

# A Strong Stable Interval System Controller Design

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## Abstract

*A method of strong stability controller design for interval system is proposed in this paper. The nominal value of the controller is designed based on  $H_\infty$  theory. Combine with Arguon's theoretical analysis the characteristics of the controller gain to get a strong stable controller. The innovation of this paper lies in separate interval system perturbation to design nominal controller utilize  $H_\infty$  theory, analyze the characteristic of the gain of interval controller apply Arguon's theory to improve the stability. At last, a numerical example is given for the design of a strong stability controller applied to an electro hydraulic position servo system.*

**Keywords:** *interval system; controller design; strong stability; Arguon's theory*

## 1. Introduction

For some systems, model parameters are constant but uncertain within a finite range. Such control systems are classified as interval systems. In order to application of classical control theory to solve practical problems, the designer must know the exact model about system before analysis and design. However, for complex systems it will be not easy to get accurately model, and all kinds of parameters will be changed slightly when system is running. This change means that there is error in the model itself. In 1978, Kharitonov [1] proposed four endpoint checking theorems for system stability based on interval polynomials. Accordingly put forward some control methods applied to interval system. However, the controller obtained by these methods are always high order[3], and limit the scope of application. Then researchers proposed some methods to reduce order for interval systems, such as[4,5] proposed applicate  $\gamma$ - $\delta$  Routh Tables. Then relevant scholars [6]utilize Pade' approximation to reduce order. To ensure performance for uncertain systems,  $H_2$ ,  $H_\infty$  and u-synthesis have been applied to design controller based on state-space representation. Many researchers [7-9] are committed to ensuring robust stability, and some controller design method are proposed to ensure that the robust performance is guaranteed. Due to the rapid developments of LMI toolboxes, LMI re-get the attention of researchers[10,11]. The polynomials where coefficients are varying has attracted the attention of many researchers. Recently [12] propose the system may exhibit unstable response under disturb condition even if all the closed-loop poles of interval system in the left half of s-plane. Generally speaking, The  $H_\infty$  norm conditions of the small gain theorem can only deal with the problem of the control system's unstructured uncertainty effectively, for structure uncertain may existence conservation. Thus, the system uncertainties involved in this paper are all unstructured uncertainties. In this paper, we try to design a strong stability interval system feedback controller.

## 2. Mathematical Preliminaries

### 2.1. Kharitonov's Theorem

Suppose that a family of polynomials is given by

$$D(s) = \sum a_i s^i, \text{ where} \quad (1)$$

$\alpha_i \leq a_i \leq \beta_i, 0 \leq i \leq n$ . According to Kharitonov's theory, the family of polynomials (1) are stable if and only if the following four endpoint polynomials are stable

$$D_1(s) = \alpha_0 + \beta_1 s + \beta_2 s^2 + \alpha_3 s^3 + \alpha_4 s^4 + \dots \quad (2)$$

$$D_2(s) = \alpha_0 + \alpha_1 s + \beta_2 s^2 + \beta_3 s^3 + \alpha_4 s^4 + \dots \quad (3)$$

$$D_3(s) = \beta_0 + \alpha_1 s + \alpha_2 s^2 + \beta_3 s^3 + \beta_4 s^4 + \dots \quad (4)$$

$$D_4(s) = \beta_0 + \beta_1 s + \alpha_2 s^2 + \alpha_3 s^3 + \beta_4 s^4 + \dots \quad (5)$$

The endpoint value that needs to be tested are

$$\begin{aligned} D_3(s) & \text{ for } i=3, \\ D_3(s), D_4(s) & \text{ for } i=4, \\ D_1(s), D_3(s), D_4(s) & \text{ for } i=5, \\ D_1(s), D_2(s), D_3(s), D_4(s) & \text{ for } i>5, \end{aligned}$$

The calculation is very complicated if  $i$  equal to five. And with the increase of the order, the calculation is more complicated.

### 2.2. Argoun's Theorem and Frequency Domain Conditions

After Kharitonov propounded the necessary and sufficient conditions for the stability of perturbed continuous time system characteristic polynomial, a similar criterion in frequency domain is proposed by Argoun. The main research of Argoun's theory is the denominator polynomial coefficients for closed-loop system in frequency domain. Consider the following polynomial

$$P(s) = a_0 + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1} + a_n s^n \quad (6)$$

whose coefficients  $a_i, i=0,1,\dots,n$  are random vary between the lower and upper limits,  $\alpha_i$  and  $\beta_i, i=0,1,\dots,n$ .  $\alpha_i < a_i < \beta_i, i=0,1,\dots,n$ . Substituting  $s = j\omega$  in (6) and letting  $\mu = \omega^2$ , we get

$$P(j\mu) = R(\mu) + j\omega Q(\mu) \quad (7)$$

$$\text{Where } R(\mu) = a_0 - a_2 \mu + \beta_4 \mu^2 - a_6 \mu^3 + \dots \quad (8)$$

$$Q(\mu) = a_1 - a_3 \mu + a_5 \mu^2 - \beta_6 \mu^3 + \dots \quad (9)$$

$R(\mu)$  contains the even power terms of  $P(j\mu)$  and  $Q(j\mu)$  contains the odd power ones. Define  $\bar{R}(\mu)$  and  $(\bar{Q}(\mu))$  are the biggest polynomial of  $R(\mu)$  and  $(Q(\mu))$ .  $\underline{R}(\mu)$  and  $(\underline{Q}(\mu))$  are the smallest polynomial of  $R(\mu)$  and  $(Q(\mu))$ . Consider the four extreme polynomials

$$\bar{R}(\mu) = \beta_0 - \alpha_2 \mu + \beta_4 \mu^2 - \alpha_6 \mu^3 + \dots \quad (10)$$

$$\underline{R}(\mu) = \alpha_0 - \beta_2 \mu + \alpha_4 \mu^2 - \beta_6 \mu^3 + \dots \quad (11)$$

$$\bar{Q}(\mu) = \beta_1 - \alpha_3 \mu + \beta_5 \mu^2 - \alpha_7 \mu^3 + \dots \quad (12)$$

$$\underline{Q}(\mu) = \alpha_1 - \beta_3\mu + \alpha_5\mu^2 - \beta_7\mu^3 + \dots \quad (13)$$

which are formed by substituting the upper and lower bounds  $\beta_i$  and  $\alpha_i$  successively for  $a_i, i=0,1,\dots,n$  in the expression for  $R(\mu)$  and  $Q(\mu)$ . In this context Argoun put forth the frequency domain counterpart of Kharitonov's theorem.

### 3. Application to Low Order Polynomials

Design  $H_\infty$  controllers are based on state-space model, and assume all states are measurable, consider an uncertain state-space model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t) \\ z(t) &= C_1x(t) + D_{11}w(t) + D_{12}u(t) \\ y(t) &= C_2x(t) + D_{21}w(t) + D_{22}u(t) \end{aligned} \quad (14)$$

where  $x(0) = x_0$  and the matrix  $A, B, C, D$  in state space(14) all possess parameter uncertainty, defined as:

$$\begin{aligned} A &= A_{nom} + \sum \delta_m A_{nom} & B &= B_{nom} + \sum \delta_c B_{nom} \\ C &= C_{nom} + \sum \delta_k C_{nom} & D &= D_{nom} + \sum \delta_n D_{nom} \end{aligned}$$

(nom is the abbreviation of the nominal value, the same below).  $\sum \delta_m A_{nom}, \sum \delta_c B_{nom}, \sum \delta_k C_{nom}, \sum \delta_n D_{nom}$  are additive parameter perturbation, suppose  $r < 0$ , and the minimal realization of  $G(s)$  is given as follows

$$G(s) = C(sI_n - A)^{-1}B + D \quad (15)$$

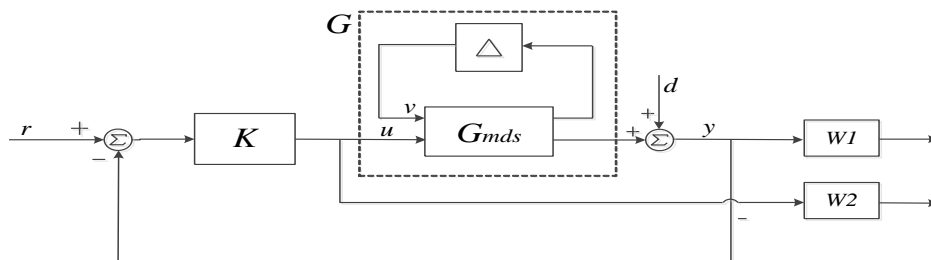


Figure 1. System Structure

$G$  is controlled object and  $K$  is the controller to be designed.

$$\begin{aligned} y &= (I + GK)^{-1}GKr + (I + GK)^{-1}d \\ u &= K(I + GK)^{-1}r - K(I + GK)^{-1}d \end{aligned} \quad (16)$$

Where  $r, y, u, v, d$  are input, output, control signal, output perturbation and disturbance signal in (16) and Figure 1. Interconnection transfer function matrix  $M$  (17) which is linear and uncertain systems can be obtained from  $H_\infty$  standard diagram coprime decomposition  $\delta_m, \delta_c, \delta_k, \delta_n$ .

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, M \in C^{2 \times 2} \quad (17)$$

(if  $(I - M_{11}\Delta)$  is invertible and exists)

Uncertainty bounds meet the following range

$$\Delta = \begin{pmatrix} \delta_m & & 0 \\ & \delta_c & \\ 0 & & \delta_k \\ & & & \delta_n \end{pmatrix}, \{\delta_m, \delta_c, \delta_k, \delta_n\} \in [-1, 1] \quad (18)$$

The output to the input matrix polynomial is as follows

$$z = \left[ M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \right] w \quad (19)$$

For perturbation block  $\Delta$ , the matrix polynomial from signal  $u$  to  $v$  is

$$T_{uv} = -K(I + GK)^{-1} \quad (20)$$

Define LFT from signal  $w$  to  $z$

$$F_u(M, \Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \quad (21)$$

For system is given by(14), robust stable conditions are

$$\left\| \Delta K(I + GK)^{-1} \right\|_{\infty} < 1 \quad \text{and} \quad (22)$$

$$\left\| K(I + GK)^{-1} \Delta \right\|_{\infty} < 1 \quad (23)$$

In (24)  $S$  and  $T$  are sensitive function and complementary sensitive function.  $W_1, W_2$  are weighting functions chosen to demonstrate the frequency index disturbance and external noise.

$$S = (I + GK)^{-1}, T = (I + GK)^{-1}GK \quad (24)$$

$$\left\| \begin{pmatrix} W_1 S \\ W_2 T \end{pmatrix} \right\|_{\infty} < \gamma \quad (25)$$

$$\left\| \begin{pmatrix} W_1 (I + F_u(M, \Delta)K)^{-1} \\ W_2 K (I + F_u(M, \Delta)K)^{-1} \end{pmatrix} \right\|_{\infty} \leq 1 \quad (26)$$

Suppose  $H_{\infty}$  controller is given as  $K(s) = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}$ , by the state-space representation

(14), closed-loop transfer matrix  $H_{CP}(s) = \begin{bmatrix} A_C & B_C \\ C_C & D_C \end{bmatrix}$ ,

where  $H_{CP} = \begin{bmatrix} A & B \\ C_1 & D_{11} \end{bmatrix} + \begin{bmatrix} B \\ D_{12} \end{bmatrix} \begin{bmatrix} D_K & C_K \\ B_K & A_K \end{bmatrix} \begin{bmatrix} C_2 & D_{21} \end{bmatrix}$

Calculation of the controller  $K(s)$  as follows:

Step 1, by solving following linear matrix inequality get  $X$  and  $Y$ .

$$\begin{bmatrix} N_X^T & \\ & I \end{bmatrix} \begin{bmatrix} AX + XA^T & XC_1^T & B_1 \\ C_1 X & -\gamma I & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{bmatrix} \begin{bmatrix} N_X & \\ & I \end{bmatrix} < 0 \quad (27)$$

$$\begin{bmatrix} N_X^T & \\ & I \end{bmatrix} \begin{bmatrix} YA + A^T Y & YB_1 & C_1^T \\ B_1^T Y & -\gamma I & D_{11}^T \\ C_1 & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} N_Y^T & \\ & I \end{bmatrix} < 0$$

In which  $N_Y = [C_2 \quad D_{21}]$ ,  $N_X = [B_2^T \quad D_{12}^T]$ , and assume  $D_{11} = [0 \quad I_{p1}]$

Step 2, get  $F$  by matrix equation  $FF^T = Y - X^{-1}$ .  $F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$

Step 3, let  $P = \begin{bmatrix} Y & F \\ F^T & I \end{bmatrix}$  substitution equation (28) get  $Q$  and  $E$ .

$$\begin{bmatrix} Q & E^T \\ F & * \end{bmatrix} = \left[ \begin{array}{ccc|c} A^T P + PA & PB & C_1^T & PB_2 \\ B^T P & -\gamma I & D_{11}^T & 0 \\ C_1 & D_{11} & -\gamma I & D_{12} \\ \hline C_2 & D_{21} & 0 & * \end{array} \right], \quad (28)$$

Step 4, suppose  $K = \begin{bmatrix} D_K & C_K \\ B_K & A_K \end{bmatrix}$  solve  $Q + E^T K F + F^T K E < 0$  (29)

$$A_K = A + BF - B_K (C_2 + D_{21} F_1)$$

$$B_K = -Z [L_2 - (B_2 + L_1 D_{12}) D_K]$$

$$C_K = F_2 - D_K (C_2 + D_{21} F_1)$$

$$D_K = -\gamma^2 (U_{122} D_{12})^{-1}$$

where  $Z = (I - \gamma^{-2} Y X)^{-1}$ ,  $\tilde{R} := D_{*1} D_{*1}^T - \begin{bmatrix} \gamma^2 I_{p1} & 0 \\ 0 & 0 \end{bmatrix}$ ,

$$L := -(B_1 D_{*1}^T + Y C^T) \tilde{R}^{-1} =: [L_1 \quad L_2], D_{*1} = \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix}$$

Specific calculation method can refer to [13]. After getting the negative feedback controller nominal transfer function, calculate the range of strong stability controller gain  $k$ . Argoun's theory indicates that, the end value of the three-order polynomial can be calculated as follows

$$\begin{aligned} \underline{R}(\mu) : \mu_{r1} &= \frac{\alpha_0}{\beta_2}, & \bar{R}(\mu) : \mu_{r2} &= \frac{\beta_0}{\alpha_2} \\ \underline{Q}(\mu) : \mu_{q1} &= \frac{\alpha_1}{\beta_3}, & \bar{Q}(\mu) : \mu_{r2} &= \frac{\beta_1}{\alpha_2} \end{aligned}$$

For closed-loop system with different orders, the conditions for the stability is, meet the following inequality. For a three-order polynomial satisfies

$$\frac{\alpha_1}{\beta_3} > \frac{\beta_0}{\alpha_2} \quad \text{or} \quad (\alpha_1 \alpha_2 > \beta_0 \beta_3)$$

For a four-order polynomial satisfies

$$\beta_1 \alpha_2 \alpha_3 > \beta_1^2 \beta_4 + \beta_0 \alpha_3^2, \quad \alpha_1 \alpha_2 \beta_3 > \alpha_1^2 \beta_4 + \beta_0 \beta_3^2$$

For a five-order polynomial satisfies

$$\begin{aligned} \mu_{q3} > \mu_{r1}, \quad \mu_{r3} > \max(\mu_{q2}, \mu_{q1}), \quad \text{and} \\ \min(\mu_{q2}, \mu_{q4}) > \max(\mu_{r2}, \mu_{r4}) \end{aligned} \quad (30)$$

Where  $\mu_{r1,2} = \frac{1}{2\alpha_4} [\beta_2 \pm \sqrt{\beta_2^2 - 4\alpha_0\alpha_4}]$

$$\mu_{r3,4} = \frac{1}{2\alpha_4} [\alpha_2 \pm \sqrt{\alpha_2^2 - 4\beta_0\beta_4}] \quad \mu_{q1,2} = \frac{1}{2\alpha_5} [\beta_3 \pm \sqrt{\beta_3^2 - 4\alpha_0\alpha_4}]$$

$$\mu_{q3,4} = \frac{1}{2\alpha_5} [\alpha_3 \pm \sqrt{\alpha_3^2 - 4\beta_1\beta_5}]$$

For the nominal negative feedback controller  $K$ , consider the proper perturbation (usually 10%~15%), we can get interval type transfer function  $k^*K$ , where  $k$  is an additional parameter, the scope of the requirements. When the open loop transfer function of controlled system is given, can get the closed-loop interval polynomial (6), the  $\alpha_i, \beta_i$  in  $P(s)$  contains unknown parameters  $k$ . Through determination condition (30),

can calculate the range of  $k$ . It is clear that the lower the  $P(s)$  order is, the easier it is to judge the stability.

## 5. Numerical Examples

An electro hydraulic position servo system's open-loop transfer function[14] is given as

$$G(s) = \frac{K_{sv} G_{sv}(s)}{s \left( \frac{s^2}{\omega_h^2} + \frac{2\xi_h}{\omega_h} s + 1 \right)}$$

The symbolic meaning of the expression is shown in the appendix.

$$\text{Where } G_{sv}(s) = \frac{1}{\frac{s^2}{\omega_{sv}^2} + \frac{2\xi_{sv}}{\omega_{sv}} s + 1}, \quad K_{sv} = \frac{K_s K_d K_a K_{sv}}{i D_m}$$

Nominal value of relevant parameters

$$K_s K_d = 1.5, K_a = 0.024, K_{sv} = 0.33, i = 216, D_m = 0.000139, \omega_h = 22, \xi_h = 5.032, \omega_{sv} = 138.16,$$

$$\xi_{sv} = 0.6, \beta_e = 7 \times 10^8, V_l = 0.0024, \text{where, } \omega_h = \sqrt{\frac{4\beta_e D_m}{J_l V_l}}; \quad \xi_h = \frac{K_{ce}}{D_m} \sqrt{\frac{\beta_e J_l}{V_l}}.$$

Open-loop nominal transfer function of servo system represent by

$$G(s) = \frac{396}{0.0001s^5 + 0.0424s^4 + 6.228s^3 + 488.7s^2 + 1000s + 396} \quad (31)$$

$$\text{Reduce to two-order } G(s) = \frac{0.87174}{s^2 + 2.192s + 0.87174}$$

Considering existence the perturbation of nominal parameter, interval servo system transfer function can be written as

$$G(s) = \frac{[0.794, 0.95]}{[0.93, 1.07]s^2 + [1.929, 2.455]s + [0.802, 0.942]} \quad (32)$$

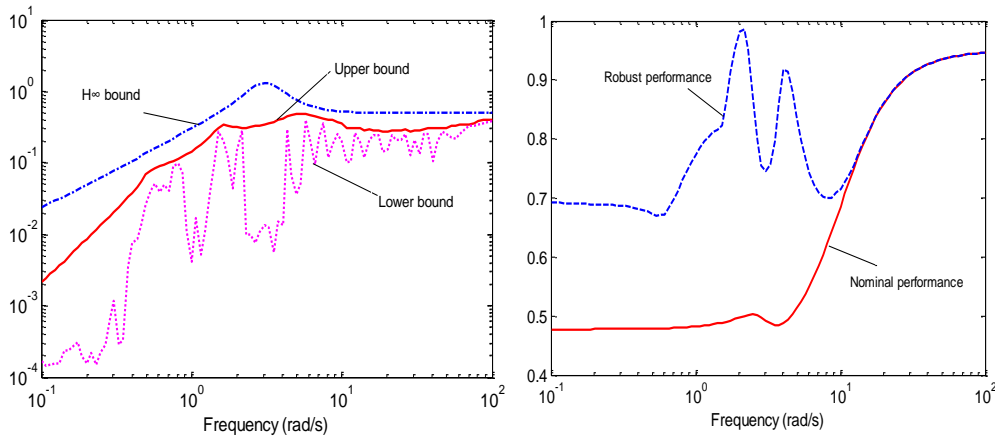
The weight functions by the method of [15][16] choice as follows

$$W_1(s) = 2 \frac{8s+1}{0.9}, W_2(s) = 0.98,$$

and through calculated we find that  $\gamma = 0.954$ . Because the controlled object has been reduced to two-order, so it is not very difficult to select the weight function, If the system is more complex then the choice of weight function may need more time. The transfer function of the nominal robust controller is obtained as

$$K(s) = \frac{0.3017s^2 + 581.2s + 226.5}{s^3 + 8.028s^2 + 79.41s + 0.09891} \quad (33)$$

Robust stability and Robust performance see Figure 2. It can be seen from figure that the system can not only meet robust stability but also meet robust performance.



**Figure 2. Robust Stability and Robust Performance of Closed-Loop System**

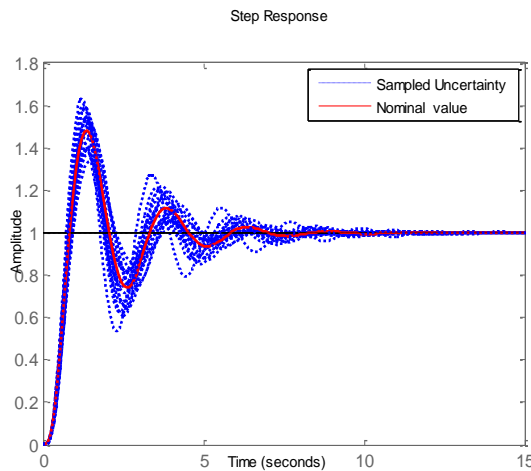
Take 15% perturbation with (33).

$$K(s) = \frac{[0.2955, 0.345]s^2 + [493.5, 668.15]s + [192.1, 259.9]}{[0.85, 1.15]s^3 + [6.8, 9.2]s^2 + [68, 92]s + [0.085, 0.115]}$$

Closed loop transfer function is obtained by adding negative feedback controller

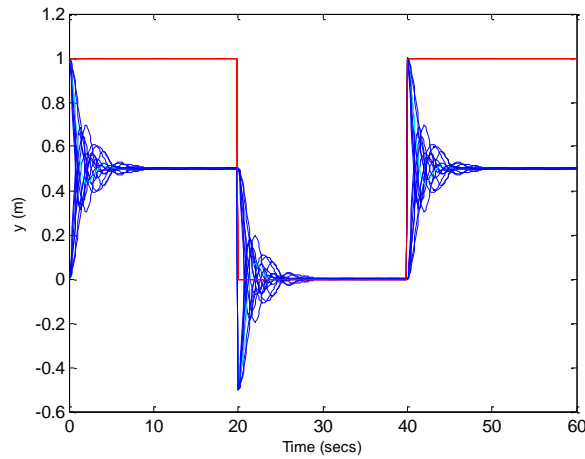
$$G(s) = \frac{[0.234, 0.328]s^2 + [391.8, 634]s + [152.5, 246.1]}{[0.79, 1.23]s^5 + [7.964, 12.67]s^4 + [77.04, 122.1]s^3 + [136.9, 235]s^2 + [446.5, 721.7]s + [152.6, 246.3]} \quad (34)$$

The step response of closed loop transfer function (34) see Figure 3. It is shown from time domain that system also has good stability.



**Figure 3. Step Response Plots of Perturbed Closed-Loop Systems**

Figure 4 shows the transient response of the closed-loop system under the influence of disturbance signal. Through input a disturbance signal every twenty seconds, we find that the system can quickly overcome.



**Figure 4. Transient Response to Disturbance Input**

The conditional polynomial for five-order closed-loop transfer function in Arguon's theorem are calculated as follows:

$$u_{r,1,2} = 0.0632 \left[ 233 + 0.329k \pm \sqrt{(233 + 0.329k)^2 - 31.96(0.0672 + 152.9k)^2} \right]$$

$$u_{r,3,4} = 0.0632 \left[ 135.8 + 0.2k \pm \sqrt{(135.8 + 0.2k)^2 - 30.28(0.107 + 247.5k)^2} \right]$$

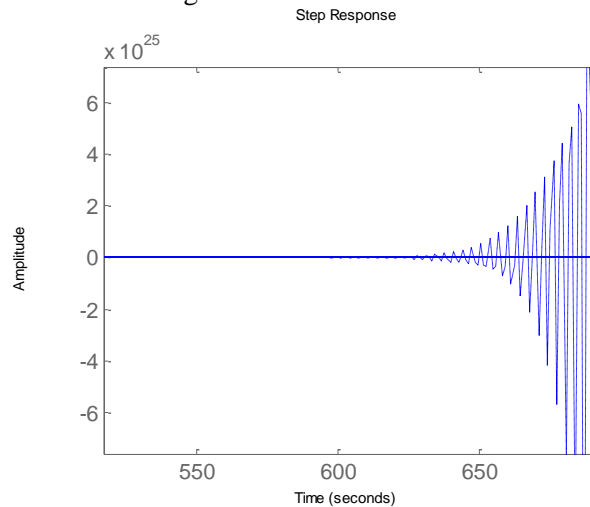
$$u_{q1,2} = 0.632 \left[ 121.4 \pm \sqrt{121.4^2 - 3.162(54.17 + 392.2k)} \right]$$

$$u_{q3,4} = 0.4 \left[ 30.66 \pm \sqrt{76.64^2 - 4.92(86.3 + 635k)} \right]$$

By solving inequality (30) we find that  $k$  close to 1, in the interval  $[0.86, 1.09]$ . It means the controller obtained by  $H_\infty$  method is very close to meet strong stability proposed by Arguon.

If  $k$  is limited to within the range  $[0.86, 1.09]$ , system will get better stability based on the original.

When the  $k$  is negative, the system is obviously unstable, and the step response for closed-loop system is shown in Figure 5 for  $k=1.5$ .



**Figure 5. Closed-Loop Step Response for  $k=1.5$**



## 5. Conclusion

In this paper a controller is designed for interval system by using  $H_\infty$  theory and negative feedback controller gain's range is calculated to guarantee strong stability. It should be pointed that, the closed-loop system has been satisfied both robust performance and robust stability with the requirements of Arguon's theorem. In addition, Arguon's theory used to judge the stability of closed-loop transfer function compared with Kharitonov's theory, can save a lot of computation when interval polynomial  $P(s)$  is less than or equal to five-order. Thus, apply Arguon's theory to design controller will obtain better stability than robust controller if the interval closed-loop system can reduce less than or equal to five-order. The proposed method can be widely applied to controller design for system in need of high stability system.

## Appendix

$K_S K_D$  gain,  $W_S W_V$  servo valve natural frequency,  $K_{SV}$  servo valve gain,  $K_a$  power amplifier gain,  $i$  transmission ratio,  $DM$  motor displacement per radian,  $\xi_{SV}$  damping ratio of electro hydraulic servo valve,  $\beta_e$  elastic modulus,  $V_t$  total motor volume.

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