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Sliding Mode Fault-tolerant Control for Uncertain Time-Delay Switched Systems

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Abstract

In this paper, a novel fault-tolerant control scheme is proposed for a class of uncertain switched systems with time-delay. The mathematical model of actuator which fault is established. Our attention is focused on the design of a sliding mode fault-tolerant controller, which guarantees the system to be asymptotically stable and has a H∞ disturbance attenuation performance in the case of actuator failure. Sufficient conditions for the existence of a desired sliding mode controller are established in terms of linear matrix inequalities (LMIs), and the corresponding sliding mode controller is designed. Simulation results verify the validity and feasibility of the proposed fault-tolerant control strategy.

***Keywords**: Switched Systems, Time Delay, Fault-tolerant Control, Sliding Mode*

1. Introduction

Switched systems is a class of hybrid systems which consist of a family of subsystems described by continuous-time (or discrete-time) dynamics and these subsystems are governed by a switching signal[1]. A large number of results have been reported for switched systems, such as, optimal performance analysis, stability analysis and control problems[2-4]. Many intelligent control strategies have been designed based on the idea of switching controllers to improve the overall performance. The reliable guaranteed-cost control problem for a class of delta operator switched linear systems is studied in [5]. Li and Zhao investigate the H∞ control problem for uncertain switched nonlinear systems with passive and non-passive subsystems [6]. These results have motivated the development of methods for control of various classes of switched systems.

Due to modern engineering systems have become more and more complex, faults and failures are more and more likely to cause unpredictable and undesired outcomes[7]. The safety and reliability issues on the complicated process receive more attention and become the most critical factors in process monitoring nowadays [8]. Fault detection and fault tolerant control techniques have received many results to ensure safe and reliable operation of the system. In [9], the guaranteed cost fault-tolerant control problem for unknown multi-input continuous nonlinear systems with loss of actuator effectiveness faults is investigated using the adaptive dynamic programming algorithm. [10] presents novel robust adaptive fault tolerant control strategies for the class of nonlinear Lipschitz systems in the presence of bounded matched or unmatched disturbances and actuator faults. Guo and Xu propose a novel ten-phase fault-tolerant permanent-magnet synchronous motor with two stators and two rotors on the same shaft [11]. Fault tolerant control strategy has also been used to make the switched system overcome some failures.

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Yang and Tong investigated the adaptive fault-tolerant control problem for a class of switched Takagi-Sugeno fuzzy systems [12]. The time delay is an important problem for discrete-time systems. Asymptotic stability for a class of discrete-time systems is investigated in [13]. Sliding mode control (SMC) is an effective robust control strategy for nonlinear systems and incompletely modeled systems. Many important results have been reported for the idea of SMC. In particular, design and analysis of sliding mode fault-tolerant control scheme[14-16]. This paper is devoted to studying fault tolerant sliding mode control problem for a class of uncertain switched systems with time-delay.

2. Problem Formulation

In this paper, the following switched system, with possibly time-delay $d \geq 0$, will be considered:

$$\begin{aligned} \dot{x}(t) &= [A_{\sigma(t)} + \Delta A_{\sigma(t)}(t)]x(t) + [A_{d\sigma(t)} + \Delta A_{d\sigma(t)}(t)]x(t-d) \\ &\quad + B[u_{\sigma(t)}(t) + f_{\sigma(t)}(x,t)] \\ x(t) &= \varphi(t), \forall t \in [-d, 0] \end{aligned} \tag{1}$$

Where $x \in R^n$ is the state vector, $u_{\sigma(t)}(t) \in \mathrm{R}^m$ is the control input. d is delay constant, and contents $d \geq 0$. $A_{\sigma(t)}$, B and $A_{d\sigma(t)}$ are known real constant matrices of appropriate dimensions. $\Delta A_{\sigma(t)}(t)$ and $\Delta A_{d\sigma(t)}(t)$ are uncertain parameter matrices of appropriate dimensions, and $f_{\sigma(t)}(x,t)$ represent nonlinear uncertainties and external disturbances of the system. $\sigma(\cdot):[0,\infty) \to \{1,2,\cdots,N\} = \bar{N}$ is the piecewise constant switching signal, and $\sigma(t) = i$ expresses the $i-th$ ($i \in \bar{N}$) subsystem which is activated at the time t . $\varphi(t)$ is a continuous function which is used to determine the initial state of the system.

Therefore, function (1) can be also expressed as following form:

$$\begin{aligned} \dot{x}(t) &= [A_i + \Delta A_i(t)]x(t) + [A_{di} + \Delta A_{di}(t)]x(t-d) + B[u_i(t) + f_i(x,t)] \\ x(t) &= \varphi(t), \forall t \in [-d, 0] \end{aligned} \tag{2}$$

For the system (2), the following assumptions are introduced:

Assumption 1. *If the matrix B is full column rank, for any $i \in \bar{N}$, the matrix for (A_i, B) is controllable.*

Assumption 2. *Uncertain system parameter matrix can be expressed as follows:*

$$\Delta A_i(\mathrm{t}) = E_i \Sigma_i(t) F_i \ , \ \Delta A_{di}(\mathrm{t}) = E_{di} \Sigma_i(t) F_{di}$$

Where, $\Delta A_i(\mathrm{t})$ and $\Delta A_{di}(\mathrm{t})$ is the i-th system time-delay uncertain parameters matrices, E_{di}, F_d and F_{di} are known constant matrices of appropriate dimensions, and $\Sigma_i(t)$ ($i \in \bar{N}$) is an unknown time-varying matrix satisfying: $\Sigma_i(t) \mid \Sigma_i^T(t)\Sigma_i(t) \leq I_k, \forall t \geq 0$.

Assumption 3. *$f_i(x,t)$ satisfies $\| f_i(x,t) \| \leq \eta_i(t)$, where $\eta_i(t)$ is a known positive scalar function.*

In the practical system, the actuator is always inevitable failure. This paper uses the model as formula (3) to show indicates actuator with failure.

$$u_i^F(t) = (I_i - \Gamma_i)u_i(t) \tag{3}$$

Where, $\Gamma_i = diag(\tau_{i1}, \cdots, \tau_{im})$ is a failure rate of actuator failures matrix, unknown parameters τ_{ik} denotes the first k of the first i subsystem failure rate of actuator failures, and τ_{ik} must satisfy the inequality $0 \leq \underline{\tau}_{ik} \leq \tau_{ik} \leq \bar{\tau}_{ik} < 1, \ k = 1, 2, \cdots, m$,Therefore, we can define

$\underline{\Gamma}_i = diag(\underline{\tau}_{i1},\cdots,\underline{\tau}_{im})$ and $\overline{\Gamma}_i = diag(\overline{\tau}_{i1},\cdots,\overline{\tau}_{im})$ as respectively failure rate of the system actuator fault lower and upper matrices.

Therefore, the time-delay switched systems with actuator failures can be described as:

$$\begin{aligned} &\dot{x}(t) = [A_i + \Delta A_i(t)]x(t) + [A_{di} + \Delta A_{di}(t)]x(t-d) + B[u_i^F(t) + f_i(x,t)] \\ &x(t) = \varphi(t), \forall t \in [-d, 0] \end{aligned} \tag{4}$$

Lemma 1 For any matrices of appropriate dimensions $F(t)$ arbitrarily satisfy $F^T(t)F(t) \le I$, there exist:

$$G_1F(t)G_2 + G_2^TF^T(t)G_1^T \le \alpha G_1G_1^T + \alpha^{-1}G_2^TG_2 \tag{5}$$

For any vector $x \in R^p$, $y \in R^q$ and constant $\alpha > 0$,they are established, where G_1 and G_2 are real matrices with appropriate dimensions.

Lemma 2 for appropriate dimensions matrices X and Y , there exist:

$$X^TY + Y^TX \le X^TPX + Y^TP^{-1}Y \tag{6}$$

For any matrix $P > 0$, it is established.

3. Main Results

Sliding surface function is defined as following:

$$s(t) = Sx(t) = B^TPx(t) = 0 \tag{7}$$

Where, matrix $P > 0$ is proposed in the design of the back, therefore, $B^TPB > 0$ is non-singular. Since the matrix B is full column rank, non-singular B^TPB can be guaranteed by P . Control goal of the system is that state trajectory can still be driven onto sliding surface (7) in the case of the system actuator failure, and along the sliding surface to zero. In this paper, define the sliding mode control law as:

$$u_i(t) = -K_ix(t) + u_{ri}(t) \tag{8}$$

Where

$$\begin{aligned} u_{ri}(t) = &-(B^TPB)^{-1}B^TPA_ix(t) - (B^TPB)^{-1}B^TPA_{di}x(t-d) \\ &-[\gamma_i + (I_i - \overline{\Gamma}_{i\max})^{-1}\eta_i(x,t)]sgn(s(t)) \end{aligned} \tag{9}$$

For all $t \ge 0$, select matrix $K_i \in R^{m\times n}$ ensures that the matrix $A_i - BK_i$ is Hurwitz stable. $\overline{\Gamma}_{i\max} = \max\{diag(\overline{\tau}_{ik}), k = 1,2,\cdots,m\}$, the scalar γ_i will be given later.

Put the formula (8) into the system (4) to obtain the overall closed-loop system as :

$$\begin{aligned} \dot{x}(t) = &[A_i + \Delta A_i(t)]x(t) + [A_{di} + \Delta A_{di}(t)]x(t-d) + B[(I_i - \Gamma_i)(-K_ix(t) \\ &-(B^TPB)^{-1}B^TPA_ix(t) - (B^TPB)^{-1}B^TPA_{di}x(t-d) \\ &-[\gamma_i + (I_i - \overline{\Gamma}_{i\max})^{-1}\eta_i(x,t)]sgn(s(t))) + f_i(x,t)] \\ x(t) = &\varphi(t), \forall t \in [-d, 0] \end{aligned} \tag{10}$$

3.1. Stability Analysis

Theorem 1. If there are a symmetric positive–definite matrix P , and positive real scalars θ_i , ε_{1i} , ε_{2i} and α_i , matrix Q_i that make linear matrix inequalities (11) and conditional expression (12) establish.

$$\begin{bmatrix} \Sigma_i & * & * & * & * & * & 0 & * \\ \tilde{\Gamma}_i^T B^T P & -\theta_i & 0 & 0 & 0 & 0 & 0 & 0 \\ PA_i & 0 & -P & 0 & 0 & 0 & 0 & 0 \\ E_i^T P & 0 & 0 & -\varepsilon_{1i} & 0 & 0 & 0 & 0 \\ E_{di}^T P & 0 & 0 & 0 & -\varepsilon_{2i} & 0 & 0 & 0 \\ B^T P & 0 & 0 & 0 & 0 & -\alpha_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -P & * \\ A_{di}^T P & 0 & 0 & 0 & 0 & 0 & PA_{di} & \varepsilon_{2i} F_{di}^T F_{di} - Q_i \end{bmatrix} < 0 \quad (11)$$

$$\theta_i I - B^T PB < 0 \quad (12)$$

Where:
$\Sigma_i = P(A_i - BK_i) + (A_i - BK_i)^T P + \varepsilon_{1i} F_i^T F_i + \alpha_i K_i^T \bar{\Gamma}_i^T \bar{\Gamma}_i K_i + Q_i$, $\tilde{\Gamma}_i = I_i - \underline{\Gamma}_i$, $\theta_i^{-1} = \theta_{1i}^{-1} + \theta_{2i}^{-1}$, the closed-loop system is asymptotically stable in the case of being designed sliding mode controller and actuator fault.

Proof. Consider the Lyapunov function as following.

$$V_i(t) = x^T(t)Px(t) + \int_{t-d}^{t} x^T(\upsilon)Q_i x(\upsilon)d\upsilon \quad (13)$$

From the derivation of $V(t)$ we can get that:

$$\begin{aligned} \dot{V}(t) &= x^T P(A_i + \Delta A_i)x + x^T P(A_{di} + \Delta A_{di})x(t-d) + x^T PB(I_i + \Gamma_i)(-K_i x + u_{ir}) \\ &\quad + x^T PBf_i(x) + x^T(A_i^T + \Delta A_i^T)Px + x^T(t-d)(A_{di}^T + \Delta A_{di}^T)x \\ &\quad + (-x^T K_i^T + u_{ir}^T)(I_i^T - \Gamma_i^T)B^T Px + f_i^T(x)B^T Px + x^T Q_i x \\ &\quad - x^T(t-d)Q_i x(t-d) \\ &= x^T(PA_i + A_i^T P - PBK_i - K_i^T B^T P + Q_i)x + 2x^T P\Delta A_i x + 2x^T PA_{di}x(t-d) \\ &\quad + 2x^T P\Delta A_{di}x(t-d) + 2x^T PB\Gamma_i K_i x + 2x^T PB(I_i - \Gamma_i)u_{ri} \\ &\quad + 2x^T PBf_i(x) - x^T(t-d)Q_i x(t-d) \end{aligned} \quad (14)$$

By Lemma 1 and Assumption 2, the following inequalities are established:

$$2x^T P\Delta A_i x \le \varepsilon_{1i}^{-1} x^T PE_i E_i^T Px + \varepsilon_{1i} x^T F_i^T F_i x \quad (15)$$

$$2x^T P\Delta A_{di} x(t-d) \le \varepsilon_{2i}^{-1} x^T PE_{di} E_{di}^T Px + \varepsilon_{2i} x^T(t-d) F_{di}^T F_{di} x(t-d) \quad (16)$$

$$2x^T PB\Gamma_i K_i x \le \alpha_i^{-1} x^T PBB^T Px + \alpha_i x^T K_i^T \bar{\Gamma}_i^T \bar{\Gamma}_i K_i x \quad (17)$$

From formula (9), and use the conditional expression (12) and Lemma 2, the following inequalities is established:

$$\begin{aligned} -2x^T PB(I_i - \Gamma_i)(B^T PB)^{-1} B^T PA_i x &\le x^T \theta_{1i}^{-1} PB(I_i - \underline{\Gamma}_i)(I_i - \underline{\Gamma}_i)^T B^T Px \\ &\quad + x^T(A_i^T P)P^{-1}(PA_i)x \end{aligned} \quad (18)$$

$$\begin{aligned} -2x^T PB(I_i - \Gamma_i)(B^T PB)^{-1} B^T PA_{di} x(t-d) &\le x^T \theta_{2i}^{-1} PB(I_i - \underline{\Gamma}_i)(I_i - \underline{\Gamma}_i)^T B^T Px \\ &\quad + x^T(t-d)(A_{di}^T P)P^{-1}(PA_{di})x(t-d) \end{aligned} \quad (19)$$

Due to $s(t) = Sx(t) = B^T Px(t)$, so we can get that:

$$\begin{aligned} &2x^T PBf_i(x,t) - 2x^T PB(I - \Gamma_i)[\gamma_i + (I_i - \bar{\Gamma}_{i\max})^{-1}\eta_i(x,t)]sgn(s(t)) \\ &\le 2|s(t)|\eta_i(x,t) - 2(I_i - \bar{\Gamma}_{i\max})[\gamma_i + (I_i - \bar{\Gamma}_{i\max})^{-1}\eta_i(x,t)]|s(t)| \\ &= -2\gamma_i(I_i - \bar{\Gamma}_{i\max})|s(t)| < 0 \end{aligned} \quad (20)$$

Substitute above formulas into formula (14) and obtain:

$$\begin{aligned}\dot{V}(t) \le\ & x^T[PA_i + A_i^T P - PBK_i - K_i^T B^T P + Q_i + \varepsilon_{1i}^{-1} PE_i E_i^T P + \varepsilon_{1i} F_i^T F_i \\ & + \varepsilon_{2i}^{-1} PE_{di} E_{di}^T P + \alpha_i^{-1} PBB^T P + \alpha_i K_i^T \overline{\Gamma}_i^T \overline{\Gamma}_i K_i \\ & + \theta_{1i}^T PB(I_i - \underline{\Gamma}_i)(I_i - \underline{\Gamma}_i)^T B^T P + (A_i^T P)P^{-1}(PA_i) \\ & + \theta_{2i}^T PB(I_i - \underline{\Gamma}_i)(I_i - \underline{\Gamma}_i)^T B^T P]x + 2x^T PA_{di} x(t-d) \\ & + x^T(t-d)[(A_{di}^T P)P^{-1}(PA_{di}) + \varepsilon_{2i} F_{di}^T F_{di} - Q_i]x(t-d) \\ & = \begin{bmatrix} x^T & x^T(t-d) \end{bmatrix} \begin{bmatrix} M_i & PA_{di} \\ A_{di}^T P & (A_{di}^T P)P^{-1}(PA_{di}) + \varepsilon_{2i} F_{di}^T F_{di} - Q_i \end{bmatrix} \begin{bmatrix} x \\ x(t-d) \end{bmatrix}\end{aligned} \tag{21}$$

Where

$$\begin{aligned}M_i =\ & PA_i + A_i^T P - PBK_i - K_i^T B^T P + Q_i + \varepsilon_{1i}^{-1} PE_i E_i^T P + \varepsilon_{1i} F_i^T F_i + \varepsilon_{2i}^{-1} PE_{di} E_{di}^T P \\ & + \alpha_i^{-1} PBB^T P + \alpha_i K_i^T \overline{\Gamma}_i^T \overline{\Gamma}_i K_i + \theta_{1i}^T PB(I_i - \underline{\Gamma}_i)(I_i - \underline{\Gamma}_i)^T B^T P + (A_i^T P)P^{-1}(PA_i) \\ & + \theta_{2i}^T PB(I_i - \underline{\Gamma}_i)(I_i - \underline{\Gamma}_i)^T B^T P\end{aligned}$$

Suppose $\xi(t) = \begin{bmatrix} x \\ x(t-d) \end{bmatrix}$, the above formula can be simplified as:

$$\dot{V}(t) \le \xi^T(t) \begin{bmatrix} M_i & PA_{di} \\ A_{di}^T P & (A_{di}^T P)P^{-1}(PA_{di}) + \varepsilon_{2i} F_{di}^T F_{di} - Q_i \end{bmatrix} \xi(t) \tag{22}$$

Therefore, it satisfy the inequality as formula (23).

$$\Omega_i = \begin{bmatrix} M_i & PA_{di} \\ A_{di}^T P & (A_{di}^T P)P^{-1}(PA_{di}) + \varepsilon_{2i} F_{di}^T F_{di} - Q_i \end{bmatrix} < 0 \tag{23}$$

Then a formula as formula (24) can be established.

$$\dot{V}(t) \le \xi^T(t)\Omega_i \xi(t) < 0 \tag{24}$$

Closed-loop system is globally stable.

By schur complement it can be seen that the formula (23) and (11) are equivalent. Theorem 1 is proved.

Defining the switching law as:

$$\sigma(t) = \arg\min_{i \in \bar{N}} [\xi^T(t)\Omega_i \xi(t)] \tag{25}$$

3.2. Reachability Analysis of Sliding Surface

In Theorem 1, it can be seen that the closed-loop system is globally asymptotically stable if it satisfies the linear matrix inequalities formula (11) and conditional expression (12) when actuator fails. Further we study the reachability of the dynamic sliding surface (7).

Theorem 2. For uncertain time-delay switched system (1), when actuator fails, the model is (4). And a function of the sliding surface is selected as $s(t) = Sx(t) = B^T Px(t) = 0$, P is a symmetric positive–definite matrix. It can be obtained from theorem 1.

Proof. Select the Lyapunov function as formula (26).

$$V_s(t) = 0.5 s^T(t)(B^T PB)^{-1} s(t) \tag{26}$$

From the derivation of $V_s(t)$ we can get that:

$$\begin{aligned}
\dot{V}_s(t) &= s^T(t)(B^TPB)^{-1}\dot{s}(t) = s^T(t)(B^TPB)^{-1}B^TP\dot{x}(t) \\
&= s^T(t)(B^TPB)^{-1}B^TP\{[A_i+\Delta A_i(t)]x(t)+[A_{di}+\Delta A_{di}(t)]x(t-d) \\
&+B[(I_i-\Gamma_i)(-K_ix(t)-(B^TPB)^{-1}B^TPA_ix(t)-(B^TPB)^{-1}B^TPA_{di}x(t-d) \\
&-[\gamma_i+(I_i-\overline{\Gamma}_{i\max})^{-1}\eta_i(x,t)]sgn(s(t)))+f_i(x,t)]\} \\
&= s^T(t)(B^TPB)^{-1}B^TP\{[A_i+\Delta A_i(t)]x(t)+[A_{di}+\Delta A_{di}(t)]x(t-d)\} \\
&+s^T(t)[(I_i-\Gamma_i)(-K_ix(t)-(B^TPB)^{-1}B^TPA_ix(t)-(B^TPB)^{-1}B^TPA_{di}x(t-d) \\
&-[\gamma_i+(I_i-\overline{\Gamma}_{i\max})^{-1}\eta_i(x,t)]sgn(s(t)))+f_i(x,t)] \\
&\le \|s(t)\|\|B^TPB)^{-1}B^TPA_i\|\|x(t)\|+\|s(t)\|\|B^TPB)^{-1}B^TPE_i\|\|F_i\|\|x(t)\| \\
&+\|s(t)\|\|B^TPB)^{-1}B^TPA_{di}\|\|x(t-d)\|+\|s(t)\|\|B^TPB)^{-1}B^TPE_{id}\|\|F_{id}\|\|x(t-d)\| \\
&+\|s(t)\|\|I_i-\underline{\Gamma}_i\|\|K_i\|\|x(t)\|+\|s(t)\|\|I_i-\underline{\Gamma}_i\|\|B^TPB)^{-1}B^TPA_i\|\|x(t)\| \\
&+\|s(t)\|\|I_i-\underline{\Gamma}_i\|\|B^TPB)^{-1}B^TPA_{di}\|\|x(t-d)\|-\|s(t)\|(I_i-\overline{\Gamma}_{i\max})\gamma_i \\
&\le -\beta\|s(t)\|+\|s(t)\|[\zeta_{1i}\|x(t)\|+\zeta_{2i}\|x(t-d)\|-((I_i-\overline{\Gamma}_{i\max})\gamma_i-\beta)]
\end{aligned}$$

Where

$$\begin{aligned}
\zeta_{1i} &= \|B^TPB)^{-1}B^TPA_i\|+\|B^TPB)^{-1}B^TPE_i\|\|F_i\|+\|I_i-\underline{\Gamma}_i\|\|K_i\| \\
&+\|I_i-\underline{\Gamma}_i\|\|B^TPB)^{-1}B^TPA_i\| \\
\zeta_{2i} &= \|B^TPB)^{-1}B^TPA_{di}\|+\|B^TPB)^{-1}B^TPE_{id}\|\|F_{id}\| \\
&+\|I_i-\underline{\Gamma}_i\|\|B^TPB)^{-1}B^TPA_{di}\|
\end{aligned}$$

And what satisfy $\beta > 0$ is a small numerical constant, γ_i satisfies $(I_i-\overline{\Gamma}_{i\max})\gamma_i-\beta > 0$.

Spatial region is defined as formula (27).

$$\Lambda_i = \{x(t): \zeta_{1i}\|x(t)\|+\zeta_{2i}\|x(t-d)\| \le (I_i-\overline{\Gamma}_{i\max})\gamma_i-\beta\} \tag{27}$$

Therefore, for all $\|s(t)\| \ne 0$, we can obtain:

$$\|s(t)\|[\zeta_{1i}\|x(t)\|+\zeta_{2i}\|x(t-d)\|-((I_i-\overline{\Gamma}_{i\max})\gamma_i-\beta)] \le 0 \tag{28}$$

The following inequality is established.

$$\dot{V}_s(t) \le -\beta\|s(t)\| < 0 \tag{29}$$

From Theorem 1 it can be seen that the status trajectories of the system can enter the area Λ_i in limited time, that is, $\zeta_{1i}\|x(t)\|+\zeta_{2i}\|x(t-d)\| \le (I_i-\overline{\rho}_{i\max})\gamma_i-\beta$ can be guaranteed in limited time. As a result, it can be concluded that the sliding mode control law designed can ensure the system trajectory overcomes the effects of the actuator failures from an arbitrary state starting and can be driven onto the default sliding surface $s(t)=0$ in limited time and remains on the sliding surface. Theorem is proved.

4. Example

Consider the actuator failures time-delay switched systems equation (4), and the parameters are following as:

$A_1 = \begin{bmatrix} 25 & -41 \\ 24 & -34 \end{bmatrix}$, $A_{d1} = \begin{bmatrix} 0.5 & 0.3 \\ 0.6 & 0.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} 26 & -42 \\ 25 & -32 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} 0.4 & 0.5 \\ 0.7 & 0.6 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$, $E_1 = \begin{bmatrix} 0.02 \\ 0.14 \end{bmatrix}$, $E_{1d} = \begin{bmatrix} 0.03 \\ 0.36 \end{bmatrix}$, $F_1 = \begin{bmatrix} 0.11 \; 0.23 \end{bmatrix}$, $F_{d1} = \begin{bmatrix} 0.41 \; 0.21 \end{bmatrix}$, $E_2 = \begin{bmatrix} 0.04 \\ 0.16 \end{bmatrix}$, $E_{2d} = \begin{bmatrix} 0.02 \\ 0.26 \end{bmatrix}$, $F_2 = \begin{bmatrix} 0.13 & 0.27 \end{bmatrix}$, $F_{2d} = \begin{bmatrix} 0.43 & 0.17 \end{bmatrix}$, $\Sigma_1(t) = \Sigma_2(t) = 0.5\sin(t)$, lower limit failure rates are respectively $\underline{\Gamma}_1 = 0.0368$, $\underline{\Gamma}_2 = 0.0421$. Upper limit failure rates are $\overline{\Gamma}_1 = 0.9728$, $\overline{\Gamma}_2 = 0.9331$ and $\overline{\Gamma}_{1\max} = \overline{\Gamma}_1 = 0.9728$, $\overline{\Gamma}_{2\max} = \overline{\Gamma}_2 = 0.9331$.

Meanwhile, external uncertain interference are $f_1(x,t) = f_2(x,t) = 0.03\mathrm{e}^{-t}\sin(\mathrm{t})$, and they satisfy that $\| f_1(x,t) \| = \| f_2(x,t) \| \le 0.03 \cdot |\sin(\mathrm{t})|$, that is, $\eta_1(x,t) = \eta_2(x,t) = 0.03 \cdot |\sin(\mathrm{t})|$. The delay constant is $d = 2$. Constants are $\gamma_1 = 0.02$ and $\gamma_2 = 0.01$. Choosing Hurwitz stable matrixes as $K_1 = \begin{bmatrix} -0.1908 & 0.0046 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.3443 & 0.0278 \end{bmatrix}$. Using the LMI toolbox of MATLAB to solve the linear matrix inequality (11) and conditional expression (12), the results obtained as follows:

$P = \begin{bmatrix} 0.2154 & -0.3190 \\ -0.3190 & 0.4868 \end{bmatrix}$, $Q_1 = \begin{bmatrix} -0.0000 & -8.4303 \\ 8.4303 & 0.0000 \end{bmatrix}$, $\varepsilon_{11} = 4.0541$, $\varepsilon_{21} = 5.4083$, $\alpha_1 = 8.6397$, $\theta_1 = 4.4446$, $Q_2 = \begin{bmatrix} -0.0000 & -3.2770 \\ 3.2770 & -0.0000 \end{bmatrix}$, $\varepsilon_{12} = 2.7728$, $\varepsilon_{22} = 4.6933$, $\alpha_2 = 2.4696$, $\theta_2 = 4.5861$.

Therefore, we can get that

$$S = B^T P = \begin{bmatrix} 0.4266 & -0.6463 \end{bmatrix}$$

By Theorem 2 we can obtain that the fault tolerant sliding mode control law of the system is

$$\begin{aligned} u_1(t) = & -\begin{bmatrix} -0.1908 & 0.0046 \end{bmatrix} x(t) - \begin{bmatrix} -0.5636 & 0.5214 \end{bmatrix} x(t) \\ & -\begin{bmatrix} -0.0203 & -0.0227 \end{bmatrix} x(t-2) - \begin{bmatrix} 0.02 + 36.7647 \cdot 0.03 \cdot |\sin(\mathrm{t})| \end{bmatrix} \mathrm{sgn}(s(t))) \\ u_2(t) = & -\begin{bmatrix} -0.3443 & 0.0278 \end{bmatrix} x(t) - \begin{bmatrix} -0.5892 & 0.3214 \end{bmatrix} x(t) \\ & -\begin{bmatrix} -0.0328 & -0.0203 \end{bmatrix} x(t-2) - \begin{bmatrix} 0.01 + 14.9477 \cdot 0.03 \cdot |\sin(\mathrm{t})| \end{bmatrix} \mathrm{sgn}(s(t))) \end{aligned}$$

Suppose the initial state of the system is $x(0) = [1 \quad -5]^{\mathrm{T}}$. Use Simulink to make simulation. Figures 4-1, 4-2, 4-3, 4-4, 4-5, 4-6 are the simulation figure of the system. It can be seen from the simulation figures that when the system is in the case of actuator fault, the designed sliding mode fault tolerant control law can overcome the external interference and the uncertainty of the system, and it can run normally. Meanwhile, the sliding surface can reach and remain on $s = 0$ in limited time.

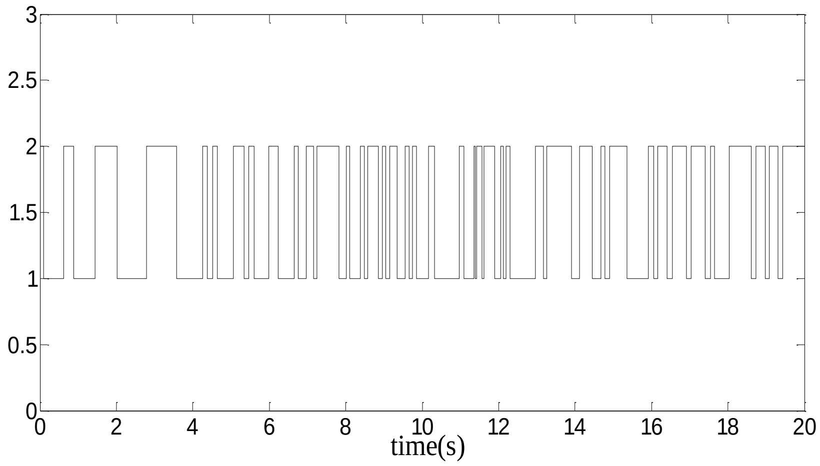

Figure 4-1. The Switching Signal

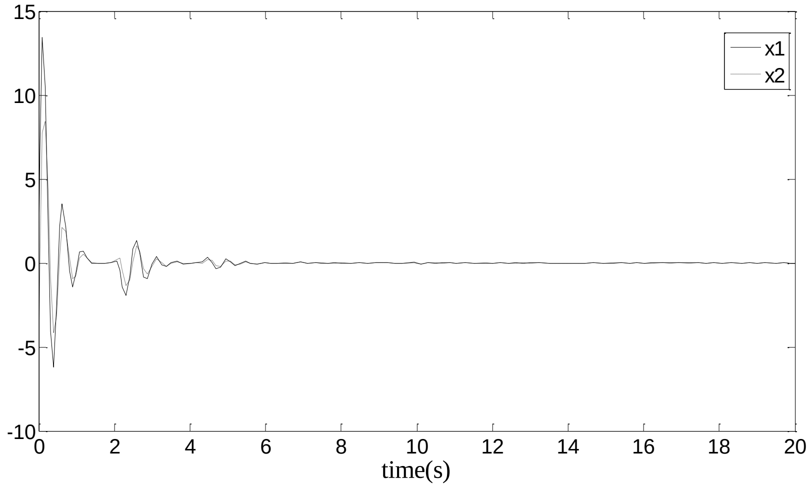

Figure 4-2. The System State Trajectories ($\Gamma_1 = 0.1$, $\Gamma_2 = 0.85$)

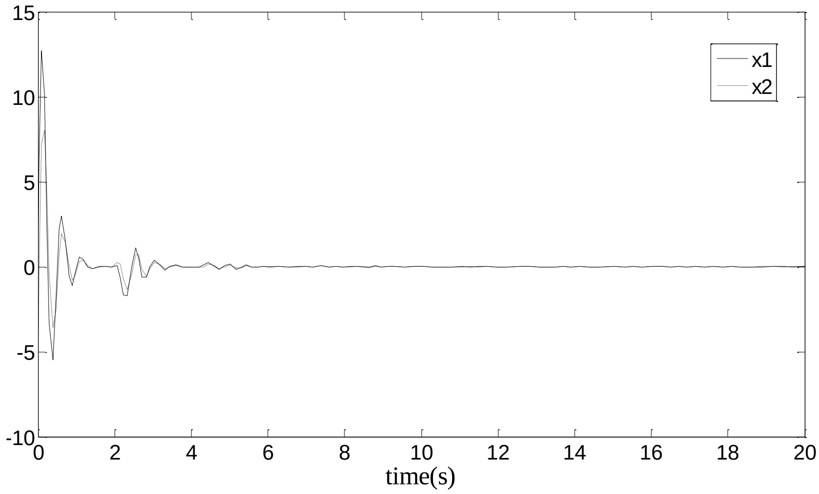

Figure 4-3. The System State Trajectories ($\Gamma_1 = 0.9$, $\Gamma_2 = 0.75$)

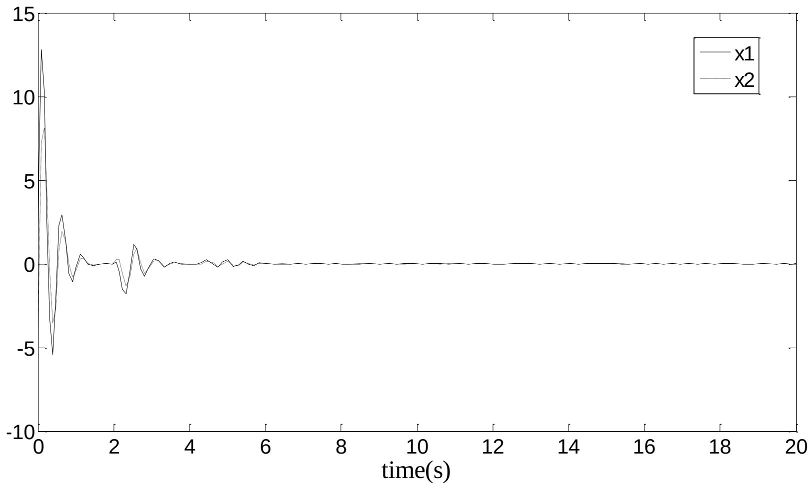

Figure 4-4. The System State Trajectories ($\Gamma_1 = 0.8, \Gamma_2 = 0.08$)

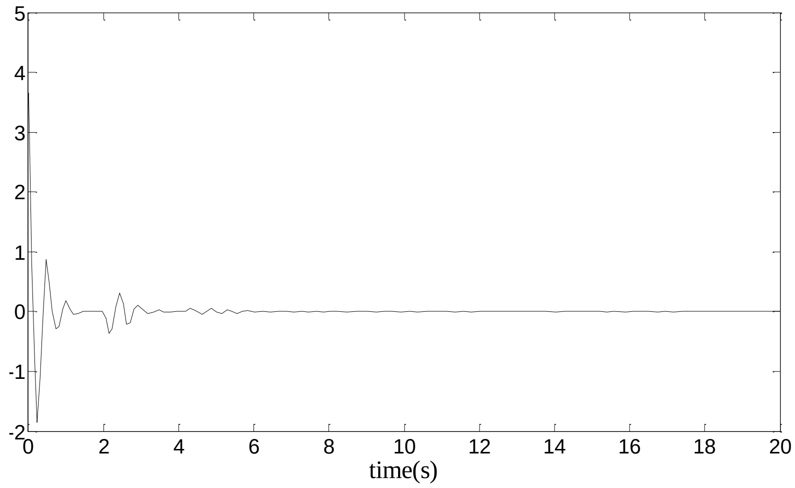

Figure 4-5. The Trajectory of the Sliding Function s

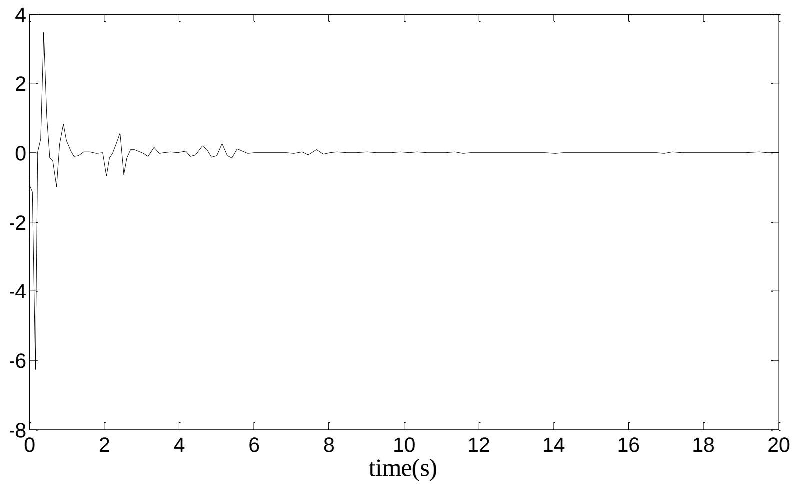

Figure 4-6. The Control Input Signal

5. Conclusion

In this paper the scheme of sliding surface and sliding mode control law for uncertain time-delay switched system in the case of actuator faults was introduced. First, actuator failure model was constructed, and then based on multiple Lyapunov function theory and linear matrix inequalities, the sufficiently conditions to ensure the stability of the system was introduced. and a common sliding surface which can ensure the system is asymptotically stable under the sliding surface was constructed. Then the designed sliding mode fault-tolerant control law to ensure the system state trajectory can reach a predetermined sliding surface in limited time and can remain on it was proved. The simulation examples illustrate the system can still hold its performance very well in case of actuator failures.

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