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Dynamic State-Derivative Feedback Controller for Uncertain Equilibrium Point Stabilization

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Abstract

We consider a local stabilization problem of an uncertain equilibrium point existed in a nonlinear continuous-time system by a state-derivative feedback controller. In previous researches, it is investigated that the uncertainty of the equilibrium point results in nonzero steady-state control input so that a different equilibrium point is stabilized. Then, a feedback controllers with steady-state blocking zeros eliminate the dependence on the steady-state. In this paper, we focus on the class of state-derivative feedback control, and develop a design method for a dynamic state-derivative feedback controller with steady-state blocking zeros. The proposed controller can reject the dependence on the uncertainty of equilibrium points. Moreover, we illustrate the effectiveness of the proposed control method by the numerical example which is the stabilization problem of the uncertain equilibrium point. In addition, we show that the poles of the closed-loop system with the proposed dynamic controller can be assigned at the desired locations.

Keywords*: state-derivative feedback, equilibrium point, steady-state blocking zero*

1. Introduction

This paper presents a dynamic state-derivative feedback controller locally stabilizing an uncertain equilibrium point of nonlinear dynamical systems. In a design of feedback controllers stabilizing an equilibrium point, it is generally assumed that its equilibrium point is accurately known. However, this assumption is undesirable in a real system, because it is difficult to get exact information of equilibrium points of the system. In stabilization of the uncertain equilibrium point, the uncertainty of the equilibrium point results in nonzero steady-state control input so that a different equilibrium point is stabilized.

As a control method for stabilizing equilibrium points without their exact information, delayed feedback control[1]–[2], adaptive feedback control [3], washout filter-aided feedback control [4], washout control [5], and state-derivative feedback control [6] have been proposed in previous researches. These feedback controllers eliminate the dependence on the steady-state by using its steady-state blocking zero. Then, steady-state blocking zeros mean blocking zeros at zero frequency.

In this paper, we focus on the class of state-derivative feedback control as with [6]. Then, there exists some practical problems where the state-derivative signals are easier to obtain than the state signals. For example, vibration suppression in mechanical systems: car wheel suspension, vibration control of bridge cables, and so on. Since the sensors used in these cases are accelerometers, if the state variables are defined by the velocities and displacement, then it is available the state-derivative as feedback.

State-derivative feedback control has been used by many researchers [7]–[10]. In [7], the decoupling of linear time-invariant (LTI) systems by proportional and state-derivative feedback was discussed. In [8],for linear descriptor systems, the design method of state-derivative feedback based on Linear Matrix Inequalities(LMIs) was proposed. In [9] and

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[10], for LTI systems with single-input, the pole assignment and the optimal control problem were considered. However, in contrast to the simple controller structure of state-derivative feedback, the design of feedback parameters is complicated.

In this paper, as an easier way, we introduce a dynamic controller with state-derivative feedback. Then, we propose the design methods of the feedback parameters stabilizing uncertain equilibrium points. Moreover, we show that the poles of the closed-loop system with the proposed controller can be assigned at the desired locations.

This paper is organized as follows: first, in section2, a local stabilization problem of an uncertain equilibrium point existed in a nonlinear continuous-time system is presented. In section 3, we propose a dynamic state-derivative feedback controller which can reject the dependence on the uncertainty of equilibrium points. Then, we give a design method of the proposed controller. In section4, we show numerical simulations which illustrate the effectiveness of the proposed method.

2. Problem Statement

We consider an nth-order linear continuous-time invariant system described by

$$\dot{\square}(\square) = \square(\square(\square), \square(\square)), \quad (1)$$

where $\square(\square) \in \mathrm{R}^{\square}$ is the state vector, and $\square(\square) \in \mathrm{R}^{\square}$ is the input vector. We assume that f is differentiable, and there exists an equilibrium point of the system (1) with $\square(\square) \equiv 0$, that is,

$$0 = \square(\square_{\square}, 0).$$

The linearized system around the equilibrium point x_e is given by

$$\square\dot{\square}(\square) = \square\square\square(\square) + \square\square(\square), \quad (2)$$

where

$$\square\square(\square) = \square(\square) - \square_{\square}, \quad \square = \left.\frac{\square\square(\square,\square)}{\square\square}\right|_{\square=\square_{\square},\square=0},$$

$$\square = \left.\frac{\square\square(\square,\square)}{\square\square}\right|_{\square=\square_{\square},\square=0}$$

Then, we assume that $(\square, \square)$ is stabilizable.

The control purpose in this paper is to stabilize the equilibrium point x_e of the system (1), that is, to design a feedback controller such that

$$\lim_{\square\to\infty} \square(\square) = 0, \quad (3)$$

where

$$\square(\square) = \begin{bmatrix} \square\square(\square) \\ \square(\square) \end{bmatrix}. \quad (4)$$

To this end, we consider the local stabilization of the equilibrium point x_e of the system (1) without changing its equilibrium point x_e.

If the equilibrium point x_e is available, it can be directly used in state feedback as

$$\square(\square) = \square(\square(\square) - \square_{\square})$$
$$= \square\square\square(\square)$$

where $\square$ is a feedback gain. However, it is generally difficult to get the exact value of the equilibrium point x_e in the real system. In this paper, it is assumed that the equilibrium point $□_□$ of the system (1) is uncertain, that is, the measured output $□(□)$ is described by

$$□(□) = □(□) - □_{□Δ} = □□(□) + Δ□_□ \quad (5)$$

where $□_{□Δ}$ is a measurable equilibrium point having a uncertain term $Δ□_□$ given by $□_{□Δ} = □_□ - Δ□_□$. Therefore, we consider the stabilization of the uncertain equilibrium point $□_□$ by using only information of $□(□)$ as feedback.

By using $□(□) = \left(□□ - □\right)^{-1}□$ and a dynamic feedback controller $□(□)$, the closed-loop system is depicted in Figure 1. For the closed-loop system, the uncertain term for the equilibrium point, $Δ□_□$, can be regarded as a steady-state disturbance. Then, a transfer function from the steady-state disturbance $Δ□_□$ to the state and input *z* is given by

$$□(□) = \begin{bmatrix} □(□)□(□)\left(□ - □(□)□(□)\right)^{-1} \\ □(□)\left(□ - □(□)□(□)\right)^{-1} \end{bmatrix}. \quad (6)$$

By this steady-state disturbance, in the steady-state, the state $□(□)$ and the control input $□(□)$ may be biased. Because, when the closed-loop system is stable, from the final value theorem, we have

$$\lim_{□\to\infty} □(□) = \lim_{□\to 0} □□(□)\frac{1}{□}Δ□_□ = □(0)Δ□_□. \quad (7)$$

This suggests that another equilibrium point $□_□^{'}$ is stabilized so that $0 = □\left(□_□^{'}, □_□\right)$where ue $□_□ \neq 0$ and $□_□^{'} \neq □_□$. Hence, in the uncertain equilibrium stabilization, it is important that the dynamic controller $□(□)$ stabilizes the linearized system (2) and eliminates the influence of the steady-state disturbance $Δ□_□$.

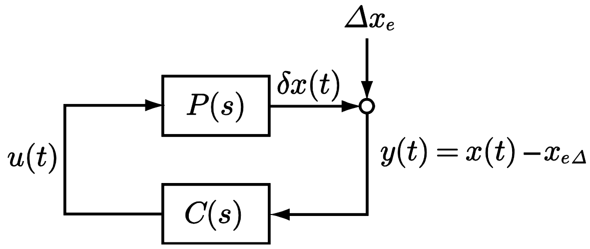

Figure 1. Closed-loop System of Linearized System and Controller

In this paper, to reject the influence of the uncertain term $Δ□_□$ in the steady-state, we focus on controllers with a blocking zero at zero frequency.

Definition 1: Blocking zeros of a transfer function matrix $□(□)$ are $□ \in \mathbb{C}$ which satisfy $□(□) = 0$. Moreover, a blocking zero at $□ = 0$ is said a *steady-state blocking zero*.

When the closed-loop system by a dynamical controller $□(□)$ having a steady-state blocking zero is asymptotically stable, from (7), we have

$$\lim_{□\to\infty} □(□) = 0.$$

Dynamic controllers with a steady-state blocking zero have been studied by several researchers [1]–[6]. In this paper, we focus on state-derivative feedback control, and propose a new dynamic state-derivative feedback controller which has steady-state blocking zeros.

Remark 1: In [6]–[10], by a state-derivative feedback controller,

$$□'(□) : □(□) = □\dot{□}(□), \tag{8}$$

the pole placement problem has been considered. In fact, since the transfer function of this controller $□'(□) = □□$, it has the steady state blocking zero.

In contrast to the simple controller structure of statederivative feedback (8), the design of feedback parameters is complicated. In the next section, we introduce a dynamic controller with state-derivative feedback.

3. Dynamic State-Derivative Feedback Controller

We consider an $\widetilde{□}$th-order dynamic state-derivative feedback controller described by

$$□(□) : \begin{array}{c} \dot{□}(□) = \widehat{□}□(□) + \widehat{□}\dot{□}(□) \\ □(□) = \widehat{□}□(□) \end{array}, \tag{9}$$

where $\widehat{□} \in R^{\widetilde{□}\times\widetilde{□}}$, $\widehat{□} \in R^{\widetilde{□}\times□}$, and$\widehat{□} \in R^{□\times\widetilde{□}}$ are design parameters.

In the vicinity of the equilibrium point $□_□$, we derive the closed-loop system with the dynamic controller (9). From (5), we obtain

$$\dot{□}(□) = \frac{□}{□□}(□□(□) + \Delta□_□) = □\dot{□}(□) \tag{10}$$

Therefore, from (2), (9), and (10), the closed-loop system is given by

$$\begin{bmatrix} □\dot{□}(□) \\ \dot{□}(□) \end{bmatrix} = □_□ \begin{bmatrix} □□(□) \\ □(□) \end{bmatrix}, \tag{11}$$

where

$$□_□ := \begin{bmatrix} □ & □\widehat{□} \\ \widehat{□}□ & □ + \widehat{□}□\widehat{□} \end{bmatrix}. \tag{12}$$

Then, we have the following lemma and theorem.

Lemma 1: The transfer function of the feedback controller (9) is given by

$$□(□) = □\widehat{□}\left(□□ - \widehat{□}\right)^{-1}\widehat{□} \tag{13}$$

Proof: It is obviously from (9).

Theorem 1: If the closed-loop system (11) is asymptotically stable by a dynamic state-derivative controller (9), then $□$ and $\widehat{□}$ are nonsingular.

Proof: When the closed-loop system (11) is asymptotically stable, $□_□$ does not have any zero eigenvalues, that is,

$$\det □_□ \neq 0.$$

Since

$$\det □_□ = \det \begin{bmatrix} □ & 0 \\ \widehat{□} & □ \end{bmatrix} \begin{bmatrix} □ & □\widehat{□} \\ 0 & \widehat{□} \end{bmatrix} = \det □ \det \widehat{□},$$

we have $\det □ \neq 0$ and $\det \widehat{□} \neq 0$.

From Theorem 1 and Lemma 1, when the closed-loop system is asymptotically stable, pole zero cancellation does not occurred. Thus, the dynamic state-derivative feedback controller (9) has a steady-state blocking zero. Therefore, if the closed-loop system is asymptotically stable by the state-derivative controller (9), we have

$$\lim_{\square\to\infty} \square(\square) = 0,$$

that is, such the controller eliminates the influence of the steady-state disturbance $\Delta\square_{\square}$.

In the following, we will give a full order dynamic state-derivative feedback controller. The term 'full order' implies that the order of the controller is the same as that of the plant. Then, we have the following theorem.

Theorem 2: If $\square$ is nonsingular, then there exists a full order dynamic state-derivative feedback controller stabilizing the closed-loop system (11). Moreover, one of the controllers is given by

$$\begin{aligned} \widehat{\square} &= \square + \square\square + \square\square + \square\square\square, \\ \widehat{\square} &= -\square, \\ \widehat{\square} &= \square, \end{aligned} \tag{14}$$

where $\square$ and $\square$ are matrices such that $\square + \square\square$ and $\square + \square\square$ are asymptotically stable, respectively.

Proof: Using (14) and a similarity transformation of Ac by the matrices

$$\square = \begin{bmatrix} \square & 0 \\ \square & -\square \end{bmatrix}, \quad \square^{-1} = \begin{bmatrix} \square & 0 \\ \square & -\square \end{bmatrix}, \tag{15}$$

we have

$$\begin{aligned} \square\square_{\square}\square^{-1} &= \begin{bmatrix} \square & 0 \\ \square & -\square \end{bmatrix} \begin{bmatrix} \square & \square\widehat{\square} \\ \widehat{\square}\square & \square + \widehat{\square}\square\widehat{\square} \end{bmatrix} \begin{bmatrix} \square & 0 \\ \square & -\square \end{bmatrix} \\ &= \begin{bmatrix} \square + \square\square & -\square\square \\ 0 & \square + \square\square \end{bmatrix}. \end{aligned} \tag{16}$$

Thus, $\square_{\square}$ is asymptotically stable by the feedback parameters (14).

From the theorems 1 and 2, it is concluded that there exists a dynamic state-derivative feedback controller (9) which stabilizes the linearized system (2) if and only if A is nonsingular. Moreover, from (16), the poles of the closed-loop system of the linearized system (2) and the state-derivative controller (9) are eigenvalues of $\square + \square\square$ and $\square + \square\square$. As a result, we can assign the poles of the closed-loop system at the desired locations.

4. Numerical Example

In this section, we show the numerical example of thestabilization problem of uncertain equilibrium points. Weconsider the second-order linearized system with uncertainparameters is described by

$$\dot{\square}(\square) = \begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix} \square(\square) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \square(\square) + \square, \tag{17}$$

where $\square \in R^2$ is an uncertain parameter. In this paper,we assume that the system (17) is the true system when$\square = 0$. Then, the true system has an equilibrium pointgiven by

$$\square_{\square} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{18}$$

Moreover, it is assumed that true equilibrium points (18)are unknown. Then, we know the equilibrium point withuncertainty,

$$□_{□\Delta} = -\begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix}^{-1} □ = \begin{bmatrix} 0.5 & -1 \\ -1 & 0 \end{bmatrix} □, \tag{19}$$

that is, the measured output □(□) is given by

$$□(□) = □(□) - □_{□□} . \tag{20}$$

The control purpose in this example is to stabilize theequilibrium point (18), that is, $\lim_{□\to\infty} □(□) = 0$ and $\lim_{□\to\infty} □(□) = 0$.

Then, we will design a stabilizing controller (9) by usingonly derivative information of (20) as feedback.

From Theorem 2, there exists a full order controller(9), because the eigenvalues of A are -0.78 and 1.28.Then, matrices □ and □ such that the eigenvalues of□ + □□are equal to $-1 \pm □$, and the eigenvalues of□ + □□ are equal to $-2 \pm 2□$, are given by

$$□ = [-3 \quad -2.5], \; □ = \begin{bmatrix} 2 & -2 \\ -1 & -3 \end{bmatrix}.$$

From (14), feedback parameters are given by

$$\widehat{□} = \begin{bmatrix} 4 & 7 \\ 4 & 3 \end{bmatrix}, \; \widehat{□} = \begin{bmatrix} 2 & -2 \\ -1 & -3 \end{bmatrix}, \; \widehat{□} = [-3 \quad -2.5]. \tag{21}$$

Figure 2 shows the poles and blocking zeros plots for the closed-loop system of the system (17) and the dynamic state-derivative feedback controller (2) with (21). Figure 2(a) is that from $□□_□$ to □, and (b) is that from $□□_□$ to□. The poles are plotted as ‘x’ and the zeros are plottedas ‘o’. It can be seen that the poles are placed at desiredlocations and the blocking zeros are placed at the origin.

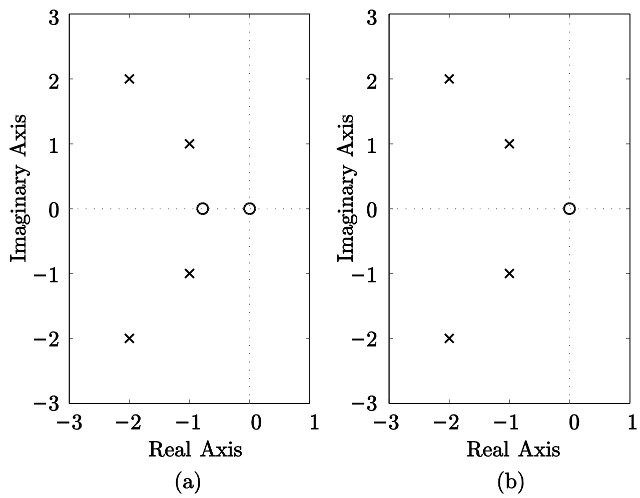

Figure 2. Poles and Blocking Zeros Plots for Closed-loopsystems

Figure 3 shows the time responses of the state $□(□) = [□_1(□) \quad □_2(□)]^T$, control input □(□) , and the measured output$□(□) = □(□) + □□_□ = [□_1(□) \quad □_2(□)]^T$ in the the system(17) with the dynamic state-derivative feedback controller(2). Then, the uncertain parameter *d* is given by

$$□ = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}.$$

The dashed line indicates □2(□) and □2(□), respectively.Figure 3 shows that both the control input □(□) and the state□(□)in the controlled system converge to 0 by the proposedstate-derivative controller. As a result, the uncertainequilibrium point xe in the system (17) can be stabilizedby the dynamic state-derivative feedback controllerwith the steady-state blocking zero.

5. Conclusions

In this paper, we have considered a local stabilizationproblem of an uncertain equilibrium point existed in anonlinear continuous-time system. Then, we have proposedthe controller which is a dynamic state-derivativefeedback controller with steady-state blocking zeros. Wehave also shown that the proposed state-derivative controllercan fully eliminate the bias of the output signal inthe steady-state. Moreover, we have illustrated the effectivenessof the proposed control method by the numericalexample which is the stabilization problem of the uncertainequilibrium point.

Future researches are to extend the proposed methodsto output feedback, applying robust control.

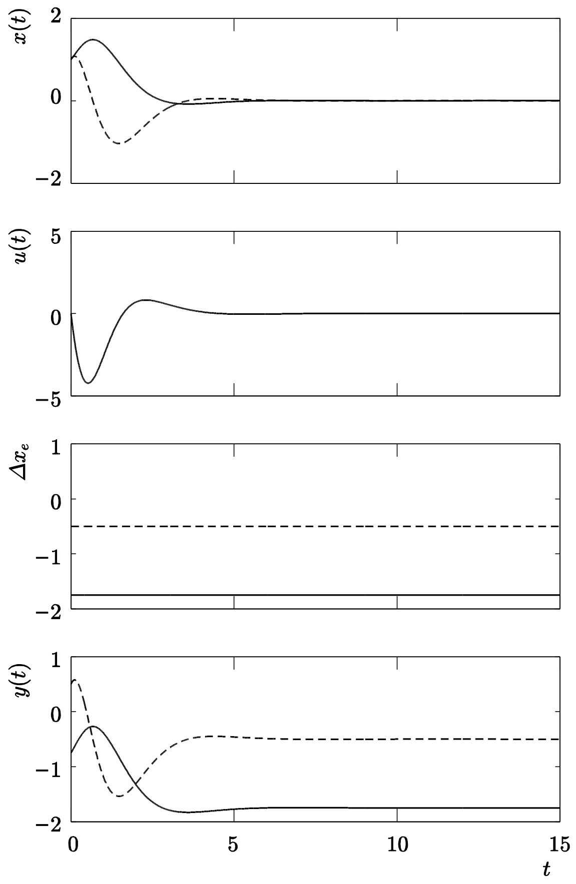

Figure 3.Time Responses for the System (17) with the Proposedcontroller

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