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Whole-Equivalent-Source Method for Electromagnetic Wave Scattering on Hybrid Impedance Plane

Tao Luo[1, 2] and Binhao Jiang[1]

[1.] *School of Electrical Engineering and Automation, Harbin Institute of Technology, Harbin 150001, China;*
[2.] *School of Physics, Changchun Normal University, Changchun 130032, China;*
jiangbh@hit.edu.cn

Abstract

To explore the analytical solution of electromagnetic scattered field for hybrid impedance plane, we studied a whole-equivalent-source method, and got the analytical solution of scattered field for two-part hybrid impedance plane with arbitrarily polarized plane wave as incident field. According to the corresponding relationship in singularity between mathematics and physics, the diffracted field expression of the impedance boundary line was separated from the reflected field generated by uniform impedance surface, and the diffraction effects was divided into first-order, second-order, third-order and higher-order diffraction effects respectively. By numerical calculation, the amplitude of electric field x-component of scattered field was studied with respect to observation position.

Keywords*: Electromagnetic scattering; Hybrid impedance plane; Impedance boundary condition; Equivalent source*

1. Introduction

Many important electromagnetic phenomena are caused by impedance mutation of electromagnetic media, the impedance boundary condition can easily describe the external macroscopic characteristics and the important relationship between the electric and magnetic fields outside the medium surface[1-2]. The electromagnetic scattering characteristics of hybrid impedance plane are studied with strong engineering requirements in aviation, maritime, applied geophysics, remote sensing, radar and other fields [3-5]. It is an important foundational research, to establish an analysis model for electromagnetic scattering of hybrid impedance plane, to explore the analytical solution of scattered field with physical meaning in electromagnetic theory and engineering[6-8].

For electromagnetic scattering problems of non-uniform impedance surface, Maliuzhinets [9] method and Wiener-Hopf[10] method are the earliest two research methods; there are also some other methods, such as mixed path theory[11], perturbation method[12], the aperture field method[13] and method of separation of variables[14], in case of plane wave or dipole source as incident field. But there are some electromagnetic scattering problems that cannot be solved at present, which restricted the all-around and deep research of the related issues to some extent. People are looking for more rational and convenient analysis method, which based on novel physical ideas.

In this paper, we studied the electromagnetic scattering characteristics of two-part impedance plane using equivalent current (charge) and magnetic flux (magnetic charge) distribution. We considered not only diffracting effect of the impedance boundary line for the rear fields, but also for the front fields, so that the scattered field expression we got is more accurate. We gave the scattered field expression in two parts, one part is the

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reflected field generated by uniform impedance surface, and another part is the diffracted field of equivalent electric and magnetic sources on boundary line.

The outline of this paper is as follows. In Section 2, the two-part impedance plane and incident field are depicted in detail. In Section 3, we derive the solution of the scattered field based on equivalent current. In Section 4, through numerical calculation for the scattered field, we analyzed the relationship between scattered fields and spatial angle or distance. In the last section, the conclusion is drawn.

2. Research Questions

The problem we considered here is the electromagnetic wave scattering by a two-part impedance plane. The Cartesian coordinate system (x, y, z) is shown in Figure 1, the upper half-space ($z > 0$) is air, the lower half-space ($z < 0$) is medium. The impedance plane $z = 0$ is divided into two parts by x-axis, and the relative impedance of left half-plane ($y<0, z=0$) is η_1, the relative impedance of right half-plane($y>0, z=0$) is η_2, each half-plane is homogeneous and isotropic. In practical problems, the impedance near the separation boundary of the two-part medium vary with gradient; we consider the case that the gradient is very large, and the width of separation boundary is very narrow compare with the wavelength of incident field.

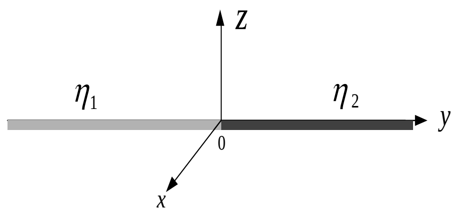


Figure 1. Geometry of Two-Part Impedance Plane

The impedance surface can be modeled by the Leontovich boundary condition [2]

$$\hat{z} \times (\hat{z} \times E) = -\eta_0 \eta (\hat{z} \times H) \tag{1}$$

Where E and H are the electric and magnetic fields in the $z \geq 0$ half-space respectively; the constant η_0 is the free space intrinsic impedance, η is the relative impedance of the surface medium. For analysis purpose, it's convenient to introduce the unit step function $u(y)$ to express the surface impedance of the two-part hybrid impedance plane. We modified the definition of the step function as bellow for analysis purposes

$$u(y)=1(y>0), \frac{1}{2}(y=0), 0(y<0). \tag{2}$$

It's obvious that we didn't change the derivative of the step function, so the impedance of two-part hybrid impedance plane can be expressed in terms of $u(y)$

$$\eta_d(y)=\eta_1+(\eta_2-\eta_1)u(y) \tag{3}$$

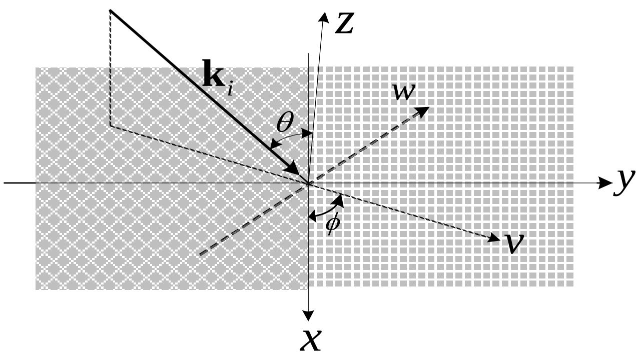


Figure 2. The Orientation Diagram of Arbitrary Polarized Plane Wave Incident on Two-Part Impedance Plane

Since any electromagnetic wave can be expressed in plane wave, it has a generality to take plane wave as incident field in our research question. The incident field assumed to be a plane wave of arbitrary polarization, obliquely incident to the x-axis as depicted in Figure 2, which can be written as [15,16]

$$E_i = \left[a^h \hat{w} + a^e \left(\hat{k}_i \times \hat{w}\right)\right] \cdot \exp\left[-ik_0\left(v\sin\theta - z\cos\theta\right)\right] \tag{4a}$$

$$H_i = \frac{1}{\eta_0}\left[a^h \left(\hat{k}_i \times \hat{w}\right) - a^e \hat{w}\right] \cdot \exp\left[-ik_0\left(v\sin\theta - z\cos\theta\right)\right] \tag{4b}$$

Where k_0 is the free space wavenumber, $\hat{k}_i$ is the wave vector, a^e and a^h are the magnitude of transverse electric (TE) wave and transverse magnetic (TM) wave of incident field respectively; where notation ϕ is the angle between $\hat{k}_i$ and the x-axis, θ is the angle between $\hat{k}_i$ and the z-axis. It is noted that all the fields in the following discussion have the $e^{i\omega t}$ time dependence.

3. Methods and Results

The total electromagnetic field $(\vec{\mathbf{E}},\vec{\mathbf{H}})$ in the half-space $z \geq 0$ satisfies the source-free Maxwell's equations

$$\nabla \times \vec{H} = \mathrm{i}\,\omega\varepsilon_0 \vec{E}, \quad \nabla \times \vec{E} = -\mathrm{i}\,\omega\mu_0 \vec{H} \tag{5a}$$

$$\mu_0 \nabla \cdot \vec{H} = 0, \quad \varepsilon_0 \nabla \cdot \vec{E} = 0 \tag{5b}$$

The field $(\vec{\mathbf{E}},\vec{\mathbf{H}})$ also satisfies the impedance boundary condition, that is

$$\hat{z} \times \left(\hat{z} \times \vec{E}\right) = -\eta_0 \eta_d\left(y\right)\left(\hat{z} \times \vec{H}\right) \tag{6}$$

It's difficult to get the solution of field $(\vec{\mathbf{E}},\vec{\mathbf{H}})$ directly in mathematics from equations (5) and (6). We should consider the scattered field of uniform impedance half-plane, also need to consider the diffracted field of impedance boundary line. According to the linear property, total electromagnetic field can be decomposed into two components as $(\vec{E}_1,\vec{H}_1)$ and $(\vec{E}_2,\vec{H}_2)$, namely $\vec{E} = \vec{E}_1 + \vec{E}_2$, $\vec{H} = \vec{H}_1 + \vec{H}_2$.

Let's introduce the equivalent sources (current $\vec{J}_e$, charge ρ_e, magnetic flux $\vec{J}_m$ and magnetic charge ρ_m), and insert pairs of negative and positive equivalent sources into the above Maxwell's equations (5) and the boundary conditions (6). Clearly the introduction of equivalent sources will not change the boundary value problem determined by the Maxwell equations (5) and the boundary conditions (6).

We can separate $(\vec{E}_1, \vec{H}_1)$ and $(\vec{E}_2, \vec{H}_2)$ from equations (5) and (6) by substituting equivalent sources, so we get the equations for field's component $(\vec{E}_1, \vec{H}_1)$, written as

$$\nabla \times \vec{H}_1 = i\omega\varepsilon_0\vec{E}_1 - \vec{J}_e \quad \nabla \times \vec{E}_1 = -i\omega\mu_0\vec{H}_1 + \vec{J}_m \tag{7a}$$

$$\mu_0\nabla \cdot \vec{H}_1 = -\rho_m \qquad \varepsilon_0\nabla \cdot \vec{E}_1 = -\rho_e \tag{7b}$$

$$\hat{z} \times \left(\hat{z} \times \vec{E}_1\right) = -\eta_0\eta_d \left(\hat{z} \times \vec{H}_1\right) \tag{7c}$$

and another component $(\vec{E}_2, \vec{H}_2)$ satisfy equations as

$$\nabla \times \vec{H}_2 = i\omega\varepsilon_0\vec{E}_2 + \vec{J}_e \qquad \nabla \times \vec{E}_2 = -i\omega\mu_0\vec{H}_2 - \vec{J}_m \tag{8a}$$

$$\mu_0\nabla \cdot \vec{H}_2 = \rho_m, \quad \varepsilon_0\nabla \cdot \vec{E}_2 = \rho_e \tag{8b}$$

$$\hat{z} \times \vec{E}_2 = 0, \quad \hat{z} \times \vec{H}_2 = 0 \tag{8c}$$

First, let's see equations (7). For field component $(\vec{E}_1, \vec{H}_1)$ are functions of the impedance η_d, and η_d is the function of the unit step function $\mathrm{u}(\mathrm{y})$, the differential operator ∇ can be written in two parts, namely

$$\nabla = \nabla_p + \vec{\mathrm{T}}, \qquad \nabla_p = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \tag{9a}$$

$$\vec{\mathrm{T}} = \hat{y}\left(\eta_2 - \eta_1\right)\delta\left(y\right)\frac{\partial}{\partial \xi}, \qquad \xi = \left(\eta_2 + \eta_1\right)/2 \tag{9b}$$

Part one ∇_{p} is just come from the differential coordinate variables, part two $\vec{\mathrm{T}}$ is from the differential impedance only, $\delta\left(y\right)$ is Dirac delta function. We substitute (9) into (7 a, b) to obtain

$$\nabla_p \times \vec{H}_1 + \vec{\mathrm{T}} \times \vec{H}_1 = i\omega\varepsilon_0\vec{E}_1 - \vec{J}_e \tag{10a}$$

$$\nabla_p \times \vec{E}_1 + \vec{\mathrm{T}} \times \vec{E}_1 = -i\omega\mu_0\vec{H}_1 + \vec{J}_m \tag{10b}$$

$$\mu_0\nabla_p \cdot \vec{H}_1 + \mu_0\vec{\mathrm{T}} \cdot \vec{H}_1 = -\rho_m \tag{10c}$$

$$\varepsilon_0\nabla_p \cdot \vec{E}_1 + \varepsilon_0\vec{\mathrm{T}} \cdot \vec{E}_1 = -\rho_e \tag{10d}$$

According to the corresponding relationship in singularity between mathematics and physics, the equivalent sources can be obtained

$$\vec{J}_e = -\vec{\mathrm{T}} \times \vec{H}_1, \quad \vec{J}_m = \vec{\mathrm{T}} \times \vec{E}_1 \tag{11a}$$

$$\rho_m = -\mu_0\vec{\mathrm{T}} \cdot \vec{H}_1, \quad \rho_e = -\varepsilon_0\vec{\mathrm{T}} \cdot \vec{E}_1 \tag{11b}$$

Obviously, when $\eta_2 = \eta_1$, the above equivalent sources are zero. And field components $(\vec{E}_1, \vec{H}_1)$ should be the scattered field of homogeneous impedance plane with the incident plane wave described as (4), which can be written as[15]

$$\begin{aligned}\vec{E}\left(\eta_c\right) = &\left[a^h R^h \hat{w} + a^e R^e \left(\hat{k}_r \times \hat{w}\right)\right] \\ &\cdot \exp\left[-ik_0\left(\left(x\cos\phi + y\sin\phi\right)\sin\theta + z\cos\theta\right)\right]\end{aligned} \tag{12a}$$

$$\vec{H}(\eta_c) = \frac{1}{\eta_0}\left[a^h R^h\left(\hat{k}_r \times \hat{w}\right) - a^e R^e \hat{w}\right] \cdot \exp\left[-ik_0\left(\left(x\cos\phi + y\sin\phi\right)\sin\theta + z\cos\theta\right)\right] \tag{12b}$$

which $\hat{k}_r$ is the wave vector of reflected fields, and $R^h = \left(\eta_c\cos\theta - 1\right)/\left(\eta_c\cos\theta + 1\right)$, $R^e = \left(\cos\theta - \eta_c\right)/\left(\cos\theta + \eta_c\right)$.

Then $(\vec{E}_1, \vec{H}_1)$ can be expressed as

$$\left(\vec{E}_1, \vec{H}_1\right) = \begin{cases} \left[\vec{E}(\eta_c), \vec{H}(\eta_c)\right]\Big|_{\eta_c=\eta_1} & y<0, \\ \left[\vec{E}(\eta_c), \vec{H}(\eta_c)\right]\Big|_{\eta_c=(\eta_2+\eta_1)/2} & y=0. \\ \left[\vec{E}(\eta_c), \vec{H}(\eta_c)\right]\Big|_{\eta_c=\eta_2} & y>0, \end{cases} \tag{13}$$

So $(\vec{E}_1, \vec{H}_1)$ represent the field produced by uniform impedance surface, $(\vec{E}_2, \vec{H}_2)$ behalf of the diffraction field caused by the impedance boundary line. Then we can use equivalent sources in above formula (11) to obtain the diffracted field from equations (8).

After doing some substitution treatment for (8a, b), we can obtain equations about $(\vec{E}_2, \vec{H}_2)$

$$\nabla\times\nabla\times\vec{E}_2 - k_0^2\vec{E}_2 = -i\omega\mu_0\vec{J}_e - \nabla\times\vec{J}_m \tag{14a}$$

$$\nabla\times\nabla\times\vec{H}_2 - k_0^2\vec{H}_2 = -i\omega\varepsilon_0\vec{J}_m + \nabla\times\vec{J}_e \tag{14b}$$

To solve these equations, we introduce the following dyadic Green function [16]

$$\vec{\vec{G}}(r,r') = \left(\vec{\vec{I}} + \frac{1}{k_0^2}\nabla\nabla\right)\cdot\left[g(r_-)\vec{\vec{I}} + g(r_+)\vec{\vec{I}}_r\right] \tag{15a}$$

where

$$\vec{\vec{I}} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}, \quad \vec{\vec{I}}_r = \hat{z}\hat{z} - \hat{x}\hat{x} - \hat{y}\hat{y}, \quad g(r_\pm) = \frac{e^{-ik_0 r_\pm}}{4\pi r_\pm} \tag{15b}$$

$$r_\pm = \left[\rho^2 + \left(z \pm z'\right)^2\right]^{1/2}, \quad \rho = \left[\left(x - x'\right)^2 + \left(y - y'\right)^2\right]^{1/2} \tag{15c}$$

$\vec{\vec{G}}(r,r')$ satisfies following equations

$$\nabla\times\nabla\times\vec{\vec{G}}(r,r') - k_0^2\vec{\vec{G}}(r,r') = \vec{\vec{I}}\,\delta(r - r') \tag{16a}$$

$$\left[\hat{z}\times\vec{\vec{G}}(r,r')\right]\Big|_{z=0} = 0 \tag{16b}$$

Then using the above dyadic Green's function $\vec{\vec{G}}(r,r')$ and vector Green's theorem, the integral solutions of (14) can be obtained

$$\vec{E}_2(r) = -\int_0^\infty dz'\int_{-\infty}^\infty \vec{\vec{G}}(r,r')\cdot\begin{bmatrix} i\omega\mu_0\vec{J}_e(x',z') \\ +\nabla'\times\vec{J}_m(x',z') \end{bmatrix} dx' \tag{17a}$$

$$\vec{H}_2(r) = -\int_0^\infty dz'\int_{-\infty}^\infty \vec{\vec{G}}(r,r')\cdot\begin{bmatrix} i\omega\varepsilon_0\vec{J}_m(x',z') \\ -\nabla'\times\vec{J}_e(x',z') \end{bmatrix} dx' \tag{17b}$$

We can see that these solutions satisfy the relevant boundary condition (8c) also. At this time, we have got the full expression of the diffraction field for two-part hybrid impedance plane.

In many cases, we only need one of the electric components of scattering field [17-18]; and the amplitude of electric field x-component can be expressed as

$$E_{\mathbf{x}} = E_{\mathbf{1x}} + E_{\mathbf{2x}} \tag{18}$$

and E_{2x} can be given as

$$E_{\mathbf{2x}} = E_{2x}^{(1)} + E_{2x}^{(2)} + E_{2x}^{(3)} + E_{2x}^{(4)} \tag{19a}$$

where

$$E_{2x}^{(1)} = -\int_0^\infty \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}\,x' \phi(\mathbf{r}') \left[\mathrm{i}\,\omega\mu_0 \Gamma'_{e,x} G_- \right]_{(y'=0)} \tag{19b}$$

$$E_{2x}^{(2)} = \int_0^\infty \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}\,x' \phi(\mathbf{r}') \left(\Gamma'_{m,z} \frac{\partial}{\partial y'} G_- \right)_{(y'=0)} \tag{19c}$$

$$E_{2x}^{(3)} = -\int_0^\infty \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}\,x' \frac{\phi(\mathbf{r}')}{k_0^2} \begin{bmatrix} \mathrm{i}\,\omega\mu_0 \Gamma'_{e,x} \dfrac{\partial^2}{\partial x \partial x} G_- \\ +\mathrm{i}\,\omega\mu_0 \Gamma'_{e,z} \dfrac{\partial^2}{\partial x \partial z} G_+ \\ + \left(\dfrac{\partial}{\partial z'} \Gamma'_{m,x} - \dfrac{\partial}{\partial x'} \Gamma'_{m,z} \right) \dfrac{\partial^2}{\partial x \partial y} G_- \end{bmatrix}_{(y'=0)} \tag{19d}$$

$$E_{2x}^{(4)} = \int_0^\infty \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}\,x' \frac{\phi(\mathbf{r}')}{k_0^2} \begin{pmatrix} -\Gamma'_{m,x} \left(\dfrac{\partial^2}{\partial x \partial z} \dfrac{\partial}{\partial y'} G_+ \right) \\ +\Gamma'_{m,z} \left(\dfrac{\partial^2}{\partial x \partial x} \dfrac{\partial}{\partial y'} G_- \right) \end{pmatrix}_{(y'=0)} \tag{19e}$$

$$\Gamma_{e,x} = -\frac{2}{\eta_0} \frac{a_e sin\theta cos\theta}{(\eta_d cos\theta + 1.0)^2} \tag{19f}$$

$$\Gamma_{m,z} = 2 \left(\frac{a_h \sin\phi \cos\theta}{\left(\eta_d \cos\theta + 1\right)^2} - \frac{a_e \cos^2\theta \cos\phi}{\left(\cos\theta + \eta_d\right)^2} \right) \tag{19g}$$

$$\Gamma_{m,x} = \frac{-2a_e \sin\phi \cos\theta}{\left(\cos\theta + \eta_d\right)^2} \tag{19h}$$

$$G_{\pm} = g_{(r_-)} \pm g_{(r_+)} \tag{19i}$$

$$\phi(\mathbf{r}') = \left(\eta_2 - \eta_1\right) \mathrm{e}^{-\mathrm{i}k_0 \left(x' \sin\theta \cos\phi + y' \sin\theta \sin\phi + z' \cos\theta\right)} \tag{19j}$$

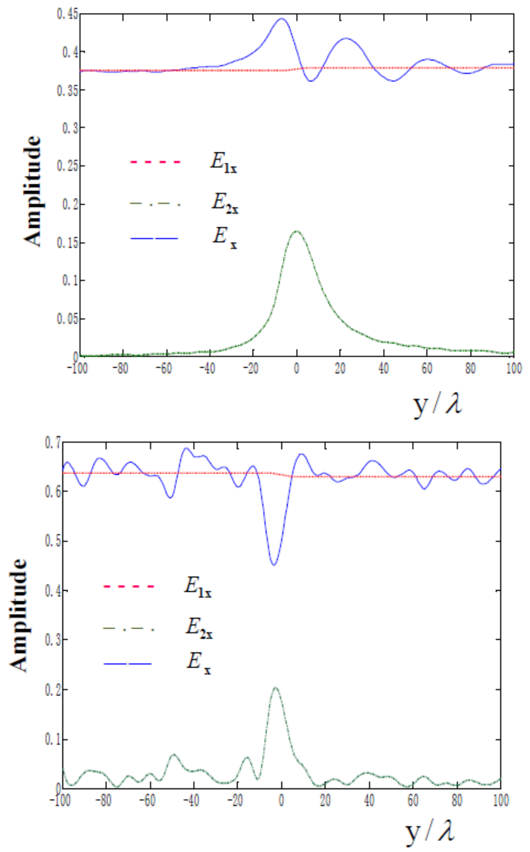
Where $E_{2x}^{(1)}$, $E_{2x}^{(2)}$, $E_{2x}^{(3)}$ and $E_{2x}^{(4)}$ represent the first-order diffraction effects, second-order diffraction effects, third-order diffraction effects and higher-order diffraction effects respectively.

4. Numerical Calculation and Analysis

Through numerical calculation with adaptive Simpson quadrature, E_{1x}, E_{2x}, and E_x were obtained, while the incident field is an obliquely incident plane wave ($\phi=40^0, \theta=60^o$), and the impedance are taken as $\eta_1 = 0.01 + 0.5i$, $\eta_2 = 4\eta_1$.

Due to electric field intensity has nothing to do with the y axis, which is about the x-z plane translational symmetry, so we are currently analyzing the change law of electric field intensity on x-z plane representatively. What depicted in Figure 3 is the relationship between the amplitude of electric field x-component and the coordinate y of observation point, and $x = 0,\ z = 1.0\lambda$, (a) is for the case of TE wave ($a^e = 1, a^h = 0$) incident and (b) is for TM wave ($a^e = 0, a^h = 1$) incident, wherein the red dotted line corresponds to uniform impedance surface, the green solid- dotted line corresponds to the impedance discontinuity surface, and the blue solid line corresponds to the total fields.

The Figure 3 shows that the diffracting effect fluctuates with the distance from the boundary line. For $\mathbf{E}_{2x}$, the maximum appears near the boundary line which $y=0$, and decreases with increasing distance quickly. But for $\mathbf{E}_x$, the amplitude of total electric field fluctuates with $\mathbf{E}_{1x}$, the maximum is not appears at the boundary line($y=0$) but for minimum, and finally approaching scattering field of uniform impedance surface. The plane waves with different incident polarization state, i.e. transverse electric(TE) and transverse magnetic(TM), diffraction effects will be different, for TE the diffraction effects can be maintained to the very far distance, attenuation is not obvious; For TM, the attenuation of diffraction effects increases significantly with distance until disappear. Also we can see that $\mathbf{E}_{1x}$ is different with the same electric field intensity of incident wave.



(a) (b)

Figure 3. The Relationship Between the Amplitude of Electric Field's X-Component aand Y-Distance, $x=0,z=1.0\lambda$.(A) TE Plane Wave Incidence ($a^e = 1, a^h = 0$), (B)TM Plane Wave Incidence ($a^e = 0, a^h = 1$)

We study the relationship between the amplitude of electric field x-component and the orientation angle α, while $x=0,\rho=20\lambda$. The Figure 4 shows that $\mathbf{E}_x$ vary greatly with α.

The polarization state of incident plane wave, *i.e.* transverse electric or transverse magnetic wave, will decidedly change the diffraction effects. Since the space angle $\alpha = 90^o$ corresponding observation point on the boundary line, the field intensity $\mathbf{E}_{2x}$ oscillate obviously. When the incident field is TE wave, the maximum of $\mathbf{E}_{2x}$ is at left half-plan, but for $\mathbf{E}_x$ the minimum appear at the same point; while TM wave incident, the maximum of $\mathbf{E}_{2x}$ and $\mathbf{E}_x$ are at right half-plan together. The scattered field decrease with the space angle with different model.

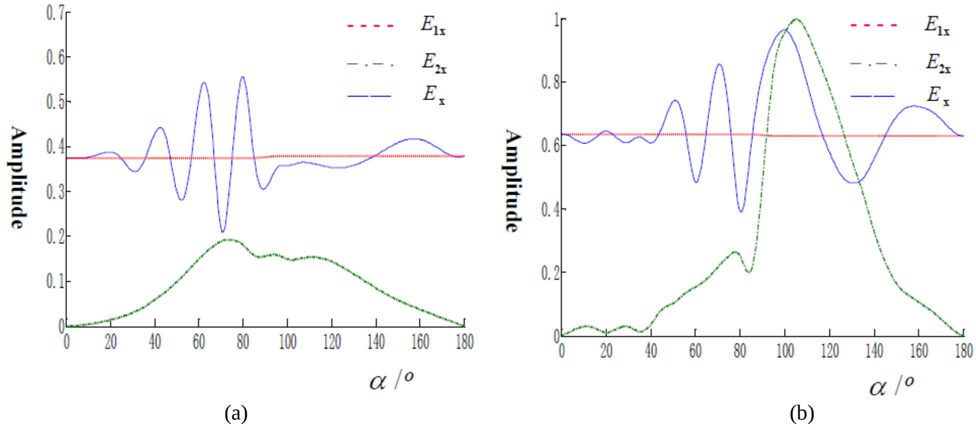


Figure 4. The Relationship Between the Amplitude of Electric Field's X-Component and the Orientation, $x = 0, \rho = 20\lambda$ **.(A) TE Plane Wave Incidence (** $a^e = 1, a^h = 0$ **), (B)TM Plane Wave Incidence (** $a^e = 0, a^h = 1$ **)**

5. Conclusion

In the case of electromagnetic scattering by hybrid impedance plane, we can use the equivalent sources to analyze the scattered field, which can be decomposed into the reflected field generated by uniform impedance surface, and the scattering field of equivalent electric and magnetic sources on separation boundary. Using the whole-equivalent-sources method to analyze the scattered field of hybrid impedance plane, we considered not only diffracting effect of the impedance boundary line for the rear fields, but also for the front fields, so that the scattered field expression we got is more accurate.

The diffracting effect fluctuates with the distance from the boundary line, and finally approaching scattering field of uniform impedance surface when the distance is about 100 wavenumber of incident wave. The maximum or minimum of the scattered field appear at the boundary line. The incident plane waves can be different polarization state, and the diffraction effects will be different. Since any electromagnetic wave can be expressed in plane wave, this method can be used to other incident field.

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Authors

T. Luo, he was born in JIANG Xi, China, in 1977. He received the M.S. degrees in Optics from the University of JILIN, China, in 2006. He is currently pursuing the Ph.D. degree in electrical engineering from Harbin institute of technology, Harbin.

Since 2011, he has been an Assistant Professor with the Physics Department, CHANGCHUN Normal University. His research interests include fiber laser and electromagnetic theory and applications. He is the author of two books, more than 10 articles, and one invention.

B. H. Jiang, he was born in CHANGCHUN, China, in 1959. He received the B.S. and M.S. degrees in applied geophysics from the JILIN University, Changchun, in 1983 and 1990, and the Ph.D. degree in electrical engineering from Harbin institute of technology, Harbin, in 2007.

From 1990 to now, he was a teacher with Harbin institute of technology. Since 2009, he has been a Professor with School of Electrical Engineering and Automation, Harbin Institute of Technology. He is the author of three books, more than 30 articles, and 5 inventions. His research interests include electromagnetic control to plasma, electromagnetic compatibility, electric propulsion technology, high power impulse technique, and electromagnetic theory and applications. He is an Associate Editor of Chinese Journal of radio science, and holds three patents.

