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A Preconditioning Based Iterative Learning Control for Systems with Unknown Initialization

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Abstract

The repeatability of system is a fundamental requirement for various iterative learning control methods, and is a necessary condition for the outcome of perfect tracking. This paper theoretically and numerically explains that how the history before the initial time of dynamic systems influences the current state and repeatability of the system. To this end, the convergence analysis of PD-type iterative learning control for initialized system is presented. A practical preconditioning strategy is added to accelerate the convergence speed, and the detailed discussions of initialization function and initialization response are shown as well. The minimum preconditioning time interval is achieved, and some unique properties of initialized system are illustrated to provide novel challenges for robust and adaptive controls. A number of numerical simulations exhibit that a simple preconditioning process can efficiently improve the performance of the initialized iterative learning control.

Keywords*: Iterative learning control, Initialized system, Repeatability, Preconditioning, Convergence speed*

1. Introduction

Just as humans need experience to master a skill, machines or robots learn autonomously (without the help of human beings) from measurement data of previous operations and make their performance of future operation better [1]. To our best knowledge, the most popular learning control strategy for both modeled and un-modeled systems is iterative learning control (ILC) [2] [3] [4], whose basic scheme is shown in Figure 1.

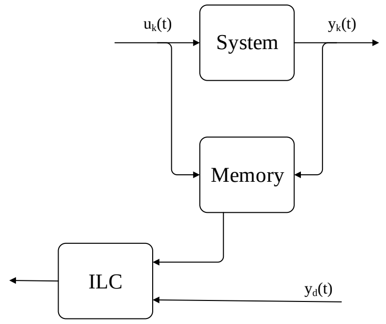

Figure 1. The Basic Scheme of ILC

The idea of ILC was first published in 1978 by Uchiyama [5]. In 1984, the widely concerned ILC paper is published in English by Arimoto [1]. One of the advantages from

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ILC schemes is the perfect tracking if certain requirements are satisfied. However, because of the complexity of the system, dropouts, time-delay [6] [7], uncertainties [8], or disturbances, the perfect tracking becomes non-achievable due to the lack of proper learning laws. Nevertheless, various novel strategies have been proposed to improve the performance of ILC methods, for example, the method of identification, the process of preconditioning and so on can be applied.

In the past three decades, ILC contributes a lot to control field [9] -[17]. This method is widely applied to robots [18]-[20], scandisk [21], networked control systems, and so on [22] [24]. The ILC scheme combined with other control strategies is also widespread, for example, the robust control can be combined with ILC to improve the robustness of the control system and the structural and parameter identification results can be applied to optimize learning law so as the convergence speed. The iterative learning law is mainly divided into the linear ones and nonlinear ones [25]. The linear learning laws are usually divided into P-type [26], PD-type [27], filtering-type [28] and so on [29]. The nonlinear learning laws are mainly Newton-type [30], Secant iterative learning law [31] and parametric learning laws [32][33]. Global Lipschitz condition provides convenience for the analysis of nonlinear systems with ILC schemes, however, more and more scholars expand the ILC to the locally Lipschitz systems by combining other methods such as Lyapunov method [34]. In this paper, to highlight the role of history of system, the controlled systems are linear ones so that the global Lipschitz condition is satisfied for sure. The 'history' discussed in this paper means that the information at previous iteration, or the period $[-L, 0]$ before one iteration begins.

The repea.gility is one of the conditions which must be satisfied to achieve the perfect tracking. In other words, the system in ILC schemes has to be assumed as unchanged in each iteration so that the perfect tracking can be achieved for proper learning laws. This assumption of strict repeatability is unpractical that the previous control inputs may change the system model very slowly, although some adaptive and robust control methods are dedicated to compensate for such uncertainties. For the batch processes, the given system may be operated for thousands of times. The systems in different iterations will be extremely distinguished for heredity properties. For example, the system model of a plane must be changed by thousands of flights due to metal fatigue, wire aging, deformation, *etc.*, so that the model in the 1000th flight is different with it in the first time. If the system changes brought by previous control process is considered, the repeatability of it cannot be guaranteed. In other words, unlike many of robust and adaptive control methods, the history of system inputs and variables may destroy its repeatability, and plays a crucial role in ILC, which cannot be fixed by traditional ways. The preconditioning process can improve the system performance in the case of model changed by the previous control process. The history of system is discussed by initialized system. The initialization response composed of initial condition (initial resetting condition) [34] [35] and initialization function provides help for the analysis of the initialized system. Therefore, in this paper, the ILC of initialized system is discussed, where a practical preconditioning procedure is added to improve the repeatability of control systems.

The main idea of this paper is summarized as:

- How does the initialization function affect the system repeatability.
- How to improve the system repeatability efficiently by preconditioning.
- How to design the initialized iterative learning control scheme.

The following of this paper is organized as: The preliminaries of Laplace transform and initialized system are shown in Section 2. In section 3, the solution of the initialized MIMO system is analyzed. The convergence conditions of the ILC scheme and the minimum preconditioning time interval are gained in Section 4. The illustrated examples are shown in Section 5 to validate the above concepts, and the conclusions are summarized in Section 6.

2. Preliminaries

Some preliminaries are introduced to extend the readability of this paper.

2.1. Laplace Transform

Some preliminaries are introduced to extend the readability. Firstly, the unit step function to be used in this paper is defined as

$$h(t) = \begin{cases} 0, t \leq 0, \\ 1, t > 0. \end{cases}$$

Moreover, the definition of λ-norm is introduced.

Definition 1 (λ-norm): The $x(t) \in R^n$ is a function on time interval $[0, T]$, a real number $\lambda > 0$, the λ-norm is shown as

$$\| x(t) \|_\lambda = max_{0 \leq t \leq T} \{ e^{\lambda t} \| x(t) \|_\infty \}.$$

It follows that

$$\| Ax \|_\lambda \leq \| A \|_\infty \| x \|_\lambda.$$

Definition 2 (unilateral Laplace transform): The Laplace transform of a function $f(t)$, defined for all real numbers $t \geq 0$, is the function $F(s)$, defined by

$$F(s) = \boldsymbol{L}\{f(t)\} = \int_{0^+}^{\infty} e^{-st} f(t) h(t) dt. \quad (1)$$

The parameter s is a complex number.

The Laplace transform is described as $\boldsymbol{L}$. The formulation of Laplace transform follows is utilized as

$$\boldsymbol{L}\{e^{-at}\} = \frac{1}{s+a}.$$

To the theoretical side, the Laplace transform can be divided into unilateral Laplace transform and bilateral Laplace transform. The unilateral Laplace transform is focused on time interval $[0, \infty]$, where all previous values before origin are assumed as zero. This is actually very popular in frequency analysis and many control strategies, but is risky in iterative learning processes. To analyze that how the history control processes influence the current system, the bilateral Laplace transform should be applied. Compared to unilateral Laplace transform, the bilateral Laplace transform is focused on time interval $[-\infty, \infty]$, the research on the extra time interval $[-\infty, 0]$ provides theoretical support to the analysis of the initialized system. The bilateral Laplace transform of a function $f(t)$ can be described as

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t) dt.$$

2.2. Initialized System and Preconditioning

To improve the readability of this paper, the initialized system, initialization function, initialization response and preconditioning are introduced in this subsection. The nominal system is

$$x(t) = \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau,$$
$$y(t) = Cx(t).$$

Definition 3 (initialized system): The initialized system denotes that the history before the initial time effects the current state of system [36]. The initialized system can be described as

$$x(t) = \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau + e^{At} x(0^+) - e^{At} x(0^+) + e^{A(t+L)} h(0^+) x(-L)$$
$$-(A+I)^{-1} x(0^+) h(t) + (A+I)^{-1} x(-L) h(t+L),$$
$$y(t) = Cx(t).$$

Definition 4 (initialization function): The value of system variables before the initial time is named as the initialization function [36], which is shown as

$-e^{At}x(0^{+}) + e^{A(t+L)}x(-L) - (A+I)^{-1}x(0^{+}) + (A+I)^{-1}x(-L).$

Definition 5 (initialization response): All terms affected by initialization function are called the initialization responses [37]. The initialization response includes the initial condition and the initialization function which is

$e^{At}x(0^{+}) - e^{At}x(0^{+}) + e^{A(t+L)}h(0^{+})x(-L) - (A+I)^{-1}x(0^{+})h(t) + (A+I)^{-1}x(-L)h(t+L).$

It is obvious that the initialization function is shown on $[-\infty, 0^{-}]$, the initial condition is defined at 0^{+}, and the initialization response is expressed on $[0^{+}, \mathrm{t}]$. Besides, the values of initialization function at 0^{-} and the initial condition at 0^{+} can be different, theoretically. Moreover, the zero initialization function corresponds to zero initialization response [38].

Definition 6 (preconditioning): For different iterations, the same control input is applied to the system before the learning procedures, the procedure of decreasing the effect of the model changes by previous control histories is called the preconditioning.

After preconditioning, the initialization functions and initialization responses can be as close as possible if the preconditioning time interval L is large enough. A simple preconditioning procedure can efficiently reduce the influences of initialization responses to ILC, and improve the repeatability of system to a great extent.

Lastly, the backgrounds of initialized system and preconditioning process can be found in physics, electrical engineering (electronic elements), material science and many other dynamic systems [36].

3. The Initialized MIMO System

In this section, the initialized multiple-input multiple-output (MIMO) system is analyzed. The solutions of initialized MIMO system in both time and frequency domains are achieved while the initialization response is also discussed.

Applying the Laplace transform to the system below

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t). \qquad (2)$$

In different time intervals of $[0^{+}, \infty]$, $[-L, \infty]$ and $[-\infty, \infty]$, the different solutions can be derived. The previous equation in subsection 2.2 is the different solutions of the above dynamics (2) in different time intervals. Then, applying the inverse Laplace transform, we can arrive at the solutions in time domain. In other words, the above three cases are classified as zero initialization function, finite time initialization function and infinite time initialization function, respectively. In the end of this section, the initialization response is discussed as well.

3.1. Zero Initialization Function

Applying the Laplace transform (1) to (2) yields

$$\mathrm{X(s)} = (\mathrm{sI} - \mathrm{A})^{-1}BU(s) + (\mathrm{sI} - \mathrm{A})^{-1}x(0^{+}),$$
$$\mathrm{Y(s)=CX(s)}. \qquad (3)$$

The inverse Laplace transform of (3) is

$$\mathrm{x(t)} = \mathrm{e}^{At}B * u(t) + \mathrm{e}^{At}x(0^{+}),$$
$$\mathrm{y(t)} = \mathrm{Cx(t)} = \mathrm{Ce}^{At}B * u(t) + C\mathrm{e}^{At}x(0^{+}),$$

which can be rewritten as

$$x(t) = \int_0^t e^{A(t-\tau)} Bu(\tau)d\tau + e^{At}x(0^+),$$

$$y(t) = Cx(t) = C\int_0^t e^{A(t-\tau)} Bu(\tau)d\tau + Ce^{At}x(0^+).$$

3.2. Finite Time Initialization Function

The state equation in (2) can be transferred as

$$\dot{x}(t)h(t+L) = Ax(t)h(t+L) + Bu(t)h(t).$$

Then,

$$\int_{-L}^{\infty} e^{-st}\dot{x}(t)h(t+L)dt = \int_{-L}^{\infty} e^{-st}Ax(t)h(t+L)dt + \int_{-L}^{\infty} e^{-st}Bu(t)h(t)dt.$$

The above equation can be expressed as

$$\int_{-L}^{0^+} e^{-st}\dot{x}(t)h(t+L)dt + \int_{0^+}^{\infty} e^{-st}\dot{x}(t)h(t+L)dt$$
$$= \int_{-L}^{0^+} e^{-st}Ax(t)h(t+L)dt + \int_{0^+}^{\infty} e^{-st}Ax(t)h(t+L)dt +$$
$$\int_{-L}^{0^+\infty} e^{-st}Bu(t)h(t)dt + \int_{0^+}^{\infty} e^{-st}Bu(t)h(t)dt \quad (4)$$

Applying the Laplace transform to (4) yields

$$\int_{-L}^{0^+} e^{-st}\dot{x}(t)h(t+L)dt + sX(s)$$
$$= \int_{-L}^{0^+} e^{-st}Ax(t)h(t+L)dt + AX(s) + \int_{-L}^{0^+\infty} e^{-st}Bu(t)h(t)dt + BU(s).$$

It's known that $u_k(t) = 0$ as $t < 0$, thus

$$e^{-st}x(t)\Big|_{-L}^{0} - \int_{-L}^{0^+} x(t)(-s)e^{-st}dt + sX(s) = \int_{-L}^{0^+} e^{-st}Ax(t)h(t+L)dt + AX(s) + BU(s).$$

The solution in frequency domain is achieved as

$$X(s) = (sI-A)^{-1}BU(s) + (sI-A)^{-1}x(0^+) -$$
$$\int_{-L}^{0^+} h(t+L)x(t)\,e^{-st}dt + (sI-A)^{-1}h(0^+)e^{sL}x(-L) - (sI-A)^{-1}x(0^+),$$

In time domain, it is shown as

$$x(t) = \int_0^t e^{A(t-\tau)} Bu(\tau)d\tau + e^{At}x(0^+)$$
$$-e^{At}x(0^+) + e^{A(t+L)}h(0^+)x(-L) + \boldsymbol{L^{-1}}\left\{-\int_{-L}^{0^+} h(t+L)x(t)\,e^{-st}dt\right\},$$

and the output $y(t)$ is expressed as

$$y(t) = Cx(t) = C[\int_0^t e^{A(t-\tau)} Bu(\tau)d\tau + e^{At}x(0^+)$$
$$-e^{At}x(0^+) + e^{A(t+L)}h(0^+)x(-L) + \boldsymbol{L^{-1}}\left\{-\int_{-L}^{0^+} h(t+L)x(t)\,e^{-st}dt\right\}].$$

3.3. Infinite Time Initialization Function

Similar to Subsection 3.2, it can be seen that, for the infinite time initialization function,

$$X(s) = (sI-A)^{-1}BU(s) + (sI-A)^{-1}x(0^+) -$$
$$\int_{-L}^{0^+} h(t+L)x(t)\,e^{-st}dt + (sI-A)^{-1}h(0^+)e^{sL}x(-L) - (sI-A)^{-1}x(0^+) +$$
$$(sI-A)^{-1}\int_{-\infty}^{-L} Ax(t)\,e^{-st}dt - (sI-A)^{-1}\int_{-\infty}^{-L} e^{-st}\,dt,$$
$$Y(s) = CX(s). \quad (5)$$

The inverse Laplace transform of (5) is

$x(t) = \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + e^{At}x(0^+) - e^{At}x(0^+) + e^{A(t+L)}h(0^+)x(-L) + \mathbf{L}^{-1}\left\{-\int_{-L}^{0^+} h(t+L)x(t)e^{-st}dt\right\} + e^{At} * \mathbf{L}^{-1}\left\{\int_{-\infty}^{-L} Ax(t)e^{-st}dt\right\} - e^{At} * \mathbf{L}^{-1}\left\{\int_{-\infty}^{-L} e^{-st}dt\right\},$

$y(t) = Cx(t) = C[\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + e^{At}x(0^+) - e^{At}x(0^+) + e^{A(t+L)}h(0^+)x(-L) + \mathbf{L}^{-1}\left\{-\int_{-L}^{0^+} h(t+L)x(t)e^{-st}dt\right\} + e^{At} * \mathbf{L}^{-1}\left\{\int_{-\infty}^{-L} Ax(t)e^{-st}dt\right\} - e^{At} * \mathbf{L}^{-1}\left\{\int_{-\infty}^{-L} e^{-st}dt\right\}].$

In Subsection 3.2 and 3.3, $dx = Ax$ because $u(t) = 0$ as $t \in (-\infty, -L)$. Then the discussion in Subsection 3.3 is reduced to it in Subsection 3.2, where $h(0^+) = 1$.

3.4. Initialization Response

The relationships of Subsections 3.1 and 3.2 will be analyzed in this part, where initialization function can be described by the difference between the solutions in them. This initialization response can influence the system performance, and how to reduce the effect of the initialization response becomes unavoidable in initialized ILC schemes. The procedure of decreasing the effect of the initialization response is called the preconditioning.

In case 2, the following equation is derived by the integrations by parts

$-\int_{-L}^{0^+} h(t+L)x(t)e^{-st}dt = -(A+I)^{-1}(-s)^{-1}[x(0^+) - x(-L)e^{sL}].$

Applying inverse Laplace transform to the above equation yields

$\boldsymbol{L}^{-1}\left\{-\int_{-L}^{0^+} h(t+L)x(t)e^{-st}dt\right\} = -(A+I)^{-1}x(0^+)h(t) + (A+I)^{-1}x(-L)h(t+L).$

Thus the solution of $x(t)$ in Subsection 3.2 can be written as

$x(t) = \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + e^{At}x(0^+)$
$-e^{At}x(0^+) + e^{A(t+L)}h(0^+)x(-L) - (A+I)^{-1}x(0^+)h(t) + (A+I)^{-1}x(-L)h(t+L).$

Theorem 1 The preconditioning effect can be indexed by

$$P(L,t) = \| e^{A(t+L)}h(0^+) + (A+I)^{-1}h(t+L) \|, \tag{6}$$

where the larger L corresponds to better preconditioning effect.

Proof 1: The difference between $x(t)$ in Subsections 3.1 and 3.2 is

$\tilde{x}(t) = -e^{At}x(0^+) + e^{A(t+L)}h(0^+)x(-L) - (A+I)^{-1}x(0^+)h(t) + (A+I)^{-1}x(-L)h(t+L).$

The function $\tilde{x}(t)$ is the so called initialization function.

It is obvious that different initial values of $x(-L)$ yield different those of $\tilde{x}(t)$.

$\hat{x}(t) = -e^{At}x(0^+) + e^{A(t+L)}h(0^+)\breve{x}(-L) - (A+I)^{-1}x(0^+)h(t) + (A+I)^{-1}\breve{x}(-L)h(t+L).$

The difference between different initialization functions is

$\| \hat{x}(t) - \tilde{x}(t) \| = \| e^{A(t+L)}h(0^+)[\breve{x}(-L) - x(-L)] + (A+I)^{-1}[\breve{x}(-L) - x(-L)]h(t+L) \|$

$\leq \| e^{A(t+L)}h(0^+) + (A+I)^{-1}h(t+L) \| \cdot \| \breve{x}(-L) - x(-L) \|.$

Given small enough positive constants ε and σ, $\| \hat{x}(t) - \tilde{x}(t) \| \leq \varepsilon$ holds if the following condition is satisfied

$\| \breve{x}(-L) - x(-L) \| \leq \sigma.$

The value of $x(-L)$ is very important and all the values of $x(t)$ is known in time domain of $t \in [-L, 0^-]$.

Thus we arrive at the following relationship

$\| \hat{x}(t) - \tilde{x}(t) \| \leq \| e^{A(t+L)}h(0^+) + (A+I)^{-1}h(t+L) \| \cdot \| \breve{x}(-L) - x(-L) \|$

$$\leq \| e^{A(t+L)}h(0^+) + (A+I)^{-1}h(t+L) \| \sigma \leq \varepsilon, \quad (7)$$

where

$\sigma \leq \| e^{A(t+L)}h(0^+) + (A+I)^{-1}h(t+L) \|^{-1} \varepsilon.$

Totally, the preconditioning procedure can be applied to reduce the effect of the initialization responses, successfully. The larger L contributes better effect of preconditioning. □

The relationship among $P(L,t)$, t and L is shown in Figure 2. From this figure, it's convenience to see the influences of time interval L of preconditioning procedure.

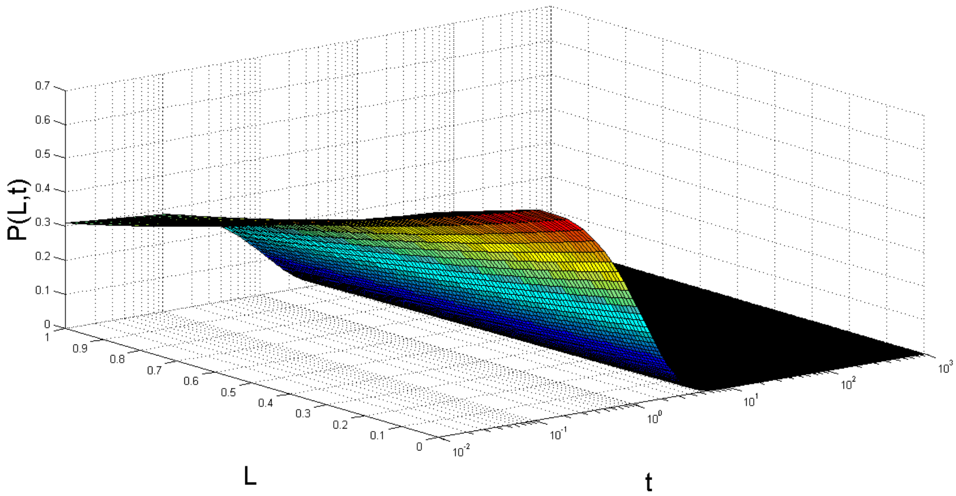

Figure 2. The Plot of $P(L,t)$ In (6) With Respect To t and L

4. Convergence Analysis

In this section, the convergence condition of initialized ILC scheme is analyzed. Besides, given small enough positive constants ε and σ, the lower bound of preconditioning time interval L is discussed based on the convergence condition.

Let the learning law be

$$u_{k+1}(t) = u_k(t) + \Gamma e_k(t) + K\dot{e}_k(t). \quad (8)$$

Each control process is on the fixed time interval $[0,T]$. But because of different initialization responses, the dynamics of the system is changed according to previous iterations. Combined with other disturbances, the output tracking errors are very hard to be disappeared. Nevertheless, it follows from (7) that this error can be restricted to a small enough neighbourhood of zero by preconditioning.

4.1. The Case of Zero Initialization Function

Assume $x_{k+1}(0^+) = x_k(0^+)$ and $t \in [0^+,T]$ the convergence condition can be achieved as follows. It follows from Subsection 3.1 and (8), the relationship between $\| e_k(t) \|_\lambda$ and $\| e_{k+1}(t) \|_\lambda$ is shown as

$$\| e_{k+1}(t) \|_\lambda \leq \left(\| I - CB\Gamma \|_\infty + \max_{0\leq t,\tau \leq T} \frac{1}{\lambda}(1-e^{-\lambda t}) \cdot \| Ce^{A(t-\tau)}B(\Gamma + KA) \|_\infty \right) \| e_k(t) \|_\lambda,$$

which can be rewritten as

$$\| e_{k+1}(t) \|_\lambda \leq (\rho_0 + \rho_1) \| e_k(t) \|_\lambda,$$

where

$\rho_0 = \| I - CB\Gamma \|_\infty$,
$\rho_1 = \max_{0\le t,\tau\le T} \frac{1}{\lambda}(1-e^{-\lambda t}) \cdot \| Ce^{A(t-\tau)}B(\Gamma + KA) \|_\infty$.
Since $0 \le \rho_0 \le 1$, it is possible to choose λ sufficiently large that
$\rho_0 + \rho_1 \le 1$,
which implies the convergence condition of zero initialized case
$\rho_0 = \| I - CB\Gamma \|_\infty < 1$.
Proof 2: The similar proof can be found in [1].

4.2. The Case of Non-Zero Initialization Function

For the initialized ILC, the system in current iteration is influenced by it in previous iterations so that the system may be non-repeatable. Nevertheless, if the initialization function in each iteration is close enough, this problem can be seemed as the traditional ILC problem similarly, the output tracking error can be restricted in a small enough neighbourhood of zero such as

$$\| \tilde{e}_k(t) \|_\lambda \le \varepsilon_1, \tag{9}$$

where there exists a k_1 such that (9) holds for all $k \ge k_1$. To achieve the above objective, it is assumed that $x_{k+1}(0^+) = x_k(0^+)$ which are known factors, while, $x_{k+1}(-L) \ne x_k(-L)$ as $k = 1,2,3,\cdots$. The output tracking error satisfies that

$$\begin{aligned}
&e_{k+1}(t) = y_d(t) - y_{k+1}(t) = y_d(t) - Cx_{k+1}(t) \\
&= y_d(t) - C\int_0^t e^{A(t-\tau)}Bu_{k+1}(\tau)d\tau - Ce^{At}x_{k+1}(0^+) + Ce^{At}x_{k+1}(0^+) \\
&-Ce^{A(t+L)}x_{k+1}(-L) + C(A+I)^{-1}x_{k+1}(0^+) - C(A+I)^{-1}x_{k+1}(-L) \\
&= e_k(t) - C\int_0^t e^{A(t-\tau)}B\big(\Gamma e_k(\tau) + K\dot{e}_k(\tau)\big)d\tau \\
&+Ce^{A(t+L)}[x_k(-L) - x_{k+1}(-L)] + C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)]
\end{aligned} \tag{10}$$

Applying λ-norm to both sides of (10), it follows from λ can be sufficiently large that

$$\begin{aligned}
&\| e_{k+1}(t) \|_\lambda \le (\rho_0 + \rho_1) \| e_k(t) \|_\lambda + \| Ce^{AL} \|_\infty \| x_k(-L) - x_{k+1}(-L) \|_\infty \\
&+ \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty \\
&= \rho_2 \| e_k(t) \|_\lambda + f_k,
\end{aligned}$$

where
$\rho_2 = \rho_0 + \rho_1$,
$f_k = \| Ce^{AL} \|_\infty \| x_k(-L) - x_{k+1}(-L) \|_\infty + \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty$.

Lemma 1: Suppose σ_1 is the maximum eigenvalue of the matrix A, ε_1 is the upper bound of $e_k(t)$, ε_2 is the upper bound of f_k, $\lambda > \| A \|_\infty$, $f_k \le (\rho - \rho_2) \| e_k(t) \|_\lambda$, k is the iteration number, $e_k(t)$ is the state and C, A are the parameters in the initialized system (2), and $e_k(t)$ is the output tracking error, then the minimum preconditioning time interval L can be achieved as

$$L \ge ln\{\| x_k(-L) - x_{k+1}(-L) \|_\infty^{-1} [(\rho - \rho_2)\varepsilon_1] - \| x_k(-L) - x_{k+1}(-L) \|_\infty^{-1} \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty\}\sigma_1^{-1} - (lnC)\sigma_1^{-1}.$$

Proof 3: Let $\lambda > \| A \|_\infty$,
$\rho_2 = \rho_0 + \rho_1$,
$f_k = \| Ce^{AL} \|_\infty \| x_k(-L) - x_{k+1}(-L) \|_\infty + \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty$.

The difference between the convergence conditions in Subsection 3.1 and 3.2 is defined in domain of $[0, \varepsilon_2]$. In fact, ε_2 is the upper bound of f_k which means that $f_k \le \varepsilon_2$.

$\| Ce^{AL} \|_\infty \| x_k(-L) - x_{k+1}(-L) \|_\infty + \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty \leq \varepsilon_2$,
then,
$(\| Ce^{AL} \|_\infty + \| C(A+I)^{-1} \|_\infty) \| x_k(-L) - x_{k+1}(-L) \|_\infty \leq \varepsilon_2$,
which implies that
$\left(\| Ce^{AL} \|_\infty + \| C(A+I)^{-1} \|_\infty\right)\sigma \leq \varepsilon_2$.
The bound of σ is shown as
$\sigma \leq \left(\| Ce^{AL} \|_\infty e^{-\lambda + \|A\|_\infty T} + \| C(A+I)^{-1} \|_\infty\right)^{-1} \varepsilon_2$.
Then, define $f_k \leq (\rho - \rho_2) \| e_k(t) \|_\lambda$ so that $\| e_{k+1}(t) \|_\lambda \leq \rho \| e_k(t) \|_\lambda$. Then the convergence condition of the non-zero initialized case becomes

$$\rho < 1.$$

In this case, $f_k \leq (\rho - \rho_2)\varepsilon_1$ means that
$\| Ce^{AL} \|_\infty \| x_k(-L) - x_{k+1}(-L) \|_\infty + \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty \leq (\rho - \rho_2)\varepsilon_1$.
Because σ_1 is the maximum eigenvalue of the matrix A, it follows that
$Ce^{\sigma_1 L} \leq \| Ce^{AL} \|_\infty$
$\leq \| x_k(-L) - x_{k+1}(-L) \|_\infty{}^{-1} [(\rho - \rho_2)\varepsilon_1] -$
$\| x_k(-L) - x_{k+1}(-L) \|_\infty{}^{-1} \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty$.
The bound of L is achieved as

$$\mathrm{L} \geq \ln\left\{\| x_k(-L) - x_{k+1}(-L) \|_\infty{}^{-1} [(\rho - \rho_2)\varepsilon_1] - \| x_k(-L) - x_{k+1}(-L) \|_\infty{}^{-1} \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty\right\}\sigma_1{}^{-1} - (\ln C)\sigma_1{}^{-1}. \quad (11)$$

Theorem 2: The convergence condition of the initialized system (2) with the ILC scheme (8) is achieved as

$$\| e_{k+1}(t) \|_\lambda \leq \rho \| e_k(t) \|_\lambda,$$

where
$\rho_2 \leq \rho < 1$,
$f_k \leq (\rho - \rho_2) \| e_k(t) \|_\lambda$,
$f_k = \| Ce^{AL} \|_\infty \| x_k(-L) - x_{k+1}(-L) \|_\infty + \| C(A+I)^{-1}[x_k(-L) - x_{k+1}(-L)] \|_\infty$,
$\rho_2 = \rho_0 + \rho_1$,
$\rho_0 = \| I - CB\Gamma \|_\infty$,
$\rho_1 = \max_{0 \leq t, \tau \leq T} \frac{1}{\lambda}(1 - e^{-\lambda t}) \cdot \| Ce^{A(t-\tau)} B(\Gamma + KA) \|_\infty$.
Proof 4: The following relationship has already been achieved

$$\| e_{k+1}(t) \|_\lambda \leq \rho_2 \| e_k(t) \|_\lambda + f_k.$$

It follows from Lemma 1 that

$$f_k \leq (\rho - \rho_2) \| e_k(t) \|_\lambda.$$

Here ends the proof. □

It is shown in (11) that the larger the L is, the better convergence performance of the initialized ILC scheme.

Remark 1: The conclusion in Subsection 4.1 can be achieved by the traditional ILC method, which is a special case of Subsection 4.2.

5. Illustrated Examples

In this section, the convergence condition of the system with initialization response is simulated. The previous ILC scheme is applied to the system with different initialization responses. The learning gains are non-negative in this paper. Nevertheless, one paper regarding the negative learning gains is cited here to extend the knowledge of this field [39].

5.1. Alternative Initialization Function

In this subsection, three groups of simulations are shown to validate the conclusion that the similar initialization responses in different control iterations contribute to better convergence performance. Let the initialized system be

$$x(t) = \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + e^{At}x(0^+) + m_k(t),$$
$$y(t) = Cx(t), \tag{12}$$

and the iterative learning law be

$$u_{k+1}(t) = u_k(t) + \Gamma e_k(t) + K\dot{e}_k(t). \tag{13}$$

The parameters of the system are $A = -0.9$, $B = 0.6$ and $C = 1$. The learning gains Γ and K equal to 1.5 and 0.01, respectively. And the initial value is $x(0^+) = 1$.

The initialization response $m_k(t)$ is defined as the following three cases

Case 1: $m_k(t) = 1,\ 0.1,\ 1,\ 0.1,\ 1,\ 0.1, \ldots,$ as $k = 1,\ 2,\ 3, \ldots,\ 10.$

Case 2: $m_k(t) = 1,\ \mathbf{0.5},\ \mathbf{1},\ \mathbf{0.5},\ 1,\ 0.5, \ldots,$ as $k = 1,\ 2,\ 3, \ldots,\ 10.$

Case 3: $m_k(t) = 1,\ 0.9,\ 1,\ 0.9,\ 1,\ 0.9, \ldots,$ as $k = 1,\ 2,\ 3, \ldots,\ 10.$

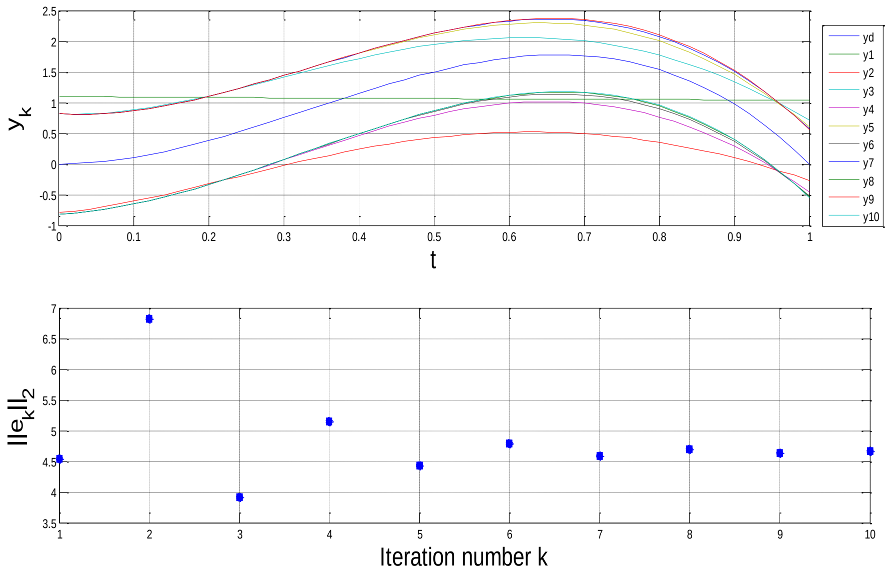

Figure 3. The Simulation Results of the Initialized System (12) With Learning Law (13), Where the Alternative Initialization Functions Are 1, 0.1, 1, 0.1

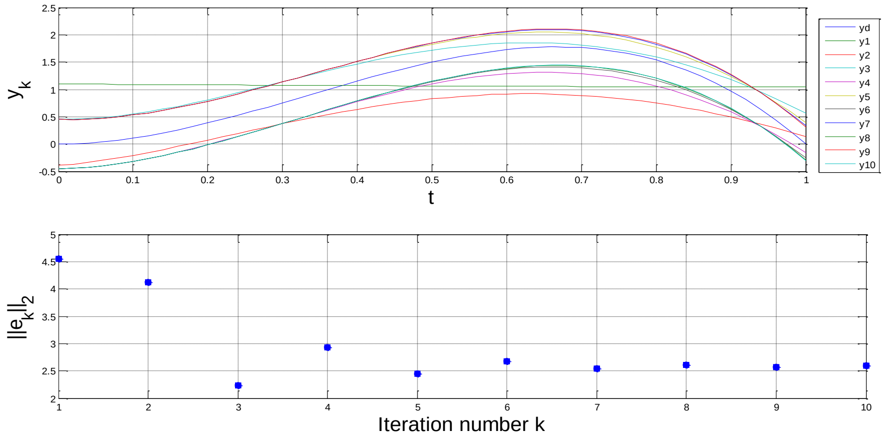

Figure 4. The Simulation Results of the Initialized System (12) With Learning Law (13), Where the Alternative Initialization Functions Are 1, 0.5, 1, 0.5

The simulations are shown in Figures 3, 4 and 5. The initial input is zero and the output reference is $y_d(t) = 12t^2(1-t)$. The plots of output $y_k(t)$, k=1, 2, 3, …, 10, and the 2-norm of tracking errors are shown in these three Figures. It can be seen from Figure 3 and Figure 4 that the 2-norm the output tracking errors can be reduced to 4.7 and 2.6 after a period of oscillation, respectively. In Figure 5, the 2-norm of the output tracking errors are monotonically convergent to a very small value, for example, $\| e_{10}(t) \|_2 = 0.5202$. Thus, the closer the initialization responses is, the better convergence performance can be achieved.

5.2. Constant Initialization Function

In this subsection, the $m_k(t)$ is defined as different unit step functions

$$h_k(t) = \begin{cases} 1, & t \in (-L, \infty), \quad as\ k\ belongs\ to\ odd\ number, \\ 1, & t \in (-L-\Delta L, \infty), \quad as\ k\ belongs\ to\ even\ number. \end{cases} \quad (14)$$

The ΔL is constant which is assumed to be 0.5 and different values of L are applied.

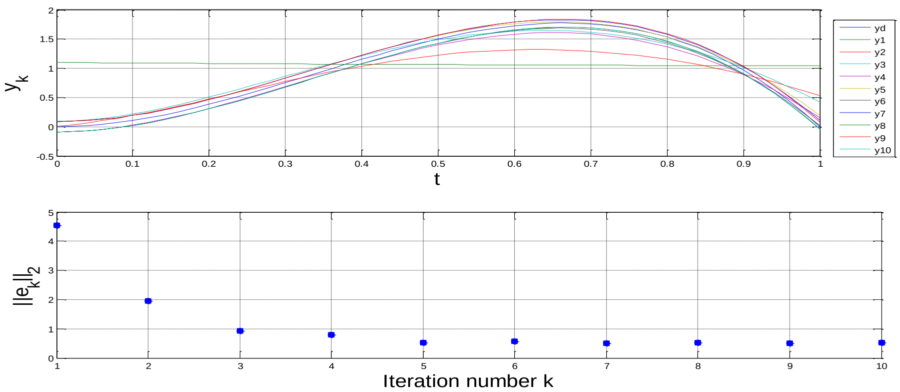

Figure 5. The Simulation Results of the Initialized System (12) With Learning Law (13), Where the Alternative Initialization Functions Are 1, 0.9, 1, 0.9

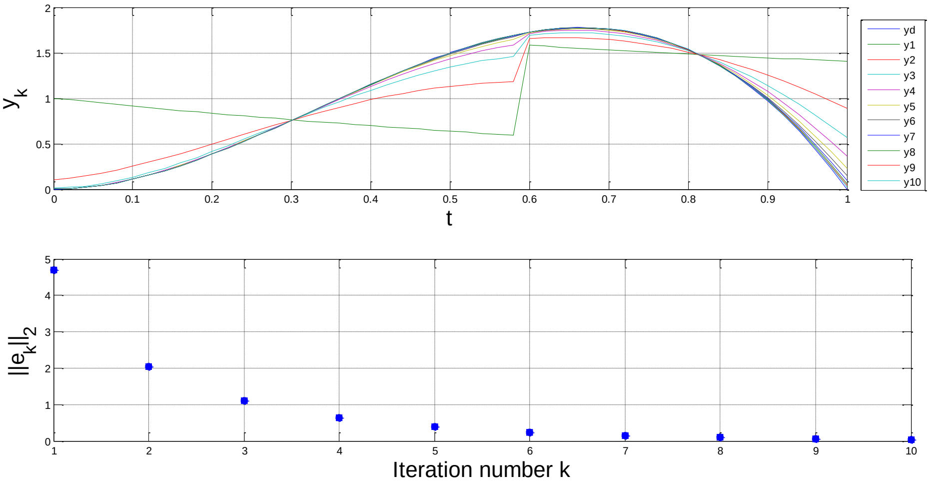

Figure 6. The Simulation Results of the Initialized System (12) With Different Unit Step Initialization Functions (14), Where the Preconditioning Time Interval Is $L + \Delta L = 0.6$

The initial input is still zero and the output reference is $y_d(t) = 12t^2(1-t)$. The simulation results are shown in Figures 6, 7 and 8. In Figure 6, $L = 0.1$, the system is convergent and $\| e_{10}(t) \|_2 = 0.0350$. In Figure 7, $L = 0.5$ and the system is convergent to a very small constant, $\| e_{10}(t) \|_2 = 0.0303$. In Figure 8, the system is monotonically convergent to a very small constant with better convergence performance, where $\| e_{10}(t) \|_2 = 0.0206$. The values of L in the above three cases are 0.1, 0.5 and 0.9, respectively, *i.e.* the larger values of L contributes faster convergence speed.

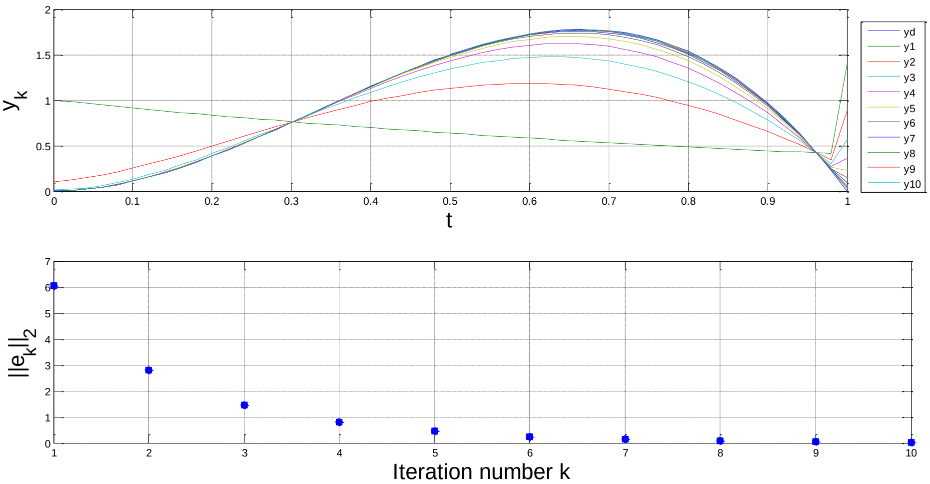

Figure 7. The Simulation Results of the Initialized System (12) With Different Unit Step Initialization Functions (14), Where the Preconditioning Time Interval Is $L + \Delta L = 1$

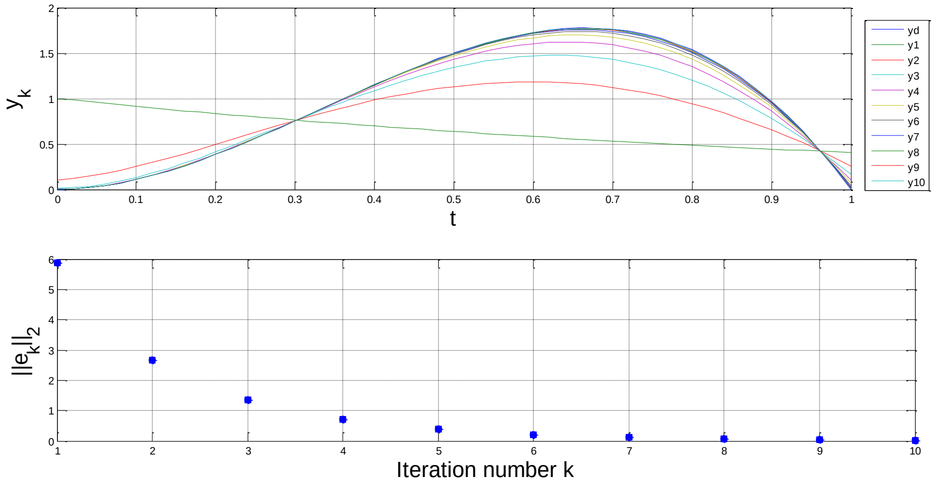

Figure 8. The Simulation Results of the Initialized System (12) With Different Unit Step Initialization Functions (14), Where the Preconditioning Time Interval is $L+\Delta L=1.4$

5.3. An Analytical Result Based Example

It follows from Subsection 3.2, the analytical solution of initialized system (2) is

$x(t)=\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau+e^{At}x(0^+)-e^{At}x(0^+)+e^{A(t+L)}h(0^+)x(-L)-(A+I)^{-1}x(0^+)h(t)+(A+I)^{-1}x(-L)h(t+L)$,

and the iterative learning law is

$u_{k+1}(t)=u_k(t)+\Gamma e_k(t)+K\dot{e}_k(t)$.

The parameters of the system A and B are -0.9 and 0.6, respectively. The learning gains Γ is 1.5 and $K=0.01$, the terminal time $T=1$. It's defined that $x_k(0^+)=x_{k+1}(0^+)=2$, and $x(-L)$ is random and the variance of $x(-L)$ is 1.

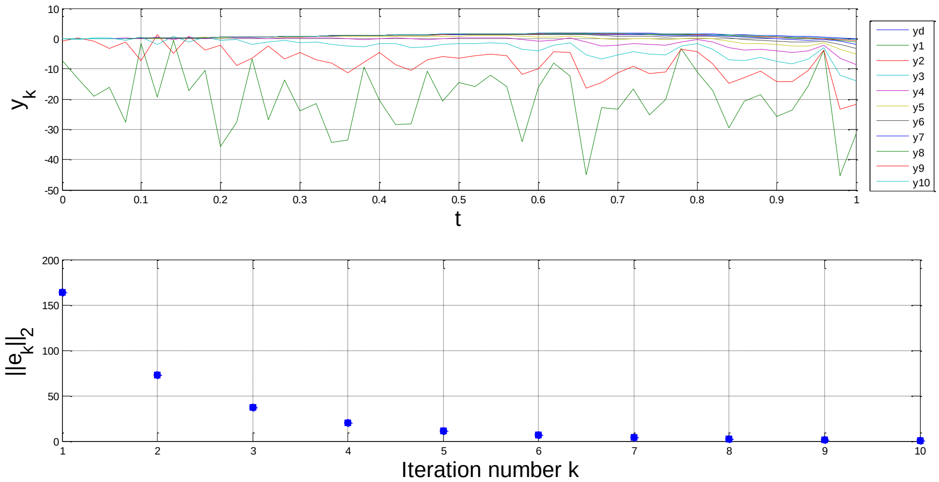

Figure 9. The Plot of An Analytical Result Based Example, Where the System Is (15), the Learning Law Is (13), The Preconditioning Time Interval is $L=0.1$, And $\| e_6(t) \|_2=6.4642$ and $\| e_{10}(t) \|_2=0.7498$

In Figure 9, the initial input is zero and the output reference is $y_d(t)=12t^2(1-t)$, and L is defined as 0.1. The plots of output $y_k(t)$, k = 1,2,3,…,10, and the 2-norm of tracking errors are shown in Figure 9. The output tracking errors of $\| e_6(t) \|_2$ and $\| e_{10}(t) \|_2$ are 6.4642 and 0.7498, respectively. In Figure 10, L = 0.9 and the output tracking errors of $\| e_6(t) \|_2$ and $\| e_{10}(t) \|_2$ are 6.4523 and 0.7492 which is smaller than the previous case. It's shown that the larger L contributes to better convergence performance.

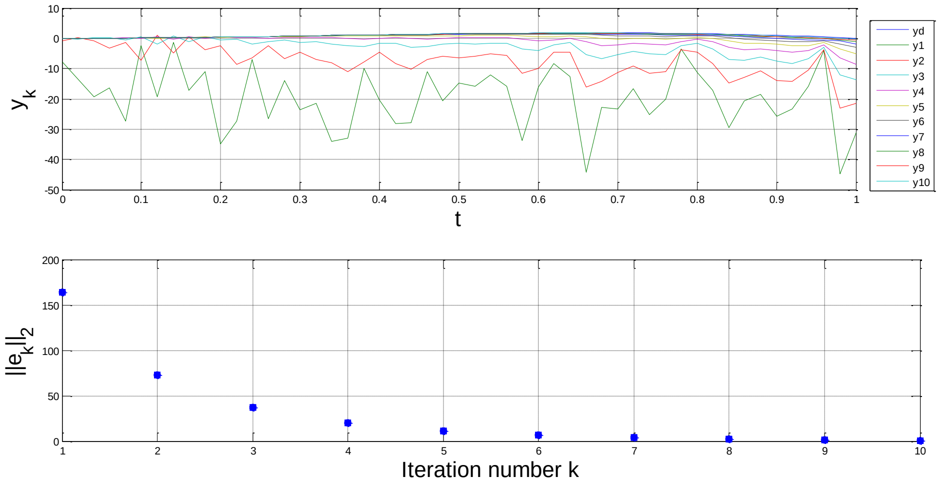

Figure 10. The Plot of an Analytical Result Based Example, Where the System Is (15), the Learning Law Is (13), the Preconditioning Time Interval Is $L = 0.9$ and $\| e_6(t) \|_2 = 6.4523$ and $\| e_{10}(t) \|_2 = 0.7492$

The tracking error with respect to iteration k and the preconditioning L can be achieved in Table 1. From this table, the conclusions are gained that the similar initialization responses in different control iterations contribute to better convergence performance and the larger preconditioning time interval L contributes to faster convergence speed.

Remark 2: The preconditioning procedure improves the convergence performance of the initialized system, and a long enough preconditioning time is necessarily required for accurate tracking.

Table 1. The Tracking Error With Respect To Iteration k and Precondition Time Interval L

(a) Numerical simulation result of the initialized system (12) with learning law (13), where the initialization functions are 1,m,1,m..., alternatively

$\| e(t) \|_2$ / m	$\| e_6(t) \|_2$	$\| e_{10}(t) \|_2$
0.1	4.7820	4.6676
0.5	2.6703	2.5938
0.9	0.5715	0.5202

(b) The numerical simulation results of the initialized system (12) with learning law (13), where the constant initialization functions are shown in (14).

L \ $\| e(t) \|_2$	$\| e_6(t) \|_2$	$\| e_{10}(t) \|_2$
0.1	0.2336	0.0350
0.5	0.2495	0.0303
0.9	0.2078	0.0206

(c) The analytical results of system (15) and learning law is (13).

L \ $\| e(t) \|_2$	$\| e_6(t) \|_2$	$\| e_{10}(t) \|_2$
0.1	4.7820	4.6676
0.9	0.5715	0.5202

6. Conclusions

In the paper, different solutions of linear system in different integral time intervals are achieved for initialized MIMO systems. The PD-type ILC scheme is applied and the convergence conditions are obtained. To meet with the convergence condition, the lower bound of preconditioning time interval L is derived. The preconditioning procedure is applied to improve the convergence performance of the control system to a large extent. The preconditioning procedure reduces the effect caused by different initialization functions. It's shown that the larger L or the closer of the initialization functions in different iterations, the better is the convergence performance. The numerical simulations validate the above conclusions.

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