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Research on Adaptive Backstepping Control of Uncertain Second Order Systems with Four Differential Algorithms

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Abstract

In order to solve differential bome problem of traditional backstepping control method, this paper studies four different differential algorithms. An adaptive backstepping control method is designed for an uncertain second order system, then four kinds of different differential algorithms are integrated with the adaptive backstepping control strategy. Through comparing the control effect of four different differential algorithms, a conclusion can be made that the comprehensive effect of one order filtering differential algorithm is the best one among above four kinds of differential algorithms.

Keywords: *differential algorithm, backstepping control, second order system, uncertainty, adaptive control*

1. Introduction

Many control methods can be used for a class of certain second-order systems, such as the classical state feedback control method and the poles placement control method. But for a class of uncertain second order systems, adaptive variable structure control methods and adaptive neural network control methods and adaptive backstepping control methods [1-9] are often used by researchers to cope with the system uncertainties.

The adaptive algorithm is an effective solution to systems with uncertain parameters problem, the construction method of backstepping control algorithm has rigor of theoretical inferences and it can be designed by Lyapunov energy function method. So there is a great deal of literatures [10-15] that have had a full discussion on the adaptive backstepping control method.

But most of backstepping control methods face a problem that the differential computing of the expectation state becomes very complicate with the increase of system order. It was called differenctical bomb problem in some references [16-19]. In this paper, based on the theory of adaptive backstepping method, four kinds of differential methods were proposed for the computing of differential state, then a numerical simulation was done and the simulation result was analyzed for the four methods. Finally the conclusion can be made as follows: the comprehensive effect of one order filtering differential algorithm is the best one, but how to choose a proper filter constant is the key of one order differential method.

2. Problem Description

The second-order system with a single control direction is a simple case in all of the second-order system. The control direction is the coefficient of the model input u which is called control coefficient. The model can be written as:

$$\dot{x} = Ax + bu \qquad (1)$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The parameters of model is unknown, the goal of
adaptive backstepping control is to design an adaptive backstepping controller such that the system state x_1 can trace the expected value x_1^d.

3. Assumption

Assumption 1: $a_{12} \neq 0$, its direction is known, without loss of generality, assume $a_{12} > 0$.

Assumption 2: the expected x_1^d is a constant, then $\dot{x}_1^d = 0$.

4. Design Adaptive Backstepping Control Law

Consider the following the first order subsystem:

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 \qquad (2)$$

Define a error variable as $e_1 = x_1 - x_1^d$, then:

$$\dot{e}_1 = a_{11}x_1 + a_{12}x_2 \qquad (3)$$

Based on assumption 1, use the backstepping design method to design the expected x_2^d of x_2 as following:

$$x_2^d = -k_1e_1 - \hat{k}_2 \qquad (4)$$

where $\hat{k}_2$ is an adaptive term which mainly is used for closing a_{11}. Define a error variable as $e_2 = x_2 - x_2^d$, then:

$$\begin{aligned} \dot{e}_1 &= a_{11}e_1 + a_{11}x_1^d + a_{12}e_2 + a_{12}x_2^d \\ &= a_{11}e_1 + a_{11}x_1^d + a_{12}e_2 + a_{12}(-k_1e_1 - \hat{k}_2) \\ &= (a_{11} - a_{12}k_1)e_1 + a_{11}x_1^d - a_{12}\hat{k}_2 + a_{12}e_2 \end{aligned} \qquad (5)$$

Define:

$$\bar{k}_1 = a_{11} - a_{12}k_1 \tag{6}$$

Obviously, there exists a big k_1 that can make $\bar{k}_1 < 0$ 。
Define:

$$\tilde{k}_2 = a_{11}x_1^d - a_{12}\hat{k}_2 \tag{7}$$

Then:

$$\dot{\tilde{k}}_2 = -a_{12}\dot{\hat{k}}_2 \tag{8}$$

Choose:

$$\dot{\hat{k}}_2 = k_2 e_1 \tag{9}$$

Then:

$$\dot{e}_1 = \bar{k}_1 e_1 + \tilde{k}_2 + a_{12}e_2 \tag{10}$$

Define:

$$V_1 = \frac{1}{2a_{12}k_2}\tilde{k}_2^2 \tag{11}$$

Then:

$$\dot{V}_1 = \frac{1}{a_{12}k_2}\tilde{k}_2\dot{\tilde{k}}_2 = -\frac{1}{a_{12}k_2}\tilde{k}_2 a_{12}\dot{\hat{k}}_2 = -\frac{1}{a_{12}k_2}\tilde{k}_2 a_{12}k_2 e_1 = -\tilde{k}_2 e_1 \tag{12}$$

Consider the second order subsystem, define a error variable as

$$\dot{e}_2 = a_{21}x_1 + a_{22}x_2 + u - \dot{x}_2^d \tag{13}$$

Design adaptive control law:

$$u = -\hat{a}_{21}x_1 - \hat{a}_{22}x_2 + \dot{x}_2^d - k_3 e_2 - k_4\int e_2 dt \tag{14}$$

Define:

$$\tilde{a}_{21} = a_{21} - \hat{a}_{21} \tag{15}$$

$$\tilde{a}_{22} = a_{22} - \hat{a}_{22} \tag{16}$$

Then:

$$\dot{\tilde{a}}_{21} = -\dot{\hat{a}}_{21} \tag{17}$$

$$\dot{\tilde{a}}_{22} = -\dot{\hat{a}}_{22} \tag{18}$$

Then:

$$\dot{e}_2 = \tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 - k_3 e_2 - k_4\int e_2 dt \tag{19}$$

Design unknown parameters weight turning law as following:

$$\dot{\hat{a}}_{21} = k_5 e_2 x_1 \quad (20)$$

$$\dot{\hat{a}}_{22} = k_6 e_2 x_2 \quad (21)$$

Choose:

$$V_2 = \frac{1}{2k_5}\tilde{a}_{21}^2 + \frac{1}{2k_6}\tilde{a}_{22}^2 \quad (22)$$

Then:

$$\dot{V}_2 = -e_2 x_1 \tilde{a}_{21} - e_2 x_2 \tilde{a}_{22} \quad (23)$$

Choose:

$$V_3 = \frac{1}{2}k_4\left(\int e_2 dt\right)^2 \quad (24)$$

Then:

$$\dot{V}_3 = k_4 e_2 \int e_2 dt \quad (25)$$

Choose:

$$V_4 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \quad (26)$$

Then:

$$\dot{V}_4 = e_1\dot{e}_1 + e_2\dot{e}_2 \quad (27)$$

For the whole system, choose a big Lyapunov function as:

$$V = V_1 + V_2 + V_3 + V_4 \quad (28)$$

Then:

$$\dot{V} = \bar{k}_1 e_1^2 + a_{12} e_2 e_1 - k_3 e_2^2 \quad (29)$$

Though the inequality transformation, it holds:

$$\dot{V} \le \bar{k}_1 e_1^2 + \frac{1}{2}a_{12} e_1^2 + \frac{1}{2}a_{12} e_2^2 - k_3 e_2^2 \quad (30)$$

Obviously, by choosing large positive numbers k_1 and k_3, it is easy to get

$$\dot{V} \le 0 \quad (31)$$

So the system is stable and synchronization can be fulfilled.

5. Numerical Simulation

Assume the model of the controlled system can be described as follows:

$$\dot{x}_1 = a_{11} x_1 + a_{12} x_2 \quad (32)$$

$$\dot{x}_2 = a_{21} x_1 + a_{22} x_2 + u \quad (33)$$

Where unknown parameters are $a_{11} = 0.3$, $a_{12} = 0.7$, $a_{21} = 5.3$, $a_{22} = 15.2$, use the above design method, design control law as bellows:

$$u = -\hat{a}_{21}x_1 - \hat{a}_{22}x_2 + \dot{x}_2^d - k_3e_2 - k_4\int e_2dt \quad (34)$$

where

$$\dot{\hat{a}}_{21} = k_5e_2x_1 \quad (35)$$

$$\dot{\hat{a}}_{22} = k_6e_2x_2 \quad (36)$$

$$e_1 = x_1 - x_1^d \quad (37)$$

$$e_2 = x_2 - x_2^d \quad (38)$$

$$x_2^d = -k_1e_1 - \hat{k}_2 \quad (39)$$

$$\dot{\hat{k}}_2 = k_2e_1 \quad (40)$$

Assume the original state of the system is zero, define expected value $x_1^d = 1$, write Matlab program, choose control parameters as

$k_1 = 5, k_2 = 1, k_3 = 5, k_4 = 0.2, k_5 = 1, k_6 = 1$

And $\dot{x}_2^d$ is considered by four situations.

First, consider using pure differential algorithm in the first situation:

```
dx2d=(x2d-x2d0)/dt;    x2d0=x2d;
```

Second, consider using a first-order filtering differential algorithm in the second situation:

```
T=0.1;
dy=(x2d-y)/T;
y=y+dy*dt;
```

The third situation is the same with the second situation, but chooses filtering constant in the third situation as bellows:

```
T=0.01;
```

In the fourth situation, the influence of $\dot{x}_2^d$ on the system is neglected.

The simulation result for the first situation is as follows:

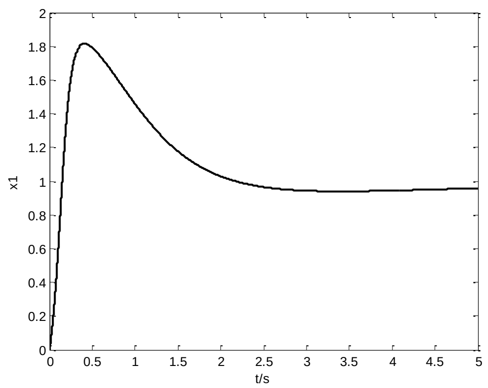

Figure 1 .Curve of State x1

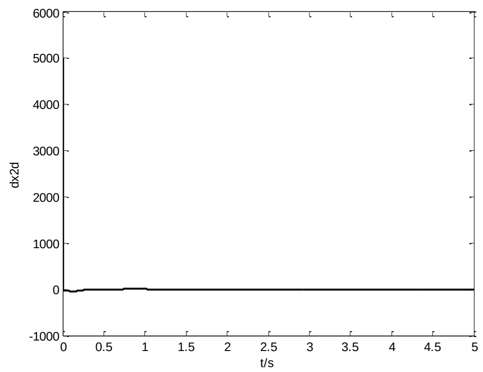

Figure 2. Curve of digital differential

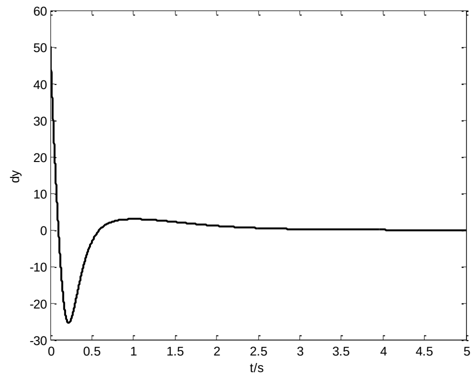

Figure 3. Curve of Filtering Differential（T=0.1）

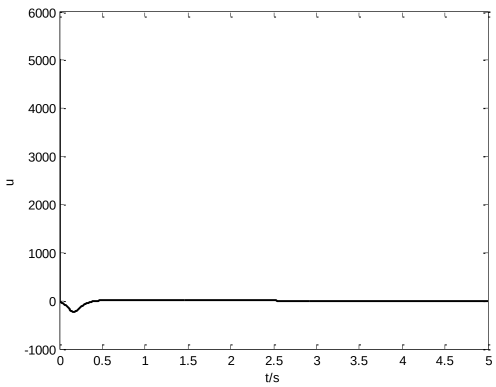

Figure 4. Curve of Control u

The simulation result for the second situation is as follows:

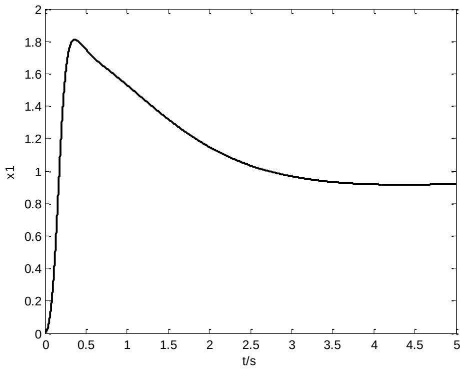

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Figure 5. Curve of State X1

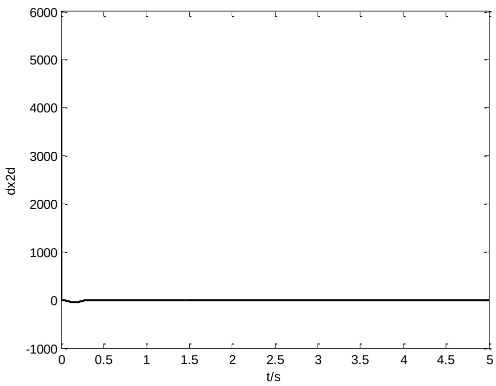

Figure 6. Curve of Digital Differential

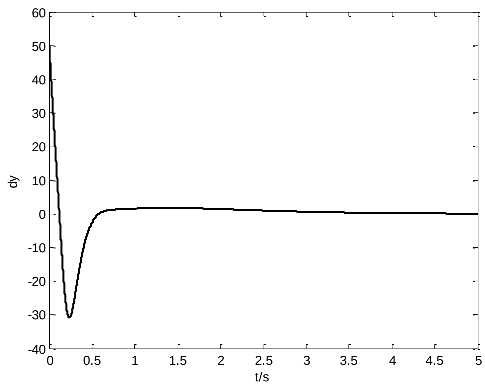

Figure 7. Curve of Filtering Differential（T=0.1）

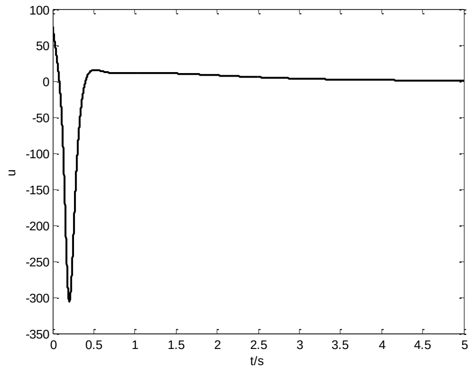

Figure 8. Curve of Control U

The simulation result for the third situation is as follows:

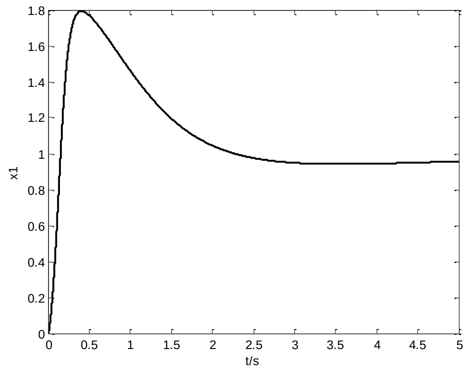

Figure 9. Curve of State X1

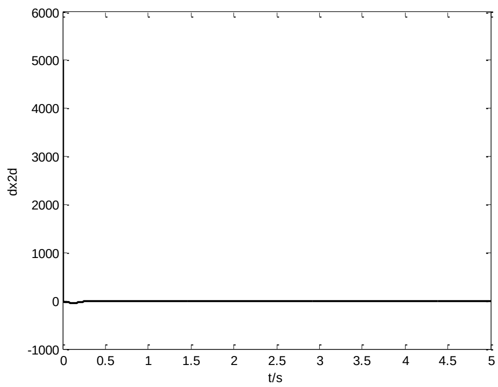

Figure 10. Curve of Digital Differential

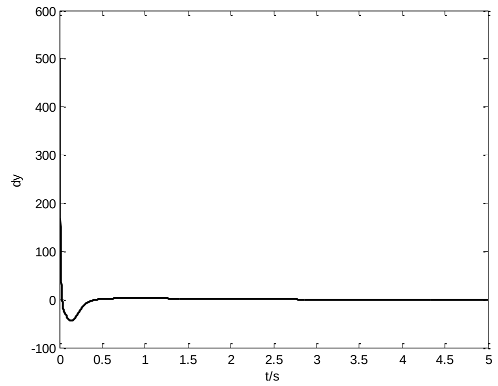

Figure 11. Curve of Filtering Differential（T=0.01）

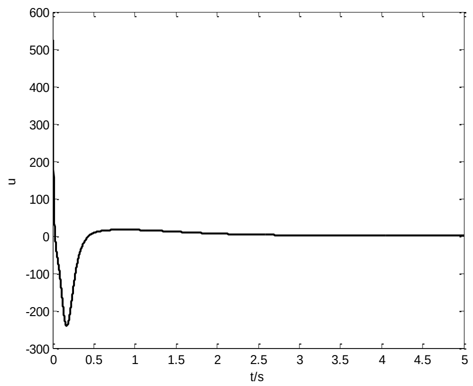

Figure 12. Curve of Control U

The simulation result for the fourth situation is as follows:

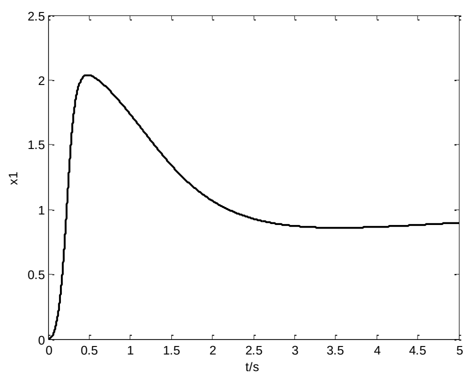

Figure 13. Curve of State X1

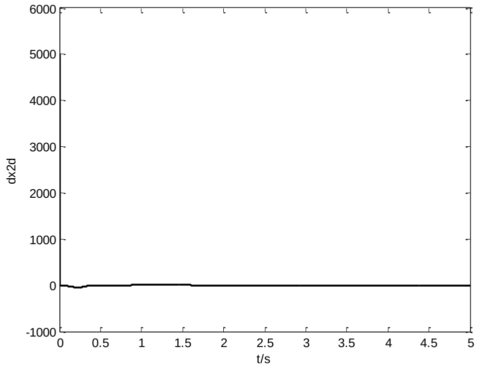

Figure 14. Curve of Digital Differential

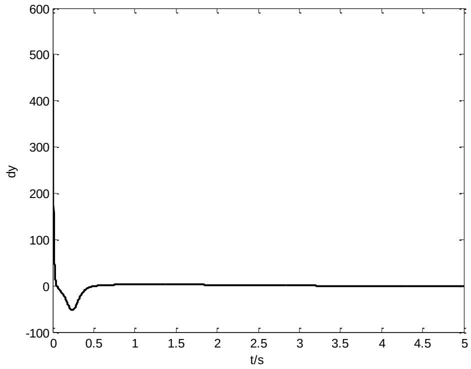

Figure 15. Curve of Filtering Differential（T=0.1）

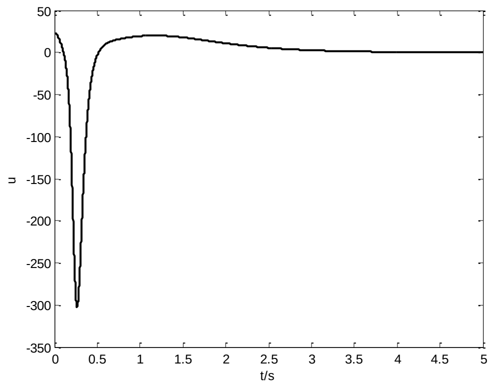

Figure 16. Curve of Control U

By comparing the simulation result of the above four situations, we can make a conclusion as follows:

First, if the numerical differentiation is used, the error of the first step is large, so the control u is large.

Second, if the filtering differentiation is used, the smaller the filtering constant is, the larger the initial differential coefficient can be chosen.

The last, if the differential coefficient is neglected, the overshoot of the system output response is very large.

So considering the above four situations, the conclusion can be made as follows: the comprehensive effect of one order filtering differential algorithm is the best and pure differential algorithm is the worst. And how to choose a proper filter constant is the key of one order differential method.

6. Conclusion

The differential bomb problem of traditional backstepping method is research in this paper and four kinds of differential methods are proposed to solve the derivative of expected state based on common adaptive backstepping design of a uncertain linear second order system. Also, detailed numerical simulations were done for four kinds methods respectively. At last, the conclusion points out that the one order filter differential method has best performance and the pure differential algorithm is the worst one.

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