International Journal of Control and Automation
Vol.9, No.1 (2016), pp.177-190
http://dx.doi.org/10.14257/ijca.2016.9.1.16

Robust H_∞ Control Strategy for Switched Dissipative Hamiltonian Systems: Controller Parameterization

Zhong Cao[1, 2], Xiaorong Hou[1] and Yingfei Zhuang[1]

[1]School of Energy Science and Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China
[2]School of Computer Science and Educational Software, Guangzhou University, Guangzhou, Guangdong 510006, China
Houxr@uestc.edu.cn

Abstract

This paper focuses on the controller parameterization method of H_∞ control for switched dissipative Hamiltonian systems (SDHSs) via multiple Lyapunov functions (MLFs) and proposes an algorithm for solving parameters of the controller with symbolic computation. An important merit of the proposed parameterization method is that it is based on the explicit construction of Lyapunov functions, which avoids solving the Hamilton-Jacobi (HJ) equations (or inequalities), sufficient conditions for the solvability of the robust H_∞ control problem are presented. A numerical example shows that the controller is effective to SDHSs and the proposed method is feasibility.

***Keywords**: Robust control, Switched dissipative Hamiltonian systems, Controller parameterization, Multiple Lyapunov functions*

1. Introduction

Switched systems consist of a family of dynamical subsystems together with a set of operating conditions under which one subsystem will be switched to the active subsystem at switching time instant arise in many areas. The widespread applications of switched systems are also motivated by increasing performance requirements in control, especially in the presence of large uncertainties or disturbances [1]. A common Lyapunov function for all subsystems was proved to be a necessary and sufficient condition for switched system to be asymptotically stable under arbitrary switching laws [2, 3]. It has been shown that the MLFs approach proposed in [4] is an effective tool for choosing such switching laws. Ref. [5] developed the sufficient conditions for exponential stability and weighted L_2-gain for a class of switching signals with average dwell time. A concept of passivity for switched systems was presented in [6] using multiple storage functions. The passivity property is invariant under compatible feedback interconnection and the asymptotic stability is reached if all subsystems are asymptotically detectable. A hybrid nonlinear control methodology for a broad class of switched nonlinear systems with input constraints was proposed in [7], which is the integrated synthesis, via MLFs, of "lower-level" bounded nonlinear feedback controllers together with "upper-level" switching laws that orchestrate the transitions between the constituent modes and their respective controllers. The switched dissipative Hamiltonian system (SDHS) is a kind of important nonlinear hybrid systems. Such system not only plays an important role in development of hybrid control theory, but also finds many applications in practical control designs for obtaining better control performance [8, 9]. The stability of switched dissipative Hamiltonian systems under arbitrary switching paths has been investigated in [10].

ISSN: 2005-4297 IJCA

On the other hand, the H_∞ control of switched systems is a valuable issue for nonlinear systems, which deserves us to pay more attention. However, H_∞ control problem has been rarely addressed for SDHSs, especially for the nonlinear case in which results mainly focus on solving HJ equations [11-13]. The robust H_∞ control problem for a class of switched nonlinear systems with neutral uncertainties has been considered in [14]. Ref. [15] focused on a class of switched nonlinear cascade systems in which stabilization and weighted L_2-gain have been achieved and H_∞ control for such system was investigated in [16], and both of them relied on the solution of the corresponding HJ equations. However, Ref. [17] solved the problem of robust H_∞ control for a class of switched nonlinear cascade systems with parameter uncertainty using the MLFs approach, which avoids solving the HJ equations.

Controller parameterization is a fundamental problem in the control theory and has aroused considerable attention in recent decades. Refs. [18, 19] proposed a family of nonlinear H_∞ controller via output feedback. Ref. [20] presented a family of nonlinear state-feedback controller, in which the system state and the external disturbance are measurable. Refs. [21, 22] extended the state-space formulas and presented a family of H_∞ state-feedback controller for n-dimensional nonlinear system. Ref. [23] proposed a family of reliable nonlinear H_∞ controller via solving the HJ inequality. The controllers obtained in [18-23] are intended to solve a class of HJ equations (or inequalities), which have actually imposed a considerable difficulty.

Therefore, how to find ways for solving the controller parameterization problem of H_∞ control for switched nonlinear systems, which do not depend on the solution of HJ equations (or inequalities), is a challenging issue. In this paper, we present a novel, straightforward and convenient strategy to design a parameterized controller to insure that the SDHSs are robust H_∞ stable via MLFs and propose a method for solving parameters of the controller by using symbolic computation. The proposed parameterization method avoids solving HJ equations (or inequalities), and thus the obtained controllers with parameters are relatively simple in form and easy in operation.

The remainder of this paper is organized as follows. In Section 2, the problem of H_∞ control for SDHSs is formulated. The main contribution of this paper is then given in Section 3, in which a controller with parameters and an algorithm for solving parameters are provided, respectively. We present a numerical example for illustrating effectiveness and feasibility of controller in Section 4 and conclusions follow in Section 5.

2. Problem Formulation

Consider the following SDHSs

$$\begin{cases} \dot{x} = [J_{\lambda(t)}(x) - R_{\lambda(t)}(x)]\nabla H_{\lambda(t)}(x) + g_{\lambda(t)}(x)u_{\lambda(t)} + \bar{g}_{\lambda(t)}(x)\omega \\ z = h_{\lambda(t)}(x)g_{\lambda(t)}^T(x)\nabla H_{\lambda(t)}(x) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector; $u = u_{\lambda(t)} \in \mathbb{R}^m$ is the controller; $\omega \in \mathbb{R}^s$ is the disturbances; $J^T_{\lambda(t)}(x) = -J_{\lambda(t)}(x) \in \mathbb{R}^{n\times n}$; $0 \le R_{\lambda(t)}(x) \in \mathbb{R}^{n\times n}$; $g_{\lambda(t)}(x) \in \mathbb{R}^{n\times m}$ and $\bar{g}_{\lambda(t)}(x) \in \mathbb{R}^{n\times s}$ are sufficiently smooth functions; $z \in \mathbb{R}^q$ is the penalty; $h_{\lambda(t)}(x)$ is a weighting matrix; $H_{\lambda(t)}(x)$ is the subsystem's Hamiltonian function (the total energy) satisfying $H_{\lambda(t)}(x) > 0$ and $H_{\lambda(t)}(0) = 0$, the map $\lambda(t):[t_0,+\infty) \to \Lambda = \{1,2,\cdots N\}$ is a piecewise constant one, called the switching law or switching path,

$\lambda(t) = i\,(i = 1, 2, \cdots, N)$ denotes that the i th subsystem is realized. For an arbitrary switching law $\lambda(t): [t_0, +\infty) \to \Lambda = \{1, 2, \cdots, N\}$, $\{t_m\}_{m=0}^{+\infty}$ is called the switching time sequence, which is assumed to satisfy $t_0 < t_1 < t_2 < \cdots < t_m < \cdots + \infty$. If $t_{m+1} = \infty$, the i th subsystem of system (1) is always realized in $[t_m, +\infty)$ and the whole system is naturally stable.

To facilitate the analysis, throughout this paper, we denote by $Z_+ = \{0, 1, 2, \cdots\}$ for the set of all nonnegative integers. We first propose two assumptions for system and one definition for system (1) as follows.

Assumption 1. For $\forall i \in \Lambda$, the Hamiltonian function $H_i(x)$ satisfies $H_i(x) \in C^2$ and the Hessian matrix $\mathrm{Hess}(H_i(x_0)) > 0$.

Remark 1. Note that $H_i(x)$ has a local minimum at the equilibrium x_0 of system (1). It is straightforward that in Assumption 1, $H_i(x) \in C^2$ guarantees the existence of $\mathrm{Hess}(H_i(x))$ and $\mathrm{Hess}(H_i(x_0)) > 0$ guarantees that $H_i(x)$ is strict convex on some neighborhood of equilibrium x_0.

Assumption 2. For $\forall i \in \Lambda$ and $\forall x, y \in \mathbb{R}^n$, the Hamiltonian function $H_i(x)$ satisfies $H_i(x) > H_i(y) \Leftrightarrow \|x\|_P > \|y\|_P$, where $\|x\|_P \triangleq \sup_{\|x\|=1} x^T P x$ and $P > 0$ is a positive definite matrix.

Remark 2. Assumption 2 implies that the Hamiltonian function (Lyapunov function) $H_i(x)$ increases with the increase of $\|x\|_P$. Obviously, this assumption can be satisfied for many Hamiltonian systems. Thus, Assumption 2 is a realistic one.

Definition 1. The problem considered in this paper is to propose an approach to parameterizing controller for systems (1), which can be described as: given a disturbance attenuation level $\gamma > 0$ and an arbitrary switching law $\lambda(t)$, we can obtain a controller with parameters of the form $u = u_{\lambda(t)}$ such that

R1: For $\forall i \in \Lambda, \forall \omega, t \in \mathrm{T}_m$ and $m \in Z_+$, the inequality $\dot{H}_i(x) + Q_i(x) \le \frac{1}{2}\{\gamma^2 \|\omega\|^2 - \|z\|^2\}$ holds along the trajectories of the closed-loop systems, which consisted of system (1) and $u = u_{\lambda(t)}$, where $Q_i(x) \ge 0$ is a scalar function;

R2: The closed-loop system (1) is asymptotically stable when $\omega = 0$;

hold simultaneously.

3. Main Results

In this section, we propose an H_∞ controller with parameters for system (1) by using MLFs and an algorithm for solving parameters with symbolic computation. The parameterization strategy suggests a framework to solve the H_∞ control problem of SDHSs.

3.1 Parameterizing Controller

Theorem 1. Assume that Assumption 2 holds for system (1), $u \equiv 0$, $\omega \equiv 0$ and $\forall i \in \Lambda$, then system (1) is asymptotically stable under the arbitrary switching law $\lambda(t)$ with its dwell time $\tau > 0$.

Proof. Assume that $\lambda(t)$ be an arbitrary switching law with its dwell time $\tau > 0$. When $u \equiv 0$, $\omega \equiv 0$, we have system

$$\begin{cases} \dot{x} = [J_{\lambda(t)}(x) - R_{\lambda(t)}(x)]\nabla H_{\lambda(t)}(x) \\ z = h_{\lambda(t)}(x) g_{\lambda(t)}^{T}(x)\nabla H_{\lambda(t)}(x) \end{cases} \tag{2}$$

When $\lambda(t) = i \in \Lambda$ for system (2), we consider the i th subsystem. We have

$$\dot{H}_i(x(t)) = -\frac{\partial H_i^T(x(t))}{\partial x} R_i(x(t)) \frac{\partial H_i(x(t))}{\partial x} \le 0 \ , \ t \in [t_m, t_{m+1})$$

So we obtain that

$$H_i(x_m) \ge H_i(x_{m+1}) \tag{3}$$

From Assumption 2 and formula (3), the following result holds

$$\|x_m\|_p \ge \|x_{m+1}\|_p \tag{4}$$

According to Assumption 2, we know that the following inequality holds for $\forall i \in \Lambda$.

$$H_i(x_m) \ge H_i(x_{m+1}), \forall m \in Z_+ \tag{5}$$

Inequality (5) shows that all the Hamiltonian functions $H_i(x)$ of the subsystems can be used as the MLFs for system (2). According to Ref. [4], system (2) is stable under the switching law $\lambda(t)$.

Remark 3. From the proof of Theorem 1, under Assumption 2, all of the Hamiltonian functions $H_i(x)$ of the subsystems can be used as the MLFs for system (1).

Theorem 2. If Assumption 1 and 2 hold for system (1) and give a disturbance attenuation level $\gamma > 0$, to $\forall i \in \Lambda$

$$R_i(x) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right) \ge 0 \tag{6}$$

$$\nabla H_i^T(x) g_i K_i(x, v) \le 0 \tag{7}$$

hold simultaneously, where $K_i(x, v) \in \mathbb{R}^{m\times 1}$ are parameterized parts of controller, v are the parameters. Then under an arbitrary switching law $\lambda(t)$, H_∞ control of system (1) can be realized by following controller which satisfies the rules R1 and R2.

$$u = u_{\lambda(t)}, u\Big|_{\lambda(t)=i} = -\frac{1}{2}\left(h_i^T h_i + \frac{1}{\gamma^2} I_m\right) g_i^T \nabla H_i(x) + K_i(x, v) \tag{8}$$

where I_m is an $m \times m$ unit matrix and

$$Q_i(x) = \nabla H_i^T(x)\left(R_i(x) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i(x)$$
$$-\nabla H_i^T(x) g_i K_i(x, v).$$

Proof. Suppose $\lambda(t) = i \in \Lambda, t \in \mathrm{T}_m, m \in Z_+$ is an arbitrary switching law. Choose Lyapunov function as $V_i(x) = H_i(x) - c \ge 0$ ($c = H_i(x_0)$).

$$\begin{aligned}
\dot{H}_i(x) &= \nabla H_i^T\left[J_i(x)-R_i(x)\right]\nabla H_i + \nabla H_i^T g_i u_i + \nabla H_i^T \bar{g}_i \omega \\
&= -\nabla H_i^T R_i(x)\nabla H_i + \nabla H_i^T g_i\left[-\frac{1}{2}\left(h_i^T h_i + \frac{1}{\gamma^2}\right)g_i^T \nabla H_i + K_i(x,\nu)\right] + \nabla H_i^T \bar{g}_i \omega \\
&= -\nabla H_i^T\left(R_i(x) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i - \frac{1}{2}\nabla H_i^T g_i h_i^T h_i g_i^T \nabla H_i \\
&\quad - \frac{1}{2\gamma^2}\nabla H_i^T \bar{g}_i \bar{g}_i^T \nabla H_i + \nabla H_i^T g_i K_i(x,\nu) + \frac{1}{2}\nabla H_i^T \bar{g}_i \omega + \frac{1}{2}\omega^T \bar{g}_i^T \nabla H_i \\
&= -\nabla H_i^T\left(R_i(x) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i + \nabla H_i^T g_i K_i(x,\nu) \\
&\quad + \frac{1}{2}\left(\gamma^2\|\omega\|^2 - \|z\|^2\right) - \frac{1}{2}\left\|\gamma\omega - \frac{1}{\gamma}\bar{g}_i^T \nabla H_i\right\|^2
\end{aligned} \tag{9}$$

From the above formula, we obtain

$$\begin{aligned}
&\dot{H}_i(x) + \nabla H_i^T\left(R_i(x) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i - \nabla H_i^T g_i K_i(x,\nu) \\
&= \dot{H}_i(x) + Q(x) \\
&= \frac{1}{2}\left(\gamma^2\|\omega\|^2 - \|z\|^2\right) - \frac{1}{2}\left\|\gamma\omega - \frac{1}{\gamma}\bar{g}_i^T \nabla H_i\right\|^2 \\
&\le \frac{1}{2}\left(\gamma^2\|\omega\|^2 - \|z\|^2\right)
\end{aligned} \tag{10}$$

So the rule R1 can be satisfied, which implies the L_2 gain of the closed-loop system (1) controlled by controller (8) (from ω to z) is bounded by γ. Next, we prove that the closed-loop system is asymptotically stable at x_0 under the arbitrary switching law $\lambda(t)$, when $\omega \equiv 0$.

When $\omega \equiv 0$, the closed-loop system can be expressed

$$\begin{aligned}
\dot{H}_i(x) &= \nabla H_i^T\left[J_i(x)-R_i(x)\right]\nabla H_i + \nabla H_i^T g_i u_i \\
&= -\nabla H_i^T R_i(x)\nabla H_i + \nabla H_i^T g_i\left[-\frac{1}{2}\left(h_i^T h_i + \frac{1}{\gamma^2}\right)g_i^T \nabla H_i + K_i(x,\nu)\right] \\
&= -\nabla H_i^T\left(R_i(x) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i - \frac{1}{2}\nabla H_i^T g_i h_i^T h_i g_i^T \nabla H_i \\
&\quad - \frac{1}{2\gamma^2}\nabla H_i^T \bar{g}_i \bar{g}_i^T \nabla H_i + \nabla H_i^T g_i K_i(x,\nu) \\
&= -\nabla H_i^T\left(R_i(x) + \frac{1}{2\gamma^2}\left(g_i g_i^T - \bar{g}_i \bar{g}_i^T\right)\right)\nabla H_i - \frac{1}{2}\left\|h_i g_i^T \nabla H_i\right\|^2 \\
&\quad - \frac{1}{2\gamma^2}\left\|\bar{g}_i^T \nabla H_i\right\|^2 + \nabla H_i^T g_i K_i(x,\nu) \le 0
\end{aligned} \tag{11}$$

From Theorem 1, the rule R2 can be satisfied.

The closed-loop system (1) controlled by controller (8) is globally asymptotically stable under the arbitrary switching law $\lambda(t)$ .This completes the proof.

Remark 4. $K_i(x)$ are polynomial vectors, which are parameterized parts of controller. They have much simpler form and are easier to realize. When $K_i(x,\nu)=0,\forall i$, the controller (8) is a controller without parameters.

So the controller (8) is a family of H_∞ controller for SDHSs. Then we will solve the parameters of controller (8) and guarantee robust stability requirements to disturbances attenuation for system (1).

3.2. Solving Parameters (SP) Algorithm

From condition (6), when $R_i(x)+\frac{1}{2\gamma^2}\left(g_i g_i^T-\bar{g}_i\bar{g}_i^T\right)=0$, we can obtain the γ_i^*. Let $\gamma\geq\max\left\{\gamma_i^*\right\}$ such that condition (6) holds.

Then we propose an algorithm to find parameters ranges of controller (8) via solving the parameters of $K_i(x,\nu)$ in condition (7). The SP algorithm now proceeds as follows.

S1. Set $K_i(x,\nu)=\left[N_1(x,\nu_1)\quad N_2(x,\nu_2)\quad\cdots\quad N_m(x,\nu_m)\right]^T$ and suppose a positive integer r, which is the degree of polynomial vector $K_i(x,\nu)$. Write $N_i(x,\nu_i)=\sum_{j=1,r}^{j=l}\nu_{ij}p_r(x)$, where $l=\sum_r c(n+r-1,r)$, $p_r(x)=\prod_{i=1}^n x_i^{r_i}$, n is the number of state variable and r_i is the integers from 1 to r.

S2. Let $S_i=-\nabla H_i^T g_i K_i(x,\nu)$.

S3. Choose all terms of $\deg(S_i)\geq 3$ and $\deg(S_i)=1$ from S_i and let the coefficients of these terms be zero. So obtain a set of equations A_i.

S3.1. Observe equations A_i. When the right-hand side is only one item with parameters and the left-hand side is zero, let these parameters be zero and substitute them into A_i. Then obtain a simplified equations A_i'.

S3.2. Obtain a set of parameters solution U_{i1} via solving A_i' by using cylindrical algebraic decompositions (CAD) algorithm [24].

S3.3. Substitute U_{i1} into S_i and obtain a new polynomial S_i', which is a quadratic form.

S4. Rewrite S_i' as coefficient matrix M_i, and all principal minors of M_i must be positive semi-definite. Choose all principal minors of M_i and obtain inequalities B_i.

S4.1. Observe inequalities B_i. Let some parameters be zero and substitute them into B_i. Then obtain the simplified inequalities B_i'.

S4.2. Obtain a set of parameters solution U_{i2} via solving B_i' by using CAD algorithm.

S5. Let $U_i=U_{i1}\cup U_{i2}$ and substitute U_i into controller (8), thus obtain the polynomial parameterized controller. This completes the algorithm.

Remark 5. (1)The SP algorithm starts from $r=1$ normally.

(2) Solve the parameters ranges of the controller $u_{\lambda(t)}$ by using SP algorithm, respectively.

(3) It is merely to simplify computation that we let some parameters be zero before using CAD algorithm. However, these parameters are not necessarily zero. So the set of parameters solution obtained by SP algorithm is a subset of solutions.

4. Numerical Experiment

Using the result proposed in this paper, this section studies an example as well as some numerical simulations to support our new results. Consider a SDHS,

$$J_1(x)=\begin{bmatrix} 0 & x_1^2-\sin(x_2x_3) & -x_2 \\ -x_1^2+\sin(x_2x_3) & 0 & x_3 \\ x_2 & -x_3 & 0 \end{bmatrix}, g_{11}=\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}, g_{12}=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$R_1(x)=\mathrm{Diag}\{0,0,1\}, H_1(x)=x_1^2+x_2^2+x_3^2,$$

$$J_2(x)=\begin{bmatrix} 0 & -x_3^2 & x_1x_2 \\ x_3^2 & 0 & -\cos(x_3)^2 \\ -x_1x_2 & \cos(x_3)^2 & 0 \end{bmatrix}, g_{21}=\begin{bmatrix} 0 & 4 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, g_{22}=\begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 1 & 0 \end{bmatrix},$$

$$R_2(x)=\mathrm{Diag}\{0,3,2\}, H_2(x)=\frac{1}{2}x_1^2+2x_2^2+x_3^2 \tag{12}$$

4.1 Controller design and solving parameters

From system (12), it is easy to get

$$\mathrm{Hess}(H_1(x_0))=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}>0, \quad \mathrm{Hess}(H_2(x_0))=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}>0$$

So Assumption 1 holds.

Then, we check that condition (6) holds for all x and given γ. From system (12), we have

$$R_1(x)+\frac{1}{2\gamma_1^2}\left(g_{11}g_{11}^T-g_{12}g_{12}^T\right)=\mathrm{Diag}\left\{0,0,1+\frac{3}{2\gamma_1^2}\right\}$$

$$R_2(x)+\frac{1}{2\gamma_2^2}\left(g_{21}g_{21}^T-g_{22}g_{22}^T\right)=\mathrm{Diag}\left\{\frac{6}{\gamma_2^2},3+\frac{1}{2\gamma_2^2},2-\frac{1}{2\gamma_2^2}\right\}$$

Let $\gamma_1^*=1$, $\gamma_2^*=\frac{1}{2}$. To ensure that condition (6) holds, the following statement should be satisfied

$$\gamma\geq\max\{\gamma_1^*,\gamma_2^*\} \tag{13}$$

Next, we consider condition (7) such that system (12) satisfies robustness in H_∞ control.

In this example, $R_1(x)\geq 0$ and $R_2(x)\geq 0$, all of the Hamiltonian functions $H_i(x)$ of the subsystems can be used as the MLFs for system (12). So choose the Lyapunov function $V_1(x)=H_1(x)$, $V_2(x)=H_2(x)$. It follows from controller (8) that

$$u_1 = \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = -\frac{1}{2}\left[h_1^T h_1 + \frac{1}{\gamma^2} I_m \right] g_{11}^T \nabla H_1 + K_1(x,\nu)$$
$$u_2 = \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = -\frac{1}{2}\left[h_2^T h_2 + \frac{1}{\gamma^2} I_m \right] g_{21}^T \nabla H_2 + K_2(x,\nu) \quad (14)$$

where $K_1(x,\nu) = \left[K_{11}(x,\nu_{11}) \quad K_{12}(x,\nu_{12}) \right]^T$, $K_2(x,\nu) = \left[K_{21}(x,\nu_{21}) \quad K_{22}(x,\nu_{22}) \right]^T$.

We know $n = 3$ in system (12) and let $r = 1$. We have

$$K_{11}(x,\nu_{11}) = a_1x_1 + a_2x_2 + a_3x_3\,,\ K_{12}(x,\nu_{12}) = b_1x_1 + b_2x_2 + b_3x_3\,,$$

$$K_{21}(x,\nu_{21}) = c_1x_1 + c_2x_2 + c_3x_3\,,\ K_{22}(x,\nu_{22}) = d_1x_1 + d_2x_2 + d_3x_3$$

where $a_i, b_i, c_i, d_i, i = 1,2,3$ are the parameters.

From system (11), we obtain that

$$\nabla H_1(x) = \left[2x_1 \quad 2x_2 \quad 2x_3 \right]^T, \nabla H_2(x) = \left[x_1 \quad 4x_2 \quad 2x_3 \right]^T.$$

Let $S_1 = -\nabla H_1^T g_{11} K_1(x,\nu)$, $S_2 = -\nabla H_2^T g_{21} K_2(x,\nu)$,we have

$$S_1 = -2b_1x_2x_1 - 4a_1x_1x_3 - 2b_2x_2^2 - (4a_2 + 2b_3)x_2x_3 - 4a_3x_3^2$$
$$S_2 = -4d_1x_1^2 - 4(c_1 + d_2)x_1x_2 - 4d_3x_1x_3 - 4c_2x_2^2 - 4c_3x_2x_3$$

S_1 and S_2 are quadratic forms and can be rewritten as coefficient matrixes,

$$M_1 = \begin{bmatrix} 0 & -b_1 & -2a_1 \\ -b_1 & -2b_2 & -2a_2 - b_3 \\ -2a_1 & -2a_2 - b_3 & -4a_3 \end{bmatrix}, M_2 = \begin{bmatrix} -4d_1 & -2c_1 - 2d_2 & -2d_3 \\ -2c_1 - 2d_2 & -4c_2 & -2c_3 \\ -2d_3 & -2c_3 & 0 \end{bmatrix}.$$

All principal minors of M_1 must be positive semi-definite. We have inequalities A_1 from M_1. From A_1, we can easy to obtain that $b_2 \le 0$ and $a_3 \le 0$. Substitute $U_{11} = \{a_1 = 0, b_1 = 0\}$ into inequalities A_1 for simplify computation, we obtain simplified inequalities B_1. Solving inequalities B_1 by using CAD algorithm, we obtain a series of sets. Choose some sets, which satisfy inequalities B_1, and organize them. We have

$$U = \{a_2 = 0, b_2 \le 0, a_3 \le 0, b_3 = 0\} \cup U_{11} \quad (15)$$

Substitute U into controller (14),

$$u_1 = -\frac{1}{2}\left[h_1^T h_1 + \frac{1}{\gamma^2} I_m \right] \begin{bmatrix} 4x_3 \\ 2x_2 \end{bmatrix} + \begin{bmatrix} a_3x_3 \\ b_2x_2 \end{bmatrix} \quad (16)$$

where, $a_3 \le 0$, $b_2 \le 0$.

Similar to that obtain u_1, we can obtain u_2 from M_2,

$$u_2 = -\frac{1}{2}\left[h_2^T h_2 + \frac{1}{\gamma^2} I_m \right] \begin{bmatrix} 4x_2 \\ 4x_1 \end{bmatrix} + \begin{bmatrix} c_1x_1 \\ d_1x_1 \end{bmatrix} \quad (17)$$

where, $c_1 \le 0$, $d_1 \le 0$.

So we have the controllers with parameters for system (12). The controller (16) and controller (17) have rather simple form.

4.2 Simulations and Results

In order to evaluate the robustness of the controller (16, 17), we set the parameters of system (11) as: $\gamma = 1$, $h_1 = \mathrm{Diag}\{0.5, 0.6\}$, $h_2 = \mathrm{Diag}\{0.7, 0.6\}$, and the parameters of controller as: $a_3 = -10$, $b_2 = -10$, $c_1 = -10$, $d_1 = -10$. We obtain the controller (18),

$$\begin{cases} u = u_{\lambda(t)} \\ u\Big|_{\lambda(t)=1} = \begin{bmatrix} -12.5x_3 \\ -11.36x_2 \end{bmatrix} \\ u\Big|_{\lambda(t)=2} = \begin{bmatrix} -0.8x_2 - 10x_1 \\ -13.2x_1 \end{bmatrix} \end{cases} \tag{18}$$

Suppose that $x(0) = (1, -1, -0.5)^T$ is the pre-assigned operating point of system (12), we impose an external disturbance $\omega = [4, 4]^T$ on system (12) during the time period 0.6~0.9s and 3.0~3.6s.

Figure 1 and Figure 2 are the response of the state and the controller (18) of system (12) under the switching law $\lambda_1(t)$.

$$\lambda_1(t) = \begin{cases} 2, & t \in [t_{2k}, t_{2k+1}), t_{2k+1} - t_{2k} = 0.15, \\ 1, & t \in [t_{2k+1}, t_{2k+2}), t_{2k+2} - t_{2k+1} = 0.15, \end{cases} \quad k = 0, 1, 2, \cdots ,$$

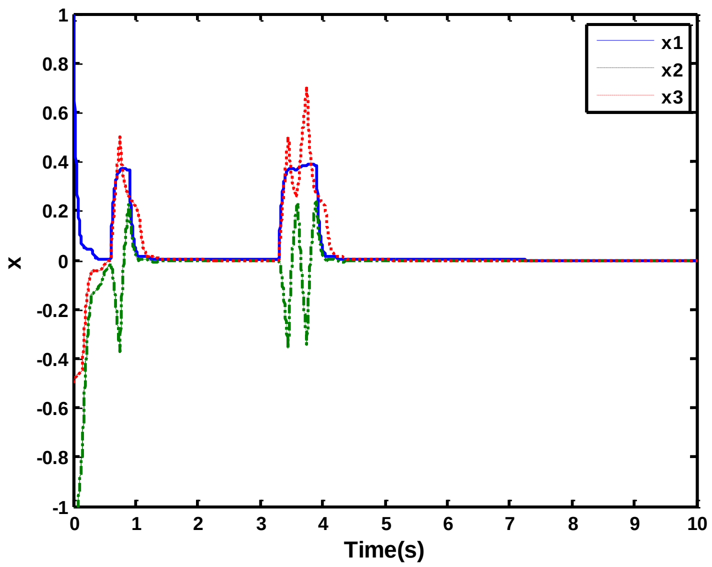


Figure 1. Swing Curves Of x in Switching Law $\lambda_1(t)$

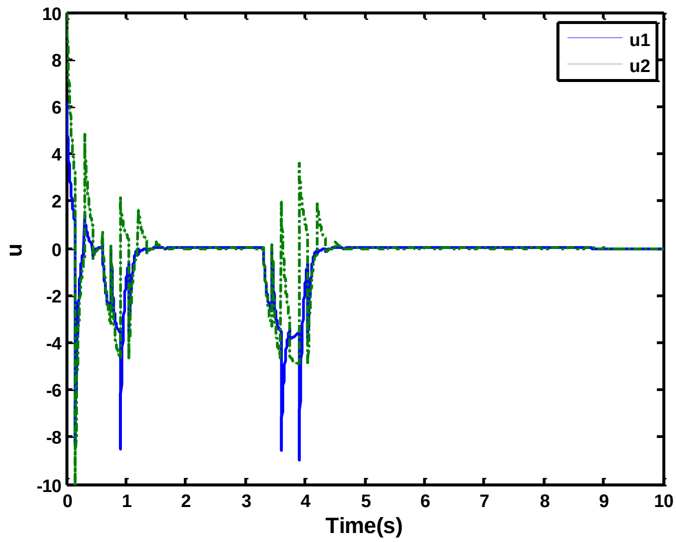


Figure 2. Swing Curves Of u in Switching Law $\lambda_1(t)$

Figure 3 and Figure 4 are the response of the state and the controller (18) of system (12) under the switching law $\lambda_2(t)$.

$$\lambda_2(t) = \begin{cases} 1, & t \in [t_{2k}, t_{2k+1}), t_{2k+1} - t_{2k} = 0.4 \times rand, \\ 2, & t \in [t_{2k+1}, t_{2k+2}), t_{2k+2} - t_{2k+1} = 0.4 \times rand, \end{cases} \quad k = 0,1,2,\cdots ,$$

where $0 \le rand < 1$.

From Figure 1~Figure 4, we can clearly see that under the switching law $\lambda_1(t)$ and $\lambda_2(t)$, the closed-loop system (12), it takes short time for system to return back to the equilibrium point. It can be seen, the obtained controller with parameters can stabilization the system (12) under external disturbance. The simulation shows that our result is correct and efficient.

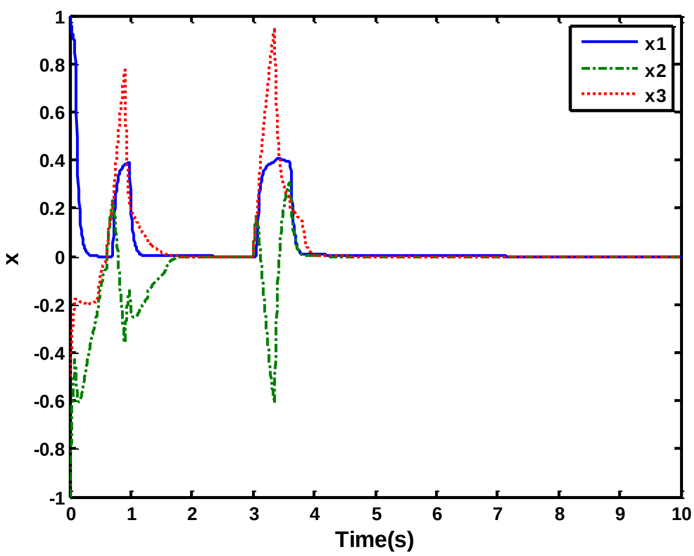


Figure 3. Swing Curves of x in Switching Law $\lambda_2(t)$

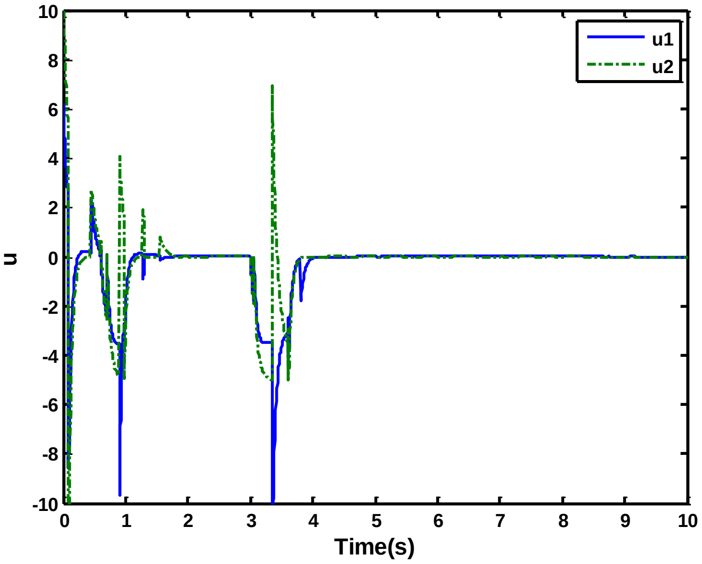


Figure 4. Swing Curves of u In Switching Law $\lambda_2(t)$

5. Conclusion

In this paper, an H_∞ control strategy to parameterizing controller for SDHSs has been considered. A controller with parameters has been obtained using MLFs method and an algorithm for solving parameters of the controller has been proposed with symbolic computation. The proposed parameterization method avoids solving HJ equations (or inequalities) and thus the obtained controllers with parameters are relatively simple in form and easy in operation. The numerical experiment and simulations show that the controller has efficient in H_∞ control.

Acknowledgments

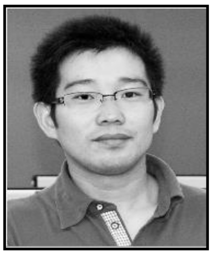
This work is supported by the National Natural Science Foundation of China No. 61074189 and No. 61374001.

References

[1] D. Liberzon and A. Morse, "Basic problems in stability and design of switched systems", IEEE Control System Magazine, vol. 19, no. 5, **(1999)**, pp. 59-70.
[2] L. Vu and D. Liberzon, "Common Lyapunov functions for families of commuting nonlinear systems", Systems & control letters, vol. 54, no. 5, **(2005)**, pp. 405-416.
[3] Y. Sun, L. Wang and G. Xie, "Stability of switched systems with time-varying delays: delay-dependent common Lyapunov functional approach", Proceeding of the 2006 American Control Conference, **(2006)**, pp. 6, American.
[4] M. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems", IEEE Transactions on Automatic Control, vol. 43, no. 4, **(1998)**, pp. 475-482.
[5] X. Sun, J. Zhao and D. Hill, "Stability and L_2-gain analysis for switched delay systems: A delay-dependent method", Automatica, vol. 42, no. 10, **(2006)**, pp. 1769-1774.
[6] J. Zhao and D. Hill, "Passivity and stability of switched systems: a multiple storage function method", Systems & Control Letters, vol. 57, no. 2, **(2008)**, pp. 158-164.
[7] N. El-Farra, P. Mhaskar and P. D. Christofides, "Output feedback control of switched nonlinear systems using multiple Lyapunov functions", Systems & Control Letters, vol.54, no.12, **(2005)**, pp.1163-1182.
[8] G. Escobar, A. van der Schaft and R. Ortega, "A Hamiltonian viewpoint in the modeling of switching power converters". Automatica, vol. 35, no. 3, **(1999)**, pp. 445-452.
[9] W.M. Haddad, S.G. Nersesor and V.S. Chellaboina, "Energy-based control for hybrid port-controlled Hamiltonian systems", Automatica, vol. 39, no. 8, **(2003)**, pp. 1425-1435.

[10] L. Zhu and Y. Wang, "Study on the stability of switched dissipative Hamiltonian systems", Science in China Series F: Information Sciences, vol. 49, no. 5, **(2006)**, pp. 578-591.
[11] G.S. Zhai, B. Hu, K. Yasuda and A. Michel, "Disturbance attenuation properties of time-controlled switched systems", Journal of the Franklin Institute, vol. 338, no. 7, **(2001)**, pp.765-779.
[12] X. Liu and S. Yuan, "On designing H_∞ default estimator for switched nonlinear systems of neutral type", Communications in Nonlinear Science and Numerical Simulation, vol. 16, no. 11, **(2011)**, pp. 4379-4389.
[13] W. Xiang and J. Xiao, "H_∞ finite-time control for switched nonlinear discrete-time systems with norm-bounded disturbance", Journal of the Franklin Institute, vol.348, no. 2, **(2011)**, pp.331-352.
[14] M. Wang, J. Zhao and G. Dimirovski, "H_∞ control for a class of cascade switched nonlinear systems", Asian Journal of Control, vol. 10, no.6, **(2008)**, pp.724-729.
[15] L. Li, J. Zhao and G. Dimirovski, "Robust H_∞ control for a class of switched nonlinear systems with neutral uncertainties", Transactions of the Institute of Measurement and Control, vol. 32, no. 6, **(2010)**, pp. 635-659.
[16] B. Niu and J. Zhao, "Stabilization and L_2-gain analysis for a class of cascade switched nonlinear systems: An average dwell-time method", Nonlinear Analysis: Hybrid Systems, vol. 5, no. 4, **(2011)**, pp. 671-680.
[17] B. Niu and J. Zhao, "Robust H_∞ control for a class of switched nonlinear cascade systems via multiple Lyapunov functions approach", Applied Mathematics and Computation, vol. 218, no. 11, **(2012)**, pp. 6330-6339.
[18] W. Lu and J. Dolye, "H_∞ control of nonlinear systems via output feedback: controller parameterization", IEEE Transactions on Automatic Control, vol. 39, no. 13, **(1994)**, pp. 831-846.
[19] A. Isidori and A. Astolfi, "Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems", IEEE Transactions on Automatic Control, vol. 37, no. 9, **(1992)**, pp. 1283-1293.
[20] A. Astolfi, "Parameterization of output feedback controller that satisfy an H_∞ norm bound", Proceedings of the 2nd European Control Conference, **(1993)**, pp. 74-78, Groningen, The Netherlands.
[21] C. Yung, Y. Lin and F. Yeh, "A family of nonlinear H_∞ output feedback Controllers", IEEE Transactions on Automatic Control, vol. 41, no. 2, **(1996)**, pp. 232-236.
[22] C. Yung, J. Wu and T. Lee, "Parameterization of nonlinear H_∞ state-feedback Controllers", Automatica, vol. 33, no. 8, **(1997)**, pp. 1587-1590.
[23] Y. Fu, Z. Tian and S. J. Shi, "A family of reliable nonlinear H_∞ state-feedback controllers", Control Theory and Application, vol. 10, no. 3, **(2001)**, pp. 350-358.
[24] B. Caviness, R. Jeremy and Johnson, "Quantifier Elimination and Cylindrical Algebraic Decomposition", Springer, Wien New York, **(1998)**.

Authors



Zhong Cao, he was born in Anhui Province, China, in 1977. He received his M.S. degree from school of computer science and educational software, Guangzhou University, Guangzhou, in 2005. He is currently pursuing the Ph.D. degree with School of Energy Science and Engineering, University of Electronic Science and Technology of China. His research interests include nonlinear control systems, and controlled Polynomial Hamiltonian systems.

E-mail: zhongc@gzhu.edu.cn

Xiaorong Hou, he was born in Shanxi Province, China, in 1966. Currently, he is a professor of School of Energy Science and Engineering at the University of Electronic Science and Technology of China. He has published over eighty research papers and two monographs. His research interests include symbolic computation, real algebraic geometry, control theory, and intelligent systems.

E-mail: houxr@uestc.edu.cn

Yingfei Zhuang, she was born in Henan Province, China, in 1988. She received her B.S. degree from School of Information Engineering, Henan Institute of Science and Technology, Xinxiang, in 2007. She is currently studying for the M.S. degree at School of Energy Science and Engineering, University of Electronic Science and Technology of China. Her research interests include symbolic computation and controller parameterization of nonlinear systems.

E-mail:zyf19882007@163.com