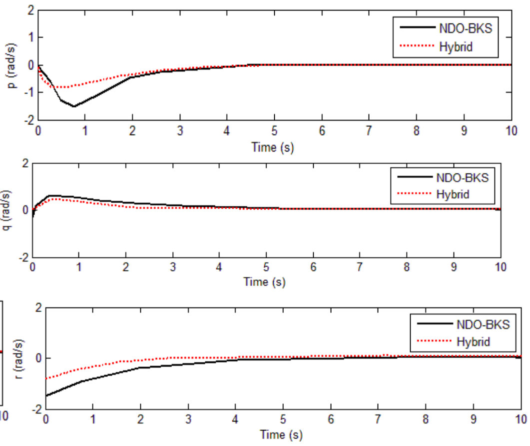

Figure 7. Angular Velocity Components

The linear and angular velocities of the aerial robot are shown in figure 6 and 7, and those responses show the similar response as the referred attitude and positions. Both velocities will converge to zero at about four seconds and behave linearly.

5. Conclusion

The stabilization of nonlinear (6 DOF) underactuated trirotor UAV is presented. The hybrid control strategy is applied on the model of UAV to control the attitude of the UAV, where the tracking error is deal by the adaptive PID controller. However, the position or altitude is control by the PD controller. The stability of the nonlinear system proof by Lyapunov stability criteria and simulation results shows the robustness of the proposed control strategy.

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