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Optimal Transport in Weighted Networks by Deleting Edges

Shoubao Su[1], Mingjuan Xu[2*], Yu Su[3*] and Fei Shao[1]

[1] School of Computer Engineering, Jinling Institute of Technology, Nanjing 211169, China

[2]School of Electronics & Information Engineering, West Anhui University, Lu'an 237012, China

[3]Xi'an Branch, Softstone Technology Ltd. Co., Xi'an 100193, China

**Corresponding author: xmj8217@wxc.edu.cn, su_yu01@163.com*

Abstract

Improve the transmission capacity of the weighted network is one of the most important issues. We propose new route strategies by different edge redefinition while the integrity of the weighted network is remaining maintained. The specified edges deleting can greatly improve the transmission capability of the BBV weighted network. In addition, in the simulations of computer generation networks and real networks, it shows that the strategy of routing policy based on the edges of the two nodes of the product is more effective especially in those large scale weighted networks. And if most edges are deleted, the strategy does not work better.

***Keywords**: Weighted network, Route strategy, Transmission capability*

1. Introduction

Since the growing importance of communications networks, such as the World Wide Web, research on the network transmission capability is becoming more and more important over the past few years. These real-world network nodes can be representatives of the interaction between the individual and their edges are properly described as a complex network. The shortest path [1] route strategy is widely applied in real communication networks because it is the most efficient. These networks are found to have the small-world character [2] and the scale free character[3] when we study their topology structure. How to make the network transmission capability maximal is very important. Now the linking strength among edges is found to have great influence on the network dynamic process [4-7]. These are called weighted networks. In some weighted networks, weights need to be reversed in shortest path algorithm because they are statistical measures of tie strength. While in some other circumstances where weights are the geographic distance, it could be applied in the shortest path algorithm directly.

Recently, lots of optimal route strategies are proposed to restrain traffic congestion and improve the network transmission capability. Some of them are based on the global information: the shortest path route strategy[1] where the packets are forwarded through the minimum number of nodes and the effective route strategy[8] which can enhance the network transmission capability more than 10 times by redistributing traffic load in central nodes to other nodes. Some others focus on local information because sometimes global information is unavailable in some large-scale networks: neighbour information[9], next-nearest-neighbour information[10] and so on. Adjusting the network topology structure is also proved to be more effective in some networks [5, 11-13]. Deleting some edges is effective

[*] Corresponding author: Ms. M.J. Xu (xmj8217@wxc.edu.cn) and Mr. Y. Su (su_yu01@163.com).

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to improve the network transmission capability of the unweight scale-free networks[12].

Based on this, in this paper, we proposed a novel route strategy for BBV weighted network. By defining the weight of edges according to the product of node degree, the product of node strength and the original weight of edges, we delete some edges according to the different definitions of edge weight. And the impacts of the total weight increase, the newly added edge number, the node number on our route strategies will be discussed.

This paper is organized as follows. In section 2 we describe the BBV weighted network model, the traffic model and our route strategies, followed by the simulations on computer generated networks (with different parameters) and real world network in section 3. The conclusions are given in section 4.

2. Models

In those previous studies on the weighted real world networks[14-16], the BBV weighted network model[15] is most widely used. A weighted adjacency matrix $\boldsymbol{W}$, whose elements w_{ij} denote the weight of the edge between node i and j, is often used to describe BBV weighted network. The generating of BBV weighted network is as follows:[17, 18]

1) Starting from N_0 nodes fully connected by edges with original weight w_0, a new node is added to the network at each time step. The newly added node will be connected to m different previously existing nodes with the same weight w_0 for every edge, choosing preferentially nodes according to the probability $\prod_{n\to i} = s_i \Big/ \sum_l s_l$, where s_i is the strength of node i and $s_i = \sum_j w_{ij}$.

2) The edge connecting to node i will introduce variations of the weight of the other edges linked to node i. We set it to be proportional to the edge weights. If the total increase is δ (δ_i=δ for simplicity), we can obtain

$$w_{ij} = w_{ij} + \Delta w_{ij} = w_{ij} + \delta * \frac{w_{ij}}{s_i} \tag{1}$$

The strength of node i is:

$$s_i = s_i + \delta + w_0 \tag{2}$$

The degree distribution of BBV weighted network $P(k) \propto k^{-\gamma_k}$ and the strength distribution $P(s) \propto s^{-\gamma_s}$ where

$$\gamma_k = \gamma_s = \frac{4\delta + 3}{2\delta + 1} \tag{3}$$

The traffic model can be described as follows:

1) All the nodes can generate packets with addresses of destination, receive packets and route the packets to their destinations according to the routing strategy.

2) At each time step t, R packets are generated in the whole network with randomly chosen sources and destinations, and placed at the end of the queue.

3) At the same time, the first C_i packets at the top of the queue of each node i are forwarded one step toward their destinations. In this paper, each node is assumed to have same packet delivery capability. Here we set C_i=1 for simplicity.

4) The delivered packets will be removed from the network as soon as they reach their destinations.

As R is increased from zero, we can obtain two phases: free flow for small R and congested phase for large R, with a phase transition at the critical value R_c. This critical value R_c can reflect the maximum network transmission capability. For $R<R_c$, the numbers of created and delivered packets are balanced, resulting in free

flow stage. For $R>R_c$, because the node delivering packet capacity is limited, traffic congestion will occur.

The betweenness b_i is often introduced which is defined as

$$b_i = \sum_{s,t} \frac{\sigma(s,i,t)}{\sigma(s,t)} \tag{4}$$

where $\sigma(s,i,t)$ is the number of paths under the given route strategy between nodes s and t that pass through node i and $\sigma(s,t)$ is the total number of paths under the given route strategy between nodes s and t and the sum is over all pairs s, t of all distinct nodes.

A created packet will pass through the node i with the probability $b_i / \sum_{j=1}^{n} b_j$. The node i will receive $R*b_i/(n*(n-1)))$ packets at each time step. Congestion occurs when the number of incoming packets is greater than the delivery packets, that is $R*b_i/(n*(n-1)) \geq C_i = 1$. So the critical packet generation rate R_c, which can best reflect the network transmission capacity, is

$$R_c = \min(n*(n-1)/b_i) \tag{5}$$

Our route strategies are described as follows:

1) We defined the weight of the edge linking two nodes i and j, W_{ij}, in different way according to different route strategies as shown in Table 1. In WWD routing strategy, we delete the edges whose definition weight W_{ij} is equal to the origin weight of the edge w_{ij} is highest; In WDP route strategy, we delete the edges whose definition weight W_{ij} is equal to the product of the node degree k_i is highest; In WSP route strategy, we delete the edges whose definition weight W_{ij} is equal to the product of the node strength s_i is highest.

2) We sort the edges by their definition weights in decreasing order and delete the edge ranked first. However, we should maintain the integrity of the network which means that, if deleting an edge will cause some nodes to be disconnected, we will not delete it, and go to deal with the edge ranked next.

3) Recalculate the definition weight W_{ij} and repeat step 2 until a fraction f_e of edges are deleted.

4) The shortest-path route strategy[1] is applied in the new network.

Table 1. The Definition Weight W_{ij} In Different Strategies

	WWD	WDP	WSP
W_{ij}	w_{ij}	k_i*k_j	s_i*s_j

In BBV weighted network, there are about $n*m$ edges. To maintain the integrity of the network, n-1 edges are necessary. So we can delete about $n*m-n+1$ edges at most. After deleting all the $n*m-n+1$ edges, the network become a treelike topology and a smaller value of packet generation rate can cause traffic congestion. So there must be a critical point where a phase transition occurs, if more edges are deleted, the effect of the route strategy is less obvious or even worse than in the original network.

We utilize the RAN route strategy, where edges are deleted randomly, to check the validity of our route strategies.

3. Simulations and Analysis

We obtain the critical packet generation rate R_c using different route strategies in BBV weighted network with n=100, δ=4, m=4 and ω_0=1. The results are shown in Figure 1.

(For every network, 10 instances are generated and for each instance, we run 10 simulations. The results are the average over all the simulations. f_e = 0 means the original network and f_e = 1 means the network with n-1 edges.)

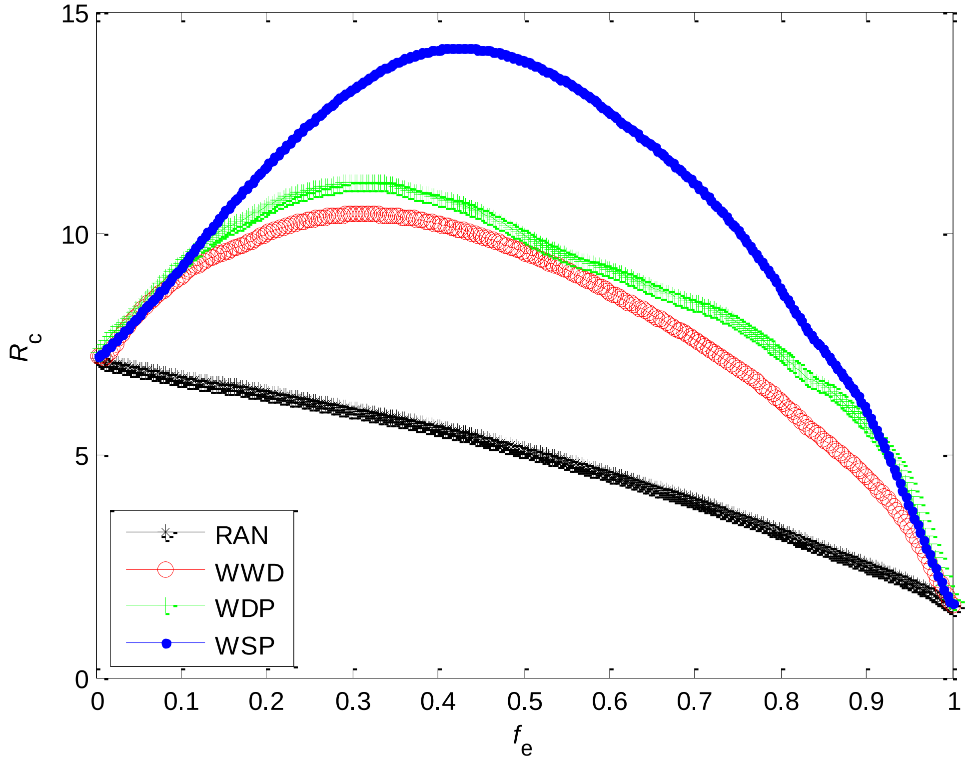

Figure 1. R_c VS f_e. BBV network with n=100, δ=4, m=4, ω_0=1, C_i=1

Figure 1 exhibits the relationship of the critical packet generation rate and the fraction of deleted edges. In four route strategies, the critical packet generation rate R_c varies when the edges are deleted. The WSP route strategy, which delete edges with highest product of the node strengths, achieve the maximum transmission capability when about 50 percent of the deletable edges are deleted. The maximum transmission capability of the WSP route strategy (when f_e is about 0.5) is 95.6% higher than in the original network (when f_e is zero) while the WDP route strategy and the WWD route strategy is 52.5% and 44.1% correspondingly. And the maximum transmission capability of the WSP route strategy is better than the other two strategies (28.26 % higher than the WDP route strategy and 35.73% higher than the WWD route strategy). In the RAN route strategy, deleting edges will result the decreasing of the maximum transmission capability.

To discover the effect of the total weight increase on critical packet generation rate, we set δ=8 and get the corresponding simulation results in Figure 2.

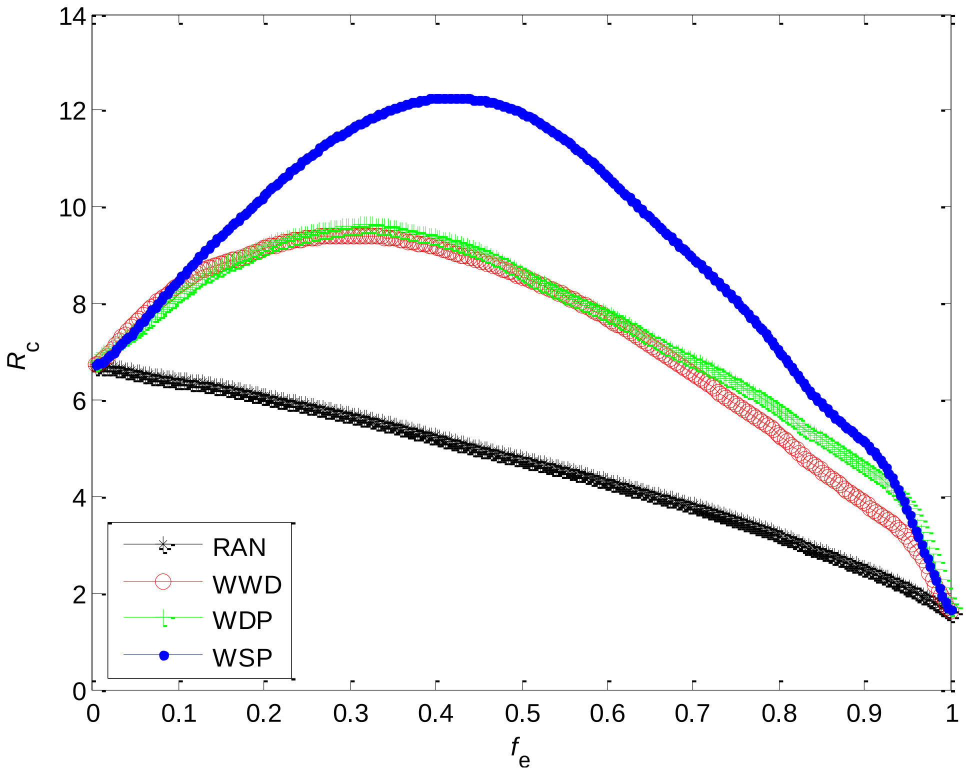

Figure 2. R_c VS f_e. BBV Network with n=100, δ=8, m=4, ω_0=1, C_i=1

Figure 2 shows almost the same result as Figure 1. The improvement of three route strategies is 82.4%, 42.6% and 40.1% correspondingly, which means the total weight increase δ almost does not affect the effective of our route strategies. And deleting edges randomly in the RAN route strategy will not enhance the network transmission capability.

Then we check the influence of the newly added edge number m.

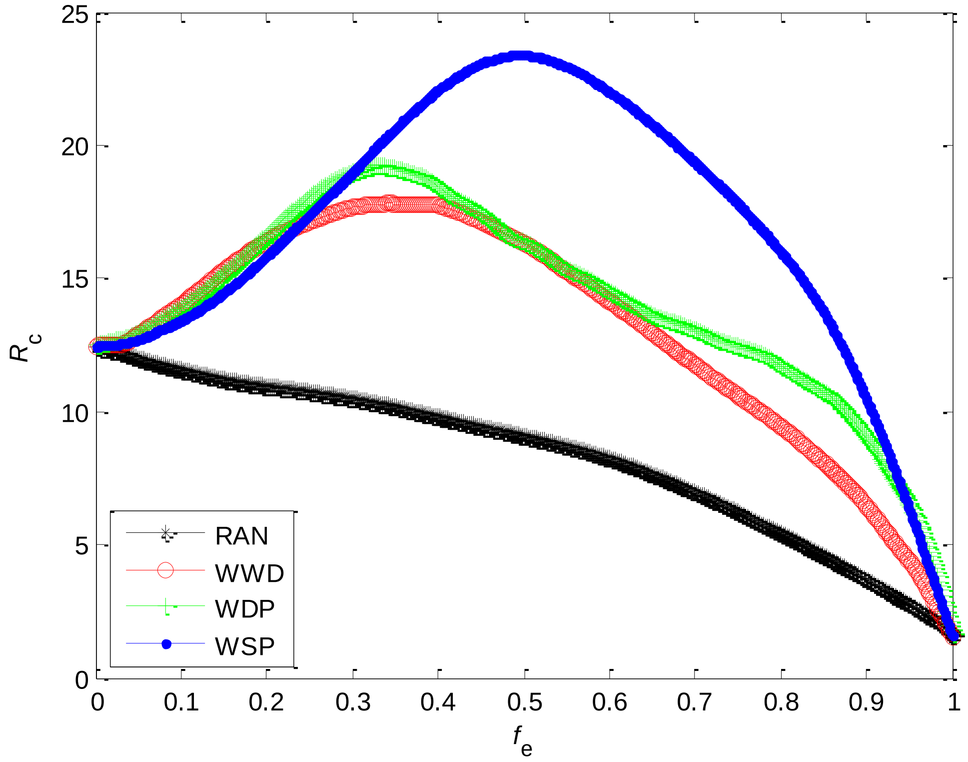

Figure 3. R_c VS f_e. BBV Network with n=100, δ=4, m=8, ω_0=1, C_i=1

Figure 3 shows the same result as displayed in Figure 1 and figure 2. The enhancement of our route strategies is 88.4%, 54.5% and 43.2%. The number of the newly added edge does not affect our route strategies, either.

Then we double the node number n to gain the results in Figure 4.

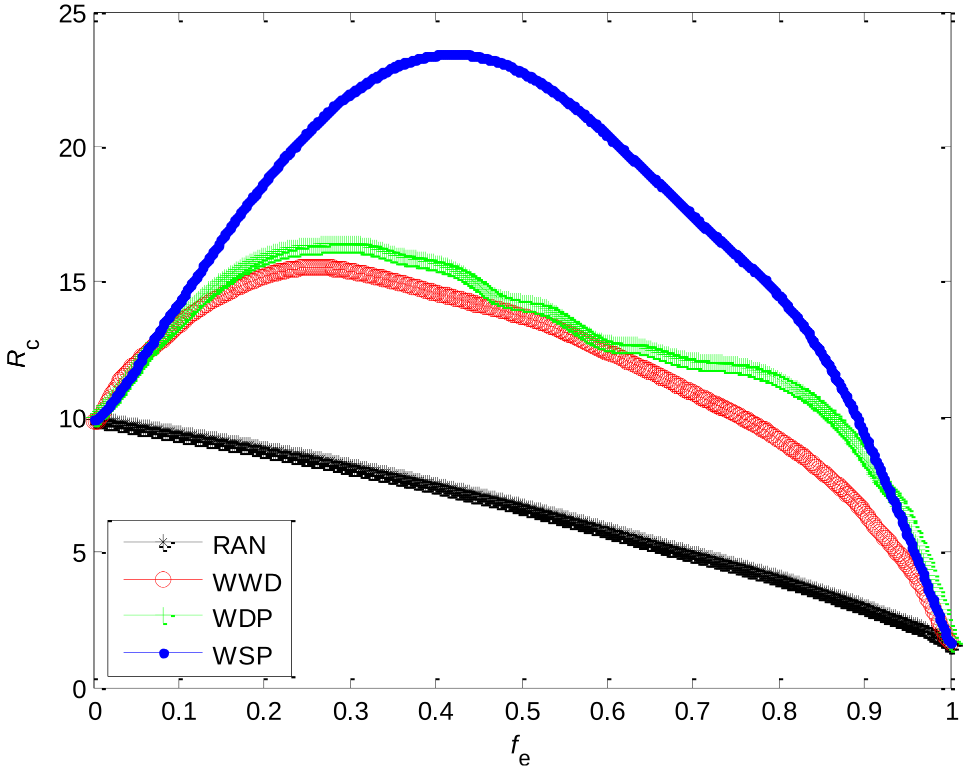

Figure 4. R_c VS f_e. BBV network with n=200, δ=4, m=4, ω_0=1, C_i=1

Comparing Figure 4 with Figure 1, we can find that the critical packet generation rate R_c of the WSP route strategy also achieves the highest position where f_e is about 0.5. The enhancement of our route strategies is 136.9%, 64.9% and 57.6%. Our route strategies are proved to be more effective in large scale networks.

We set the total weight increase δ to 8 and the new added edge number m to 8 with the node number n=200 to get the results shown in Figure 5 and Figure 6. These two figures also display the same feature that the WSP route strategy which deleting edges with highest product of the node strengths achieve the maximum network transmission capability and the more nodes there are in the weighted network, the more effective our route strategies are.

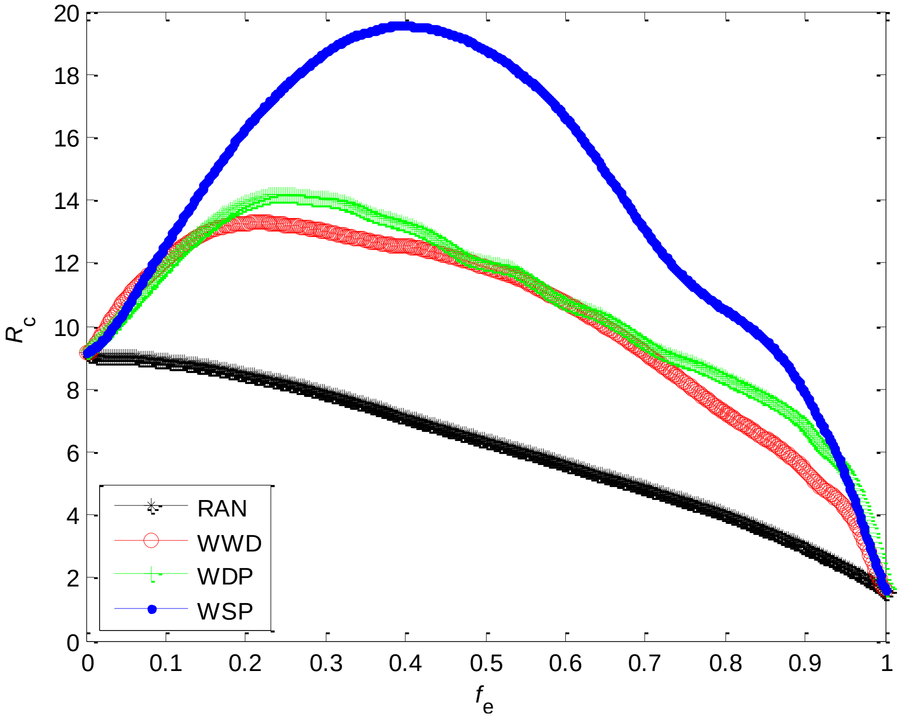

Figure 5. R_c VS f_e. BBV Network with n=200, δ=8, m=4, ω_0=1, C_i=1

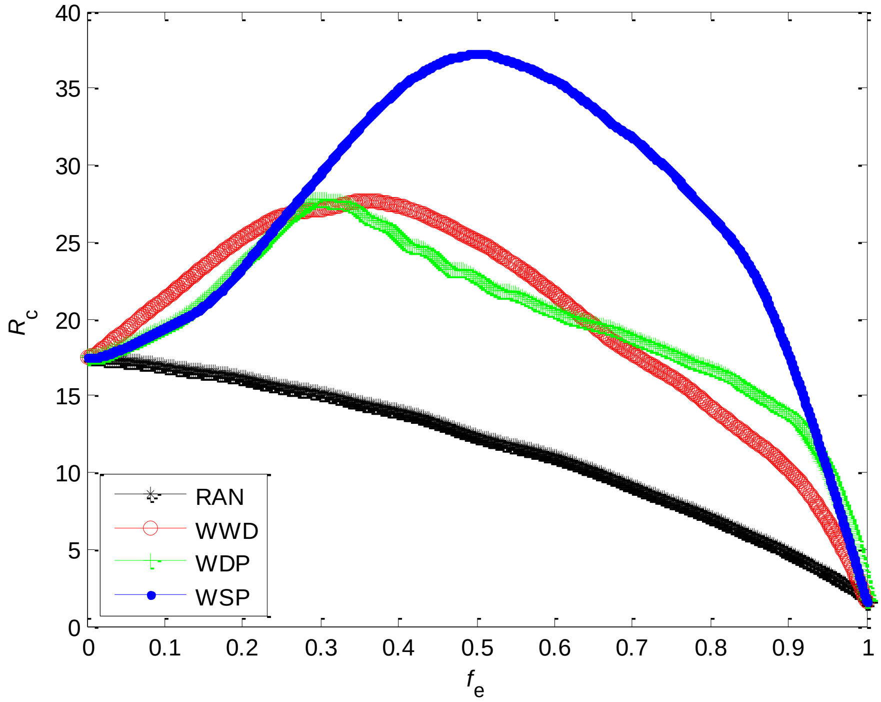

Figure 6. R_c VS f_e. BBV Network with n=200, δ=4, m=8, ω_0=1, C_i=1.

Finally, we choose the USAir 97 network (http://vlado.fmf.uni-lj.si/pub/networks/data/) with 332 nodes and 2126 edges to check the validity of test our route strategies on real world network. Simulation results are shown in table 2.

Table 2. The Critical Packet Generation Rate R_c of the Usair 97 Network

	RAN	WWD	WDP	WSP
f_e=0.1	3.42	3.66	3.74	4.12
f_e=0.2	3.34	3.84	4.02	4.68

Table 2 also certifies that our route strategies also work well in weighted real-world network.

The relationship between the average weighted average length[19] L_{AVE} versus the node number n is shown in Figure 7.

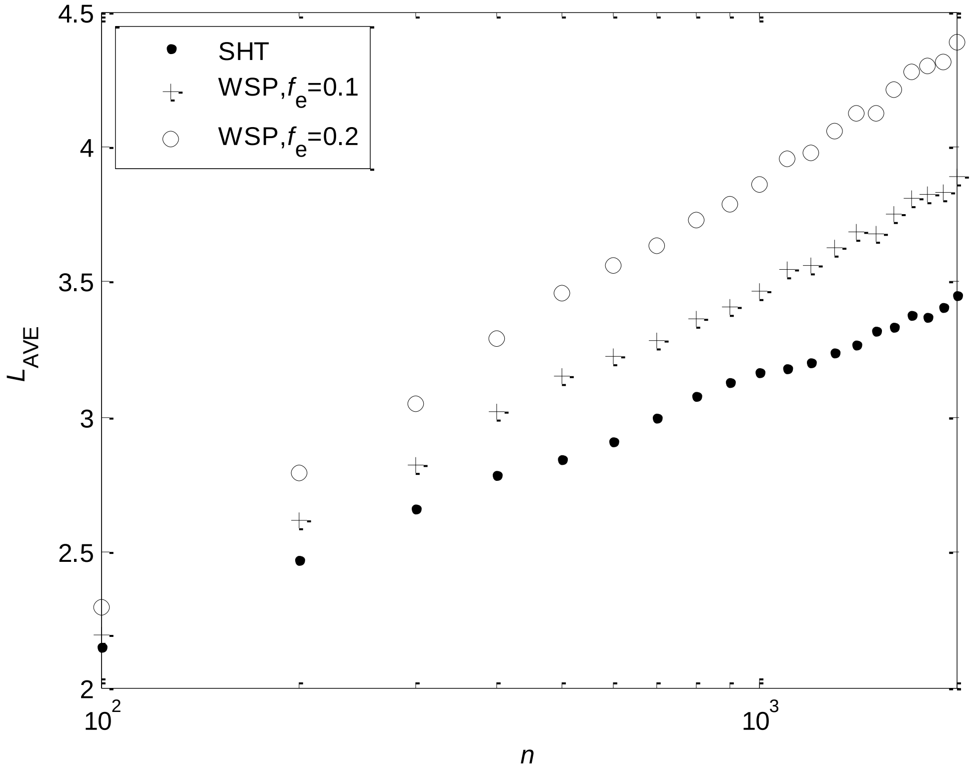

Figure 7. L_{AVE} VS n. BBV Network with δ=4, m=4 and ω_0=1

Although after deleting some edges, the weighted average length of WSP route strategy are higher than that of the traditional shortest path route strategy, the small-world character, *i.e.* $L_{AVE} \propto \ln n$, is still maintained. The network transmission capability of weighted network is enhanced at the cost of increasing the average weighted average length slightly. It means our route strategies are also of great practical importance in designing real world networks.

4. Conclusions

This paper has proposed three route strategies to enhance the transmission capacity of the BBV weighted networks. Simulations indicated that the critical packet generation rate is enhanced greatly after deleting some edges. Both the total weight increase and the newly added edge number will not affect the efficiency of our route strategies. The WSP route strategy which deletes the edges according to the product of the node strengths is proved to be the most effective, especially in the large scale network. Meanwhile, the small-world character of the average weighted average length is still maintained. The validity of our route strategies on the real world networks is checked. And the whole network integrity is still maintained.

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