

On the Active Disturbance Rejection of SWATH Ship based on Terminal Sliding Mode Control

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Abstract

For the motion control problem of a SWATH ship, a terminal sliding mode controller is designed based on extended state observer (ESO). Firstly, considering the motion control problem under random wave disturbances, the nonlinear model of SWATH ship with compound disturbances is established. Then, considering the parameters perturbation, complexity and randomness of the wave, the control system is divided into inner loop observer and outer loop controller respectively. An ESO is implemented to estimate and compensate the compound disturbances in the terminal sliding mode controller. At last, the asymptotical stability is achieved based on Lyapunov theory. Simulations show the effectiveness of the proposed method. Linear ESO can estimate the compound disturbances successfully, meanwhile, the terminal sliding mode controller can enable the system states converge quickly.

Keywords: *Small waterplane area twin hull (SWATH), motion control, extended state observer (ESO), terminal sliding control*

1. Introduction

Small waterplane area twin hull (SWATH) is a kind of catamaran. Since its displacement volumes are mainly underwater, the SWATH has very good performance on superior sea keeping. Consequently, in recently years, the applications of SWATH have attracted much attention in both military and marine research. However, according to the special characteristic, the SWATH has a large tendency to pitch and heave motions which may affect the performance during the sailing. Hence, fins is needed to ensure the stability of the SWATH motions [1, 2].

During the sailing, the SWATH may be influenced by the parameters perturbation, attitude variation during motions, wave disturbances, etc. These unmeasured uncertainties, which may decrease the control accuracy or even make the system unstable, largely limited the application of SWATH [3]. Most previous researches on SWATH are mainly carried out based on nominal parameters, whereas the influence on control performance caused by system uncertainties is usually neglected [1-4]. Several strategies have been investigated for system uncertainties, such as adaptive control [5], neural networks [6], fuzzy systems [7], and nonlinear disturbance observer [8]. However, the analysis and description of all the strategies mentioned above are based on time domain, whereas the frequency domain performance is rarely concerned. For the practical applications, it is very hard for

these methods to be used directly according to the measurement noise and vibration. The active disturbance rejection (ADRC) is an effective way in solving the control problem with system uncertainties [9]. The extended state observer (ESO) is the most important part in ADRC. It can estimate and compensate the influence caused by system uncertainties, which has been widely used in motion control area [10, 11].

From the analysis above, we combine the ESO and terminal sliding mode control (SMC) [12] technique together to dealing with the system uncertainties in the motion control of SWATH. First, aiming at the SWATH motion control problem with wave disturbances, the nonlinear system model is established with composite disturbances. Then, considering the complexity and stochastic of the wave disturbances, parameters perturbation, the control system is divided into the inner loop observer and outer loop controller, respectively. For the inner loop, a linear ESO is applied to estimate the composite disturbances. Then, with the estimation of disturbance, the terminal sliding mode technique is employed as the outer loop for desired control performance. The Lyapunov theory is applied to analyze the stability of the closed-loop system.

This paper is organized as follows. In Section II, the system model of SWATH with composite disturbances is established. In Section III, the terminal SMC scheme with ESO is proposed. In Section IV, stability of the closed-loop system is analyzed based on Lyapunov theorem. Simulations are carried to show the effectiveness of the proposed control scheme in Section V, followed by Conclusion in Section VI.

2. System Model

As depicted in Figure 1, the right hand coordinate system with z axis upwards is applied in the motion control of SWATH. The direction around x and y axes are defined as the pitch and heave motion of the SWATH.

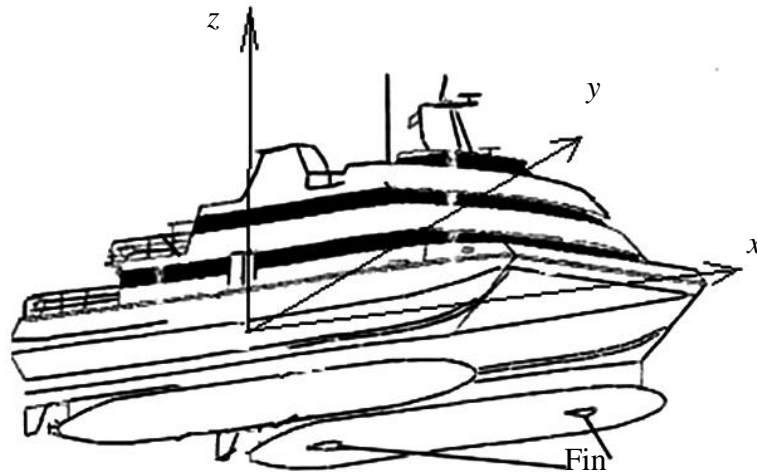


Figure 1. The Principle of the SWATH

From the Newton method, the dynamics of the SWATH is given as

$$\begin{cases} (M + A_{33}(t))\ddot{\xi}_3 + B_{33}(t)\dot{\xi}_3 + C_{33}(t)\xi_3 + \\ A_{35}(t)\ddot{\xi}_5 + B_{35}(t)\dot{\xi}_5 + C_{35}(t)\xi_5 = F_3 + F_{fin}(\xi) \\ (I_5 + A_{55}(t))\ddot{\xi}_5 + B_{55}(t)\dot{\xi}_5 + C_{55}(t)\xi_5 + \\ A_{53}(t)\ddot{\xi}_3 + B_{53}(t)\dot{\xi}_3 + C_{53}(t)\xi_3 = F_5 + M_{fin}(\xi) \end{cases} \quad (1)$$

where the subscript 3 and 5 represent the pitch and heave motion, respectively, M is the mass of the ship, A_{ik} , B_{ik} and C_{ik} ($i, k=3,5$) are the generalized mass, damping coefficient, and restoring force of the SWATH. F_3 and F_5 are the disturbance force and torque of the ship. $F_{fin}(\xi)$ and $M_{fin}(\xi)$ are the control thrust and torque of the fins. It is clear that the dynamics in Eq. (1) is a MIMO system with coupling property.

Generally, the thrust and torque can be obtained as

$$\begin{cases} L = \frac{1}{2} \rho U^2 S C_L \alpha(\xi) \\ M = \frac{1}{2} \rho U^2 S C_L \alpha(\xi) \cdot L_p \end{cases} \quad (2)$$

where ρ is the density of the water, U is the velocity of the ship, S is the area of the fins, C_L is the lift coefficient of the fins, $\alpha(\xi)$ is the attack angle of the fins, L_p is the length of the center of gravity from fins to the ship.

By defining the generalized states of the system as: $\xi = [\xi_3 \quad \xi_5]^T$, then the dynamics of the ship can be rewritten as

$$A(\xi) \ddot{\xi} + B(\xi) \dot{\xi} + C(\xi) \xi = F + F_{fin} \quad (3)$$

where $F = [F_3 \quad F_5]^T$, $F_{fin} = [F_{fin}(\xi) \quad M_{fin}(\xi)]^T$,

$$A(\xi) = \begin{bmatrix} M + A_{33}(\xi) & A_{35}(\xi) \\ A_{53}(\xi) & I + A_{55}(\xi) \end{bmatrix}, \quad B(\xi) = \begin{bmatrix} B_{33}(\xi) & B_{35}(\xi) \\ B_{53}(\xi) & B_{55}(\xi) \end{bmatrix}, \quad C(\xi) = \begin{bmatrix} C_{33}(\xi) & C_{35}(\xi) \\ C_{53}(\xi) & C_{55}(\xi) \end{bmatrix}.$$

Here, take the parameters perturbation of the matrices of $A(\xi)$, $B(\xi)$ and $C(\xi)$ into consideration. Assume that the nominal value of the above three matrices are: $\Delta A(\xi) = A(\xi) - A_0(\xi)$, $\Delta B(\xi) = B(\xi) - B_0(\xi)$, $\Delta C(\xi) = C(\xi) - C_0(\xi)$, then we get the finally form of the system dynamics as

$$\ddot{\xi} = -A_0^{-1} B_0 \dot{\xi} - A_0^{-1} C_0 \xi + A_0^{-1} F_{fin} + F' \quad (4)$$

where F' is the composite disturbances caused by both internal uncertainties and external disturbances.

Assume that there is no hysteresis in the motors, such that the motor speed can achieve desired speed immediately. Hence, the trajectory tracking control of the quadrotor aircraft equals to design the controller T and τ to stabilize the system.

3. Control System Implementation

3.1. Control Objective

During the sailing of SWATH, the unknown internal uncertainties and random wave disturbances acting on the dynamics may decrease the control accuracy. In this work, the closed-loop control system is divided into the inner loop for the dynamics and outer loop for the whole control system, respectively. For the inner loop, an ESO is introduced to estimate the composite disturbance, and thus compensate in the controller. Then, for the overall control system, a terminal SMC scheme is then proposed to stabilize the closed-loop system. The sliding surface the corresponding reaching law are designed to increase the control accuracy of the SWATH.

3.2. Observer Configuration

The system uncertainties and the random wave disturbances acting on the dynamics will decrease the control accuracy, or even make the system unstable. By using the estimation of "external disturbances" and "internal disturbances" caused by controlled

objective online, the ESO can compensate the estimation in the controller. The existence of ESO can bring the system with strong disturbance rejection performance and robustness against system uncertainties.

Assuming the first order derivative of the composite exists, and then we get the following system model

$$\begin{cases} \dot{\xi} = -A_0^{-1}B_0\dot{\xi} - A_0^{-1}C_0\xi + A_0^{-1}F_{fm} + F' \\ \dot{F}' = h(t) \end{cases} \quad (5)$$

where $h(t)$ is the first order derivative of the composite disturbances.

The second order of a linear ESO is designed as

$$\begin{cases} \dot{\hat{z}}_1 = -A_0^{-1}B_0\dot{\xi} - A_0^{-1}C_0\xi + A_0^{-1}F_{fm} + \hat{z}_2 + 2\omega_0(\dot{\xi} - \hat{z}_1) \\ \dot{\hat{z}}_2 = \omega_0^2(\dot{\xi} - \hat{z}_1) \end{cases} \quad (6)$$

By using the Laplace transformation on Eq. (6), and then substitute Eq. (5) into Eq. (6), we have

$$\begin{cases} s\hat{z}_1 = (\hat{z}_2 - F') + s\dot{\xi} + 2\omega_0(\dot{\xi} - \hat{z}_1) \\ s\hat{z}_2 = \omega_0^2(\dot{\xi} - \hat{z}_1) \end{cases} \quad (7)$$

According to Eq. (7), the transfer function from the composite disturbances F' to its estimation \hat{z}_2 is given as

$$\hat{z}_2 = \frac{\omega_0^2}{s^2 + 2\omega_0s + \omega_0^2} F' \quad (8)$$

where ω_0 is cutoff frequency of the ESO. It is clear that the transfer function from the composite disturbances F' to its estimation \hat{z}_2 is a Hurwitz transfer function. That is, the estimation \hat{z}_2 can converge to the composite disturbances F' asymptotically. We also have

$$\lim_{\omega_0 \rightarrow \infty} \hat{z}_2 = \lim_{\omega_0 \rightarrow \infty} \frac{\omega_0^2}{s^2 + 2\omega_0s + \omega_0^2} F' = F' \quad (9)$$

Notice that the composite disturbances contain internal uncertainties caused by parameters perturbation and external disturbances caused by random wave, that is, with the estimation of \hat{z}_2 , the control system can eliminate the influence caused by both internal uncertainties and external disturbances.

3.3. Terminal SMC Strategy

The concept of SMC is to design the sliding surface based on low order model, and the reaching law is selected to force the system converge to the sliding surface in finite time. The sliding mode controller can maintain the system state on the sliding surface in the future moment. The SMC method is an effective way in dealing with the control problem such as stabilization, trajectory tracking, and robust control with model error and external disturbances. The switching function is applied in traditional SMC method to deal with uncertainties, which may bring the system with chattering phenomenon. However, the SMC control output is discrete, which is hard for implementation in a practical ship control system. Consequently, in this paper, an ESO is employed to eliminate the influence caused by composite disturbances instead of the switching function.

The sliding surface is defined as

$$s = \dot{\xi} + \Gamma \xi^{q/p} = 0 \quad (10)$$

where $s=[s_1 \ s_2]^T$, $\Gamma = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, λ_1 and λ_2 are positive constant. The parameters p and q are odd numbers that satisfy $q < p < 2q$. By introducing the nonlinear function, the tracking error can converge to 0 in finite time.

The exponentially reaching law is selected as

$$\dot{s} = -Ls \quad (11)$$

where $L = \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix}$, l_1 and l_2 are positive constant.

Finally, we get the terminal sliding mode controller is given as

$$F_{fin} = B_0 \dot{\xi} + C_0 \xi - A_0 \Gamma \xi^{q-p/p} \dot{\xi} - A_0 Ls - A_0^{-1} \hat{z}_2 \quad (12)$$

4. Stability Analysis

In this section, the stability of the overall closed-loop system is analyzed.

Theorem 1. Considering the dynamics of SWATH in Eq. (1), with the terminal sliding mode controller (12) and the ESO (9), the control error of closed-loop system is asymptotical stable.

Proof.

Define the control error of the SWATH as ξ . For the overall closed-loop control system, define the Lyapunov function as

$$V = \frac{1}{2} s^T s \quad (13)$$

Then, the first order derivative of the Lyapunov function is expressed as

$$\dot{V} = s^T \dot{s} = s^T (-A_0^{-1} B_0 \dot{\xi} - A_0^{-1} C_0 \xi + A_0^{-1} F_{fin} + F' + \Gamma \xi^{q-p/p} \dot{\xi}) \quad (14)$$

Define the estimation error of the ESO as: $\tilde{F} = F' - \hat{z}_2$, and substitute the controller (12) into Eq. (14), we have

$$\dot{V} = s^T \dot{s} = -s^T Ls + s^T \tilde{F} \quad (15)$$

Assuming the estimation error of ESO as the input of the closed-loop system, and then the unforced system is exponentially stable at the equilibrium point. Consequently, for the input \tilde{F} , the system is input to state stable. Considering that the estimation \hat{z}_2 can converge to the composite disturbances asymptotical. From Lemma 4.7 in [13], the system can converge to the sliding surface asymptotically. Then, according to the definition of the sliding surface, the overall closed-loop is globally uniformly stable.

5. Simulations and Analysis

5.1 Simulations of Wave Disturbances

The ITTC single wave parameter spectrum is applied to acquire the wave disturbances in the simulations. The specified form is shown as

$$S_{\xi}(\omega) = \frac{8.11 \times 10^{-3} g^2}{\omega^5} \exp\left(-\frac{3.11}{h_{1/3}^2 \omega^4}\right) \quad (16)$$

where $h_{1/3}$ is the significant wave height, ω is the frequency of the wave.

According to the microtomy [14], we can get the disturbance thrust and torque of the wave with different frequencies. Then, by using the data fitting and superposition methods, the disturbance thrust and torque acting on the SWATH can be acquired.

In the simulations, the significant wave height is selected as 3m, and the speed of SWATH is selected as 18kn. The obtained disturbance thrust and torque is shown in Figure 2. The verification of power spectrum is depicted in Figure 3 [15]. It is clear that the obtained disturbance thrust and torque satisfy the requirement of power spectrum.

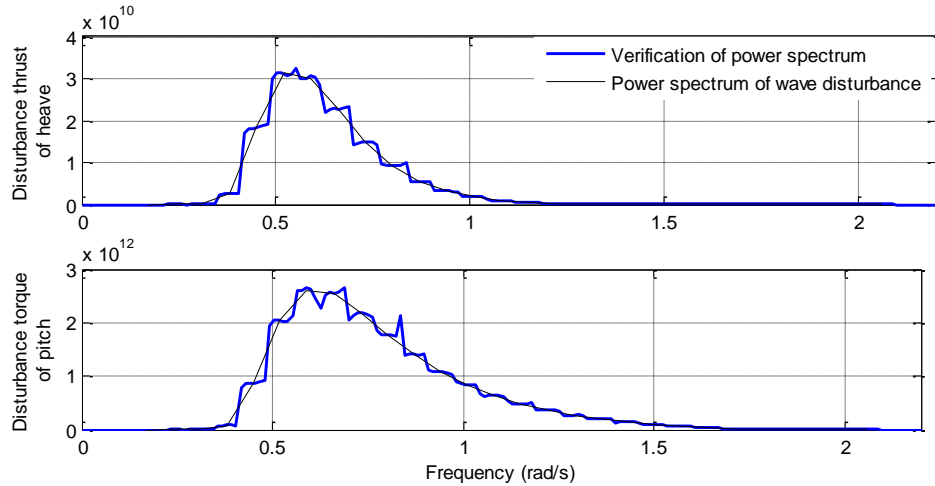


Figure 2. Wave Disturbance Thrust and Torque

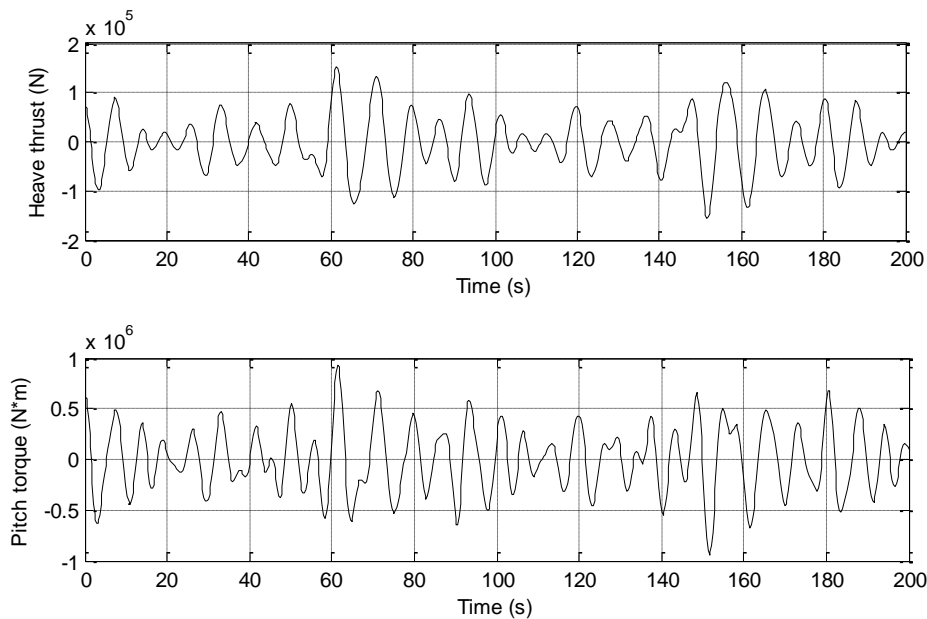


Figure 3. Verification of Power Spectrum

5.2 Simulation Results

The simulation of the SWATH is carried out in MATLAB/SIMULINK. The parameters of the SWATH is given as:

$$A = \begin{bmatrix} 6.92 \times 10^5 & -2.03 \times 10^5 \\ -8.62 \times 10^4 & 6.16 \times 10^7 \end{bmatrix}, B = \begin{bmatrix} 2.64 \times 10^4 & 2.47 \times 10^6 \\ -2.58 \times 10^6 & 2.49 \times 10^6 \end{bmatrix}, C = \begin{bmatrix} 5.68 \times 10^5 & -2.03 \times 10^5 \\ -2.03 \times 10^5 & 4.37 \times 10^7 \end{bmatrix}$$

Assuming that the parameter error is within 10%, the nominal value of the previous three matrices is given as

$$A_0 = \begin{bmatrix} 6.2 \times 10^5 & -1.5 \times 10^5 \\ -8.0 \times 10^4 & 5.6 \times 10^7 \end{bmatrix}, B_0 = \begin{bmatrix} 2.1 \times 10^4 & 1.9 \times 10^6 \\ -2.0 \times 10^6 & 2.1 \times 10^6 \end{bmatrix}, C_0 = \begin{bmatrix} 5.1 \times 10^5 & -1.5 \times 10^5 \\ -1.5 \times 10^5 & 3.7 \times 10^7 \end{bmatrix}$$

The control parameters of the terminal SMC is selected as $L = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$, $p=1$, $q=1.5$. The cutoff frequency of the linear ESO is selected as 1.5rad/s.

Figure 4 shows the disturbances estimation effect of the designed ESO. We can see that the second order ESO can estimate the composite disturbances with small estimation error. Figure 5 shows the control effect comparison of the proposed ESO based terminal SMC and optimal control. It is clear that with the system uncertainties and the wave disturbances, the proposed control scheme can acquire better control performance, since the ESO estimate and compensate the composite disturbances effectively. However, the optimal controller cannot obtain desired control performance with the existence of the composite disturbances. It is clear that the proposed control scheme in this paper can provide the system with better control performance, it can increase the control accuracy of pitch and heave motion of the SWATH and bring the system with more robustness.

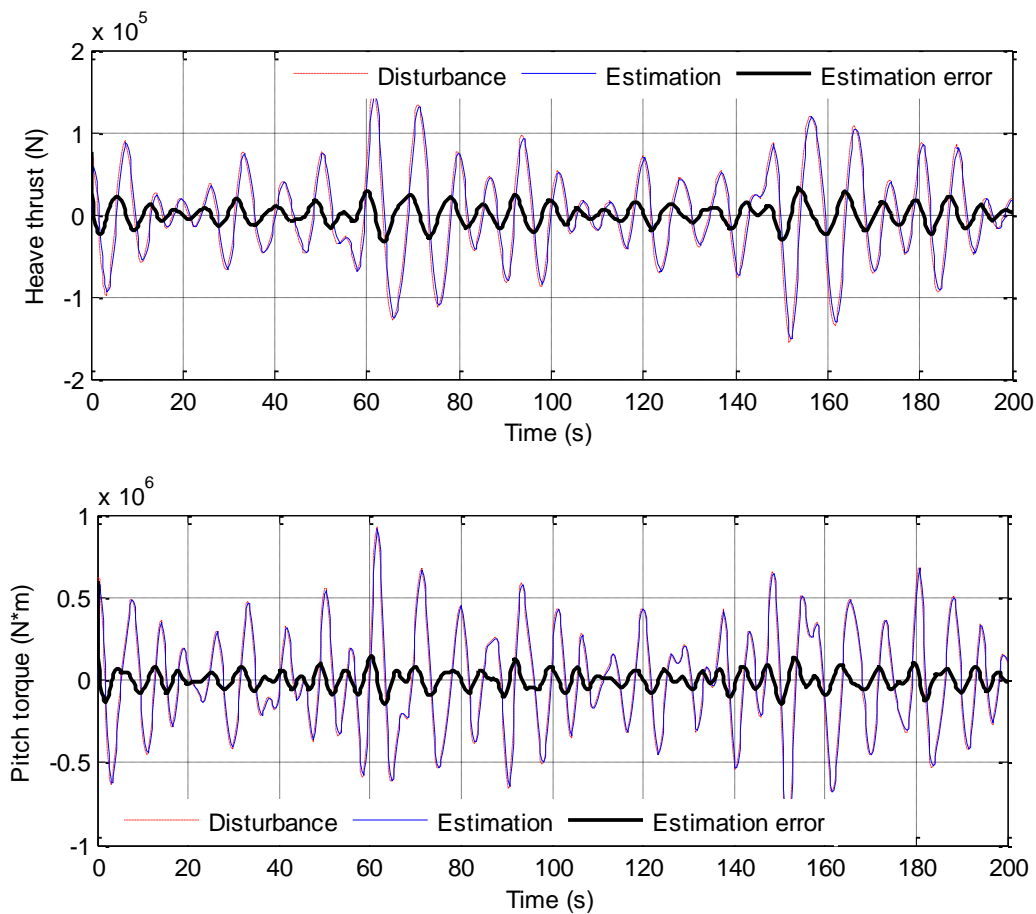


Figure 4. Disturbance Estimation Effect Of ESO

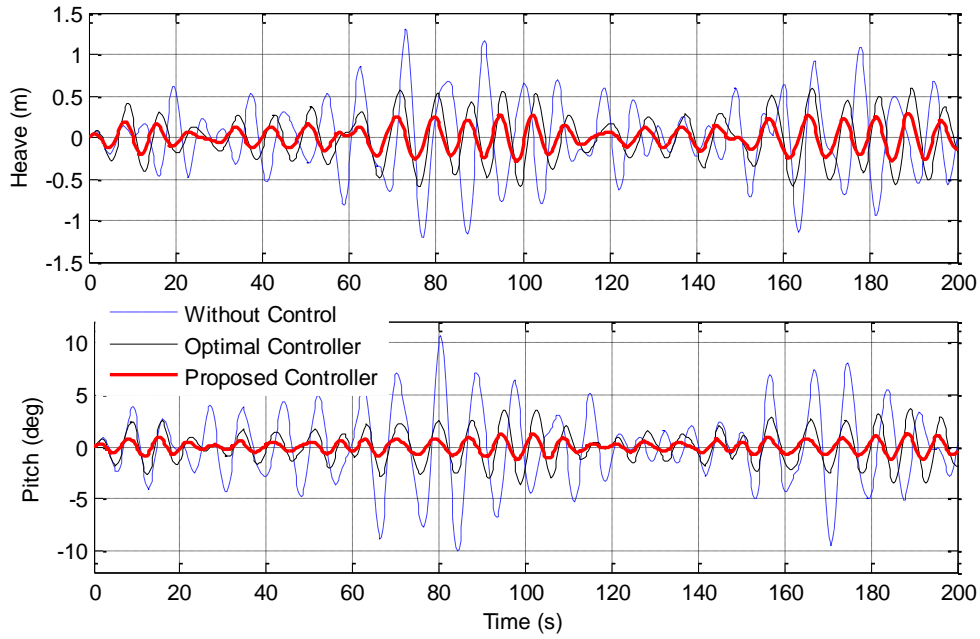


Figure 5. Control Effect Comparison

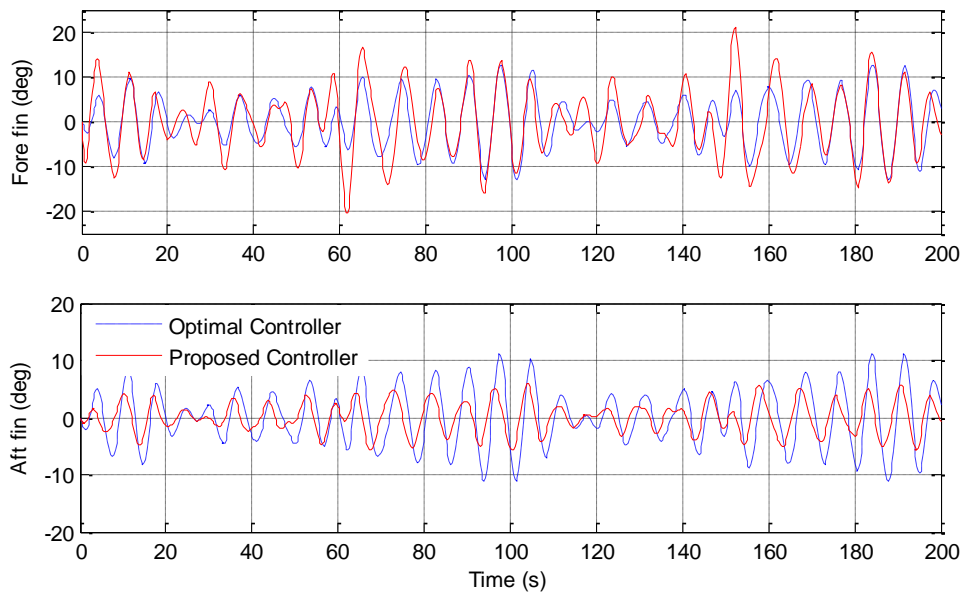


Figure 6. Control Input

6. Conclusions

The control problem of SWATH with both system uncertainties and random wave disturbances is investigated in this paper. The terminal SMC scheme with ESO is proposed. The control system consists of the inner loop observer and outer loop controller. The ESO is applied as inner loop to estimate the composite disturbances that contains both internal uncertainties and wave disturbance, and thus compensated the real controlled objective as the nominal plant. Then, the terminal SMC technique is employed to acquire desired control performance. Simulation results show that the proposed control

scheme can increase the accuracy of the motion control of SWATH. The desired ESO can estimate the composite disturbances online. The designed terminal SMC can provide the system with high convergence speed.

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