

## Resource Allocation in Multiple Product Design Projects: A Bi-level Programming Approach

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### **Abstract**

*This paper addresses the problem that the inherently hierarchical nature of resource allocation is usually ignored in the research on multiple design projects. We propose a bi-level programming mathematical model to solve this problem and seek its approximately satisfactory solution with simulated annealing algorithm. A simple numerical example is presented to demonstrate the method. We wrote the article to propose a reasonable possible research direction and attract more interests in our engineering design community for the resource allocation in multiple design projects.*

**Keywords:** *Bi-level Programming, Resource Allocation, Multi-projects, Design Project, Engineering Design*

### **1. Introduction**

With the intensification of global competition, product design process is increasingly complex while enterprise must respond more quickly to customer needs, which makes the product design projects must be more efficient. Enterprises generally deal with more than one product design order and each order corresponds to a product design project. Multiple projects have certain similarities, and influence each other.

Similar design projects correspond to different customer orders, product technology demands, product delivery time and so on, but the task structure, constraint relations, planning process of them are similar. In order to take full advantage of the similarity of the products and processes to improve efficiency, we should plan multiple product design projects in parallel.

Nonetheless, even in project management community, multi project management is a complex problem since resource conflicts often occur among projects. Vermaas [1] suggests that one design project should consider of five steps: goals, actions, functions, behaviors, and physical structures. It means that, even in one design project, there are lots of actions. Resource conflicts also occur among actions. All actions in each project need resources but resources are limited in the enterprise, so how to allocate the resources is the key to the complex multiple product design projects planning.

Resource constraints have very large impact on the projects, directly affecting the completion duration and quality of all projects. The efficiency of resource allocation to some extent is affected by resources organizational structure. Nowadays, most of extant research on resource allocation in multiple design projects usually combine multiple projects into a large project by adding virtual actions and artificially setting priority, then apply some proven methods of single project resource allocation problem, such as CPM (Critical Path Method) or PERT (Program Evaluation and Review Technique),etc, to solve it.

However, these methods ignore the inherent hierarchical nature of resource allocation in multiple design projects, which have been noted by some researchers. Winkofsky, Baker and Sweeny [2] represent a R&D resource allocation process at three hierarchical

levels. Reppening [3] proposed a dynamic model of multi-project R&D resource allocation process at two levels between current and future projects. Kai Pan, Xinjian Gu, etc. [4] proposed a three-level approach for assigning proper staff to special tasks in MRO (Maintenance, Repair and Operation) activities. Zandt [5] dealt with hierarchical computation of the resource allocation problem. Tianran Zhou, etc. [6] presented a two-level hierarchical resource allocation approach for integrated modular avionics systems.

Also, these research ignore the point that there are stakeholders at each level who have their own value appeal. When lower-level stakeholders make decision on resource allocation, upper-level stakeholders' value also are influenced and need to update their decisions. So the process in fact is a dynamic feedback iteration process. But traditional methods usually combine multiple projects into a large project by adding virtual actions and artificially setting priority, which are static and don't reflect the nature of the problem. So it is necessary for us to study the problem from the perspective of system hierarchy using bi-level programming, which can correctly reflect the dynamic nature of the problem.

## 2. Research Status on Resource Allocation in Multiple Design Projects

Multi-projects resource allocation problem means how to allocate limited resources to different projects. Every project consists of a group of sequence actions and each of them needs a certain amount of resources so that all the projects could be accomplished and their expected objectives reached. The possible objectives are the least construction period delay, the least resources consumption, the maximum net present value of the project, etc.

As the certain precedence relationship among actions in the projects doesn't completely exist among several concurrent projects, the multi-projects resources allocation problem is different from the resources allocation among the actions in one project. Because of the difference of the inner constraint relation, resources allocation problem in multi-projects is much more complicated than that in a single project.

In the environment of multiple design projects, the workload of the enterprise is heavy, while the resources scale is relatively stable. Concurrent multiple design projects share to exploit the enterprise's resource set so that there are varied competitions among them. To prevent the projects from vying the resources and the mutual interference, the core problem in the multiple design projects planning is the resources allocation problem among multiple projects. Multiple design projects planning should comprehensively consider the enterprise resource, timely adjust according to the requirements and characteristics of each project, optimally allocate the resources using scientific methods, take full advantage of the enterprise available resources ability, assure the achievement of all projects objectives, and finally achieve the maximum benefit of the whole enterprise.

Some of research achievements on the multiple design projects resource allocation problem are picked up as follows. Pritsker, Watters, and Wolfe [7] proposed a 0-1 programming model to allocate the human resources in multiple design projects, which has a good effect in the case of a simple project. Yongyi Shou [8] studied the iterative algorithm of the resource-constrained multi-projects scheduling. Wei Fang and Lixiong Ou [9] converted the products design multi-projects resource allocation problem into a multi-queue queuing problem, and built a simulation model. Speranza and Vercellis [10] proposed multi-projects resource allocation grading model, and brought the concept of compact scheduling in the branch-and-bound method to improve the efficiency of the algorithm, but sometimes wrong results will come out of this method. Kurtulus and Davis [11] classified the functions of the heuristic rules in the multi-projects environment, and pointed out the deficiency of artificially adding virtual work to convert the multi-projects into single project to solve the multi-projects resource allocation problem, but they hadn't provided the mathematical model description of the problem.

### 3. Hierarchical Nature of Resource Allocation in Multiple Projects

Each project manager focuses on how to achieve the goal of his project, only concerning resources, schedule and costs of his project. Sometimes the effort to maximize the profits of each single project will damage enterprise's overall profits. So enterprise should plan, allocate and control project resources from a holistic perspective. In multi-project resource allocation, to achieve the overall target, enterprise management should allocate the resources to each project in accordance with an appropriate proportion. In the meantime, in order to achieve their objectives, each project manager reasonably allocates resources to the actions of his project. This means that single project resource allocation problem is embedded in the multi-project resource allocation problem. So bi-level programming problem may be a good way to deal with the problem. It's worth mentioning that Ye Tan, Weijun Zhong and Nanrong Xu [12] have done initial exploration in the bi-level decision-making model for multi-project resource allocation problem. But there are still some room to construct a better and dynamic bi-level decision-making model.

### 4. Introduction of the Bi-level Programming

The bi-level programming studies the planning and management (control) problems of the systems with two levels. Such decision problem is made of two levels of hierarchical deciders who are relatively independent. The upper level deciders just guide the lower level deciders via their decisions and never intervene directly, while the lower deciders just need to take the upper deciders' decisions as parameters or constraints, and they can make decisions freely in their own possible range.

The process of the bi-level programming system is as follows. The upper level provides some information to the lower level, and with the information, the lower level makes a response (decision) according to the benefits and preference. Then according to the response, the upper level adjusts the decision to make sure better general benefits.

The general characteristics of bi-level programming problem are as follows.

- (1) The system is hierarchically managed, deciders at each level make decisions sequentially, the lower level is submitted to the upper one, but it has certain autonomy.
- (2) Deciders at each level have their own different objectives that sometimes are mutually contradictory.
- (3) Each level deciders control a part of decision variables to optimize their own objectives.
- (4) The upper level deciders make decisions firstly, and the lower cannot oppose the upper level deciders' decisions when they are selecting strategies to optimize their own objectives.
- (5) The upper level decisions may affect the lower level decisions set, and then to some extent affect the achievement of the lower level objectives, but the upper level cannot completely control the lower level decisions.
- (6) The lower level decisions not only determine the achievement of its own objectives, but also affect the achievement of the upper level objectives. So when the upper level is making the optimal decision to achieve its own objectives, it must take the adverse effect from the lower level decisions into consideration.
- (7) Each level deciders' admissible strategy sets are usually inseparable, and they usually form a correlative whole. [13] [14]

We can see that these characteristics are entirely consistent with the interactive relationship between enterprises high-level leaders and project managers in multi-projects resources allocation problem. So building bi-level programming mathematical models of

multi-projects resources allocation is consistent with the abstract of the problem essential characteristic.

### 5. Bi-level Programming Mathematical Model for Resource Allocation in Multiple Design Projects

The article established a simple bi-level programming mathematical model for resource allocation in multi design projects to propose a reasonable possible research direction and attract more interests in our community. The objective function in the model is set as the minimum tardiness to expected delivery time. So the process of the problem could be described as follows: At first, enterprise executive allocates the resource scale to each project. Then each project manager reasonably allocates all kinds of resources to the actions in the resource scale restraint according to the project network plan for the shortest duration. Then the project managers will feed back the results to enterprise executive who will readjust the resource scale plan, and each project manager will make optimal decisions in accordance with the re-allocation of the resource scale. Iterating like this, until achieving the overall minimum tardiness to expected delivery time for all projects. Figure 1 shows the interaction between upper-level and lower-level decision-making.

The problem can be stated as follows:

(1) Assuming that the total scale of the resources assigned to the projects by enterprise executive is  $Z$ , its lower limit is  $a$  and upper limit is  $b$ , which represent the resources amount range to be devoted to the projects by the enterprise, so  $a \leq Z \leq b$ . Assuming there are  $n$  projects, the resources scale of the  $i$ -th ( $i=1,2,\dots,n$ ) project is  $x_i$ , its lower limit is  $a_i$  and upper limit is  $b_i$ , so  $a_i \leq x_i \leq b_i$ .  $x_1, x_2, \dots, x_n$  are the upper-level decision variables in the bi-level programming mathematical model of the problem.

$Z = \sum_{i=1}^n x_i$ , so  $a \leq \sum_{i=1}^n x_i \leq b$ . The upper-level problem is that how enterprise

executive allocates the resources scale  $Z$  to  $n$  projects,  $x_i$  is the resource amount assigned to the  $i$ -th project, so the decision variables to the upper level programming are  $\{x_i\}$  ( $i=1,2,\dots,n$ ).

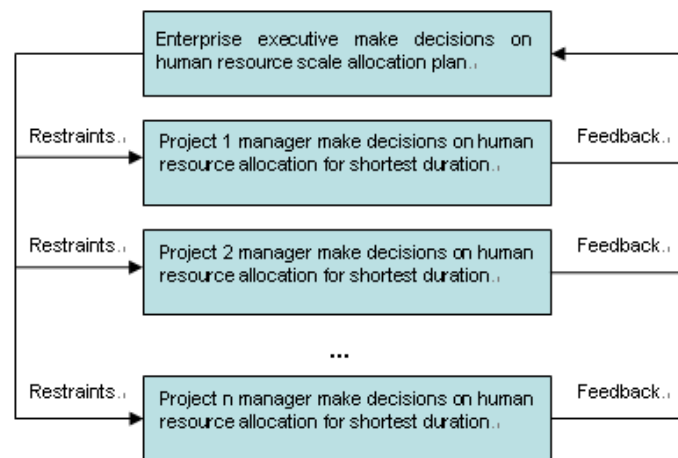


Figure 1. Interaction between Upper-level and Lower-level Decision-making

(2) Because of the similarity of multiple design projects, assuming each project consists of the same amount of actions. So the lower-level problem is that how the  $i$ -th project manager allocates  $x_i$  scale of resources to the corresponding  $m$  elastic actions. (Elastic

action is defined as the action whose completion time is affected by the amount of resources. The more resources, the faster completion. In contrast, stable action is defined as the action whose completion time depends on its inherent characteristics, won't be shortened by allocating more resources.) Assuming  $y_{ij}$  is the the resources scale assigned to the  $j$ -th ( $j=1,2,\dots,m$ ) elastic action in the  $i$ -th project, its lower limit is  $a_{ij}$  and upper limit is  $b_{ij}$ , so  $a_{ij} \leq y_{ij} \leq b_{ij}$ , ( $i=1,2,\dots,n; j=1,2,\dots,m$ ). The decision variables of lower-level problem are  $\{y_{ij}\}$ , and the upper-level and lower-level decision variables have the following relationship:  $\sum_{j=1}^m y_{ij} \leq x_i$  ( $i=1,2,\dots,n$ ).

(3) Assuming there are  $M_i$  actions in the dual-code network plan of the  $i$ -th project, which contain  $m$  elastic actions and  $M_i - m$  stable actions. Assuming all resources requirement to stable action could be met, its duration is a fixed value  $d_{ig}$  ( $i=1,2,\dots,n; g=1,2,\dots,M_i - m$ ). Assuming all other resources requirement to elastic action could be met and the same kind of resource has the same productivity. Its duration depends on the amount of resources, the more resources, the faster completion. This relationship could be stated as follows:  $D_{ij} = W_{ij} / (y_{ij} \cdot v_j)$  ( $i=1,2,\dots,n; j=1,2,\dots,m$ ).  $D_{ij}$  is the duration of the elastic action,  $W_{ij}$  is the workload of the elastic action,  $y_{ij}$  is the amount of resources allocated to the elastic action and  $v_j$  is the efficiency of the  $j$ -th kind of resource. The goal of the upper-level problem is achieving the overall minimum tardiness to expected delivery time for all  $n$  projects, the objective function is represented as  $F$ . And expected duration of the  $i$ -th project is represented as constant  $T_i^*$ , which is usually determined by the delivering date asked by the customer or market. In the lower-level problem, the goal of each project is the fastest completion, the objective function of the  $i$ -th project is represented as  $F_i(x_i, y_{i1}, y_{i2}, \dots, y_{im})$ . Using dual-code network plan to represent each project schedule and obtain its duration  $T_i(x_i)$  by Critical Path Method. So the bi-level programming mathematical model for resource allocation in multiple design projects could be described as follows:

$$\min F = \min \left\{ \sum_{i=1}^n (\max(T_i(x_i) - T_i^*, 0)) \right\}$$

$$\text{s.t. } a_i \leq x_i \leq b_i$$

$$a \leq \sum_{i=1}^n x_i \leq b$$

$$x_i \geq 0, \text{ and is integer.}$$

$$(i=1,2,\dots,n)$$

$$\min F_i(x_i, y_{i1}, y_{i2}, \dots, y_{im})$$

$$= \min T_i(x_i)$$

$$\text{s.t. } a_{ij} \leq y_{ij} \leq b_{ij}$$

$$\sum_{j=1}^m y_{ij} \leq x_i$$

$$y_{ij} \geq 0, \text{ and is integer.}$$

$$(i=1,2,\dots,n; j=1,2,\dots,m)$$

## 6. Algorithm Explanation

The model solution is resolved by Simulated Annealing (SA). SA has been primarily applied in combinatorial optimization field by Kirkpatrick etc. [15]. It is a random optimization algorithm based on Monte-Carlo iterative solution strategy, and its starting point is based on the similarity between the annealing process of the solid physical matter and general combinatorial optimization problem. The SA starts at a higher initial temperature, and as the temperature parameter continuously decreases, it randomly searches for the global optimal solution of the objective function combining with the probabilistic jumping property namely the local optimal solution can jump and finally tend to the global optimal solution at an uncertain probability. The model solution steps of the SA can be described as follows.

- (1) Initialization: initial temperature  $T$  (sufficiently large), initial solution state  $X_0$  (the start point of the algorithm iteration), the iteration times of each  $T$  are  $K_{\max}$ .
- (2) For  $k=1, k=2, \dots, K_{\max}$  executes from the step(3) to step(6).
- (3) Generate new solution  $X'$ .
- (4) Calculate the increment  $\Delta t' = C(X') - C(X)$ , herein  $C(X)$  is the evaluation function.
- (5) If  $\Delta t' < 0$ , then accept the  $X'$  as the new current solution, otherwise accept the  $X'$  as the new current solution at a probability of  $\exp(-\Delta t'/T)$ .
- (6) If  $f(X^{k+1}) < f_{\min}$ , then  $X_{\min} = X^{k+1}$ ,  $f_{\min} = f(X^{k+1})$ .
- (7) If the termination conditions are met, then output the current solution as the optimal solution, and terminate the procedure.  $X_{\min}$  can be seen as the approximate global optimal solution, and the  $f_{\min}$  is the corresponding optimal value. Otherwise, get to the step(8).
- (8) Update the function according to the given temperature, and generate a different temperature  $T_{k+1}$ , set  $k=k+1$ , and goes to the step(2).

According to this problem, both levels are nonlinear integer programming, so it's possible to conduct the respective solution calculation for both the upper level problem and the lower level problem in variables space via SA. But this calculation doesn't repeat simply, instead it sees the upper and lower level as an organic whole. The solution steps are shown as follows:

Step1: Simulate the initialization of the SA, and set the algorithm parameters.

Step2: According to the resource scale constraints and each sub-project resource programming constraints, allocate the resource scale  $X = (x_1, \dots, x_i, \dots, x_n)$  of each subproject via SA.

Step3: The  $i$ -th project decider updates resource planning of  $m$  elastic actions  $Y_i = (y_{i1}, \dots, y_{ij}, \dots, y_{im})$  according to the resource planning  $x_i$  which is made by upper level deciders via SA.

Step4: Estimate that whether the project construction period of the lower level  $F_i(x_i, y_{i1}, \dots, y_{ij}, \dots, y_{im})$  is the optimal or not. If it is the optimal, then go to Step5, otherwise, go to the Step2.

Step5: Estimate whether the  $n$  upper level projects delay is the optimal or not. If it is the optimal, then go to Step6, otherwise, go to the Step2.

Step6: Terminate the algorithm, and exit.

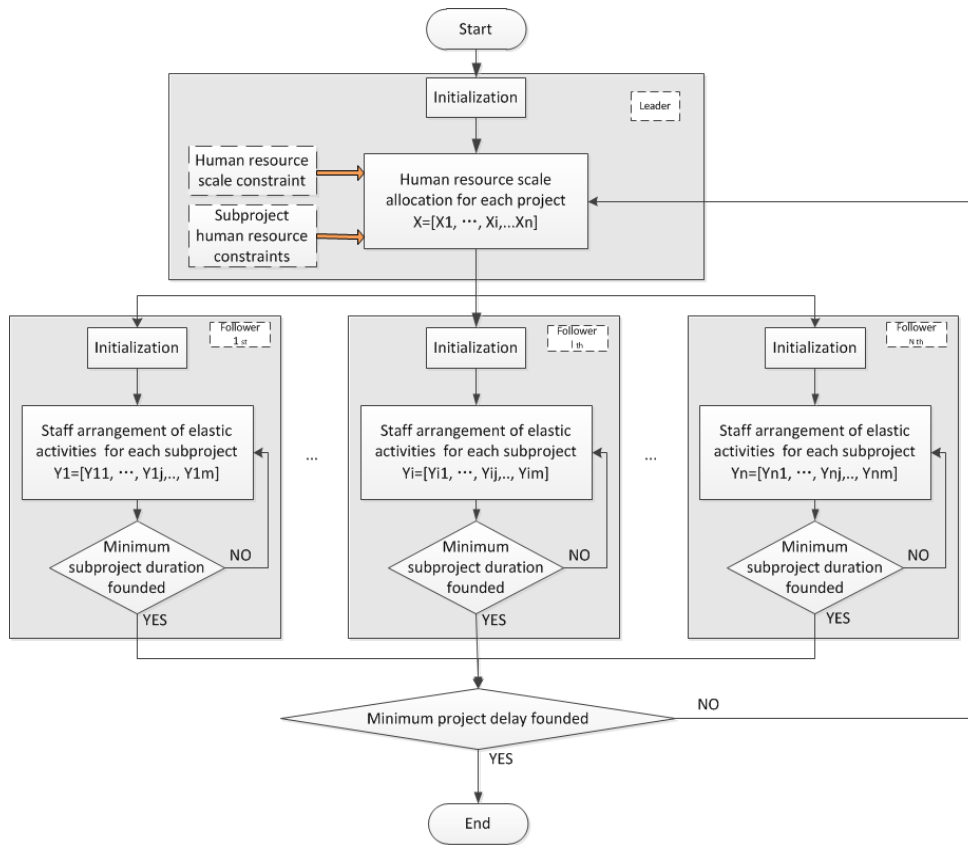


Figure 2. The Algorithm Process

### 7. Demonstration Numerical Example

A vacuum cleaner enterprise need to deal with four design projects at the same time: upright vacuum cleaner, pusher-bar vacuum cleaner, hand-held vacuum cleaner and vacuum cleaner robot. Since the resources allocated in multiple design projects are mainly human resources (such as market analysts, design engineers, process engineers, etc.) [16] and delivering speed to seize market is the priority factor in household appliance industry. So in this case, we allocate the human resources using bi-level programming to achieve the overall minimum tardiness to expected delivery time for all projects, to demonstrate the approach proposed in Section 5 and 6. The conditions and constraints of the projects are shown in Table 1.

Table 1. Conditions and Constraints of the Four Projects

Project	$T^*$	Limits	Project staffs	Elastic action 1	Elastic action 2	Elastic action 3	Elastic action 4	Elastic action 5	Elastic action 6	Elastic action 7
A	52	upper limit	16	4	5	5	6	4	3	3
		lower limit	10	1	1	1	2	2	1	1
B	51	upper limit	18	4	5	5	6	4	3	3
		lower limit	12	1	1	1	2	2	1	1
C	52	upper limit	17	4	5	5	6	4	3	3
		lower	13	1	1	1	2	2	1	1

		limit								
D	51	upper limit	18	4	5	5	6	4	3	3
		lower limit	13	1	1	1	2	2	1	1
Total		upper limit	65							
		lower limit	51							
Action workload/Staff efficiency			20	24	30	36	15	22	15	

The dual-code network plans to project A (upright vacuum cleaner), B (pusher-bar vacuum cleaner), C (hand-held vacuum cleaner) and D (vacuum cleaner robot) are shown in figure 3 to figure 6. Duration of actions are also shown in the figures, the **constant numbers** are the duration of each stable action and the ratio numbers are the duration of each elastic action:

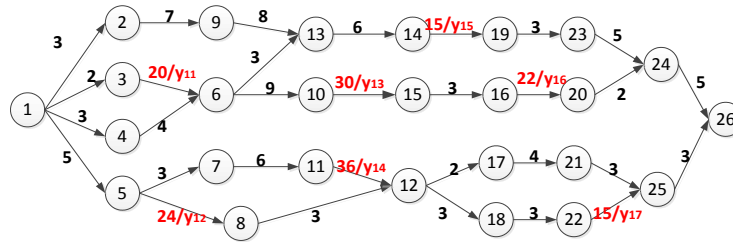


Figure 3. The Dual-code Network Plans to Project A

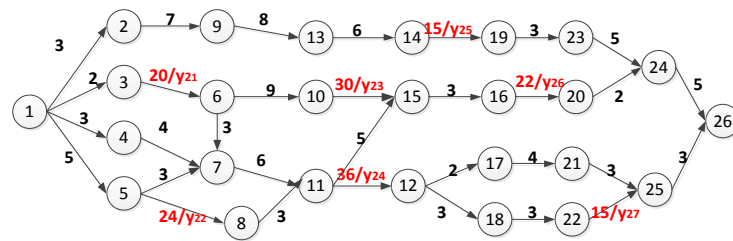


Figure 4. The Dual-code Network Plans to Project B

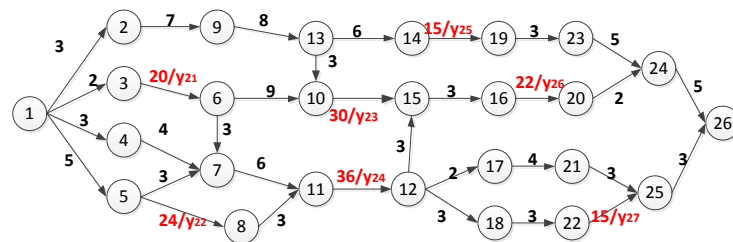


Figure 5. The Dual-code Network Plans to Project C

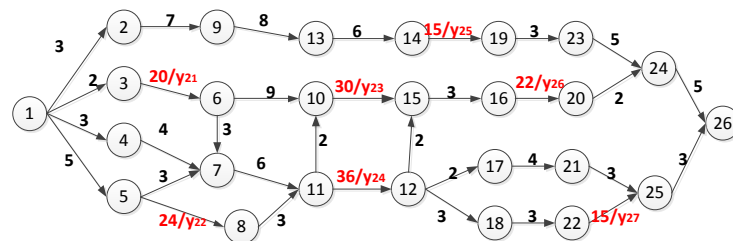


Figure 6. The Dual-code Network Plans to Project D



Now based on these data, we use Matlab to compute the problem according to the resource constraints and algorithm procedures demonstrated by Fig.2, set  $T_0=10000$ ,  $\sigma=3$ ,  $\beta=2.0$ ,  $K_{max}=1000$ , computing following the algorithm process and the optimal results are shown in Table 2.

These results indicate that using simulated annealing algorithm to solve the bi-level programming mathematical model for the problem could find out the approximately satisfactory solution at a faster rate, so it is feasible.

**Table.2. The Optimal Results**

	Staffs number	Staffs in action 1	Staffs in action 2	Staffs in action 3	Staffs in action 4	Staffs in action 5	Staffs in action 6	Staffs in action 7	Duration
Project A	15	2	1	3	2	2	3	2	48.5
Project B	17	3	2	3	3	2	2	2	48.7
Project C	16	2	2	2	3	2	3	2	53.3
Project D	17	2	2	3	4	2	2	2	50.3
Total	65	Overall minimum tardiness for all four projects							1.3

## 8. Summary and Future Research

In the environment of multiple design projects, multiple design projects planning should reasonably allocate the resources to prevent the projects from contending for the resources. Most of the extant research ignore the inherent hierarchical nature of resource allocation in multiple design projects. In consider of the principal and subordinate relationship and the strategy sets interactions between enterprise executive and project managers, the article constructs a bi-level programming mathematical model to the multiple design projects resources allocation problem and seeks its approximately satisfactory solution with simulated annealing algorithm, then using a numerical example simply demonstrate this method.

In this paper, the complexity of the problem has been simplified, but the factors not considered would be studied in the later work. For example, in this paper we assume there is a fixed relationship among the project network planning, but in real environment project network planning is usually dynamic, further research should be done in this aspect. And the paper addresses only single kind of resource, using bi-level programming to solve multiple resources allocation in multiple projects needs further research.

Recent years, a new theory called critical chain project management [17] attracts lots of attention in the project management community. It also tries to deal with the hierarchical nature of the resource allocation in multi project management, but from the another perspective considering human nature and using management method. So how to integrate the critical chain project management method and the bi-level programming algorithm method when dealing with the resources allocation in multi design project planning will be the direction of our efforts.

## Acknowledgement

The authors acknowledges the financial support by the National Natural Science Foundation of China (No. 51275456), and the National High Technology Research and Development Program of China (No. 2013AA041304).

The authors are grateful to the anonymous referees for the careful checking of the paper and for helpful comments that improved this paper.

## References

- [1] P. E. Vermaas, "The flexible meaning of function in engineering", DS 58-2: Proceedings of ICED 09, the 17th International Conference on Engineering Design, Design Theory and Research Methodology, (2009); Palo Alto, CA, USA.
- [2] E. P. Winkofsky, N. R. Baker and D. J. Sweeney, "A decision process model of R&D resource allocation in hierarchical organizations", *Management Science*, vol. 27, no. 3, (1981), pp. 268-283.
- [3] N. P. Repenning, "A dynamic model of resource allocation in multi-project research and development systems", *System Dynamics Review*, vol. 16, no. 3, (2000), pp. 173-212.
- [4] K. Pan, X. Gu, N. Chen, Y. Ji and C. Le, "Three-phase approach to service staff assignment for MRO tasks", *International Journal of Services Operations and Informatics*, vol. 7, no. 2, (2012), pp. 96-116.
- [5] T. Van Zandt, "Hierarchical computation of the resource allocation problem", *European Economic Review*, vol. 39, no. 3, (1995), pp. 700-708.
- [6] T. Zhou, H. Xiong and Z. Zhang, "Hierarchical resource allocation for integrated modular avionics systems", *Systems Engineering and Electronics Journal*, vol. 22, no. 5, (2011), pp. 780-787.
- [7] A. A. B. Pritsker, L. J. Watters and P. M. Wolfe, "Multi-project scheduling with limited resources: A zero-one programming approach", *Management Science*, vol. 16, no. 1, (1969), pp. 93-108.
- [8] Y. Shou, "Iterative technique for scheduling resource constrained multiple projects", *Journal of Zhejiang University (Engineer Science)*, vol. 38, no. 8, (2004), pp. 1095-1099.
- [9] W. Fang and L. Ou, "Study on Multiple New Product Projects Resource Allocation", *Journal of Industrial Engineering and Engineering Management*, vol. 19, (2005), pp. 6-10.
- [10] M. G. Speranza and C. Vercellis, "Hierarchical models for multi-project planning and scheduling", *European Journal of Operational Research*, vol. 64, (1993), pp. 312-325.
- [11] I. Kurtulus and E. W. Davis, "Multi-project scheduling: Categorization of heuristic rules performance", *Management Science*, vol. 28, no. 2, (1982), pp. 161-172.
- [12] Y. Tan, W. Zhong and N. Xu, "Bi-level Decision Method for Multi-Resource Constrained Multi-project Scheduling", *Systems Engineering — Theory & Practice*, vol. 14, no. 3, (1999), pp. 290-295.
- [13] X. Wang and S. Feng, "The Bi-level System Optimization Theory", Science Press, (1995).
- [14] C. Teng and Z. Li, "Theory and Application of Bi-level Programming", Science Press, (2002).
- [15] S. Kirkpatrick S, D. G. Jr. and M. P. Vecchi, "Optimization by simulated annealing", *Science*, vol. 220, no. 4598, (1983), pp. 671-680.
- [16] X. Fang, "Configuration-Based Multiple Design-Projects Planning for ETO Product", Zhejiang University, Hangzhou, China, (2011).
- [17] C. L. Huang, R. K. Li and Y. C. Chung, "A Study of Using Critical Chain Project Management Method for Multi-Project Management Improvement", *International Journal of Academic Research in Economics and Management Sciences*, vol. 2, no. 3, (2013).

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