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Nussbaum Gain Control for Limit Input Systems with Unknown Control Direction

Junwei Lei, Di Liu and Lingling Wang

Department of Control Engineering, Naval Aeronautical and Astronautical University, Yantai Shandong 264001, China
leijunwei@126.com, liudi5388466@163.com, lingling0711@163.com

Abstract

The limited input is constraint condition that prevails in actual physical system. The saturated that is caused by limited input is nonlinear factor that need to be considered in the design of control system. The input of control system is limited by saturation in many past research papers. The limited input Nussbaum gain control algorithm is proposed in this paper. On the basis of limited gain, the problem of actuator saturation is considered in this paper. The theoretical basis is provided for Nussbaum gain control experiencing from academic research to practical application. At last, detailed numerical simulation is done to testify the rightness and robustness of the proposed method. The algorithm greatly improves the stability range of the Nussbaum gain control and makes it depend on the initial value less and less.

Keywords: *gain control; constraint condition; dynamic property; adaptive control; robustness; unboundedness*

1. Introduction

The limited input is constraint condition that prevails in real systems, and it is studied by many learned men in recent years. Strictly speaking, any real systems are constrained by limited energy. for example, aircraft uses steering engine as actuator and the maximum deflection angle of steering engine does not exceeding 30 deg[1-4],the system use electric sign as drive and the maximum current or voltage is constrained by circuit physics. The above limited input is the basic saturated limit that form in practice. So the saturated that is caused by limited input is nonlinear factor that need to be considered in the design of control system.

There is a specialization of study for input that is limited by saturation. Huang Xian-lin researched robust gain control method for air-breathing hypersonic vehicle model which input is limited. In order to deal with saturated nonlinearity in the aircraft model, integral quadratic function is used to restrain calibration linear matrix inequality form to give variable gain algorithm of customer service actuator saturation. literature[17] reduced advent effects of the gain switching and saturating actuators by introducing compensator.

Nussbaum gain control method is a nonlinear adaptive control method, because of nonlinearity of Nussbaum function, especially unboundedness of most Nussbaum function, the gain is so big that will cause control saturation if gain is misdesigned or controlled objects face disturbance. so it is necessary meaningful to study Nussbaum gain problem with limited input. It is indicated in this paper that Nussbaum gain method with limited input improve robustness of original Nussbaum gain method in a certain extent [5-8].

2. Problem Description

Considering the following typical one order system with unknown control directions:

$$\dot{x} = f(x) + bu \tag{1}$$

where x is state variable of system, u is control, or call input., $f(x)$ is known nonlinear function, b is unknown control direction, also call it control coefficient or input coefficient.

The control objective of limit input Nussbaum gain control is to design a control law $u = q(x,k)$, $\dot{k} = p(x,k)$, where k is called Nussbaum gain and u satisfies $|u| \le u_{sat}$, such as the system state x can trace the expected value x^d whether the sign of control coefficient\ b is change or not.

On the basis of the strategy of the gain is limited, a solution for the limited input is studied in this article, a saturated ideal controlled variable design method is used and an ideal controlled variable is designed, and due to the final controlled variable is the product of ideal controlled variable and the limited Nussbaum, so bounded controlled variable can be ensured [9-12].

3. Basic Assumptions

The hypothesis of limited control is as follows:

Assumption 1:The given saturation value of input u_{sat} is big enough, at least it can satisfies the basic energy demand for making the system to be stable.

Assumption 2:The given saturation value of gain k_{sat} is big enough, at least it can satisfies the basic energy demand for making the system to be stable.

Assumption 3:Without loss of generality, assume the expected value is a constant, so its derivatives is 0, then $\dot{x}^d = 0$.

Assumption 4: The control direction b is bounded, and it belongs to an unknown closed interval $I = [l_1^-, l_1^+]$, where $0 \notin I$, so it means that the sign of b is unknown.

Assumption 5: The bound of b is known, so it means that there exists a known constant $b_{\max}$ such that $|b| \le b_{\max}$.

Assumption 6: The times of changing of control direction is limit [13-16].

4. Nussbaum Gain Design for Limit Input Situation

Define the error variable as $z = x - x^d$, then the error system can be written as:

$$\dot{z} = f(x) + bu \tag{2}$$

Use the poles placement method to design the virtual control as:

$$u^{d1} = -f(x) - k_1 z - k_2 \int z dt \tag{3}$$

Considering the input is limit by $|u| \le u_{sat}$, so design the ideal control as follows:

$$u^d = \begin{cases} u_{sat} & u^{d1} > u_{sat} \\ u^{d1} & |u^{d1}| < u_{sat} \\ -u_{sat} & u^{d1} < -u_{sat} \end{cases} \tag{4}$$

With the assumption 1, it is easy to prove that the system is stable when the system input is equal to ideal input. So this question is neglected here.

So considering the unknown control direction of control, design the Nussbaum gain control law as follows:

$$u = -N(k) u^d \tag{5}$$

then

$$\dot{z} = -k_1 z - k_2 \int z dt - bN(k)u^d - u^d \tag{6}$$

Design the Nussbaum gain turning law as:

$$\dot{l} = k_l z u^d \tag{7}$$

And choose a limit Nussbaum gain function as:

$$k = f_s(l) \tag{8}$$

where $f_s(l)$ can choose a sine function.

Then the error equation can be written as:

$$z\dot{z} = -k_1 z^2 - k_2 z \int z dt - \frac{1}{k_l}(bN(k)+1)dl \tag{9}$$

And construct a kind of integral Lyapunov function as follows:

$$V = \frac{1}{2}z^2 + \frac{k_1}{2}\int z^2 dt + \frac{k_2}{2}\left(\int z dt\right)^2 \tag{10}$$

Solve it derivatives as:

$$k_l \dot{V} = -(bN(k)+1)dl \tag{11}$$

Solve its integration along both sides of equation, it holds:

$$\begin{aligned} k_l V(t) - k_l V(0) &= \int -(bN(k)+1)dl \\ &= \int_{l(0)}^{l(t)} -bN(k)dl + l(0) - l(t) \end{aligned} \tag{12}$$

To make it easy to understand, choose limit gain function as $k = 10\sin(l)$, and choose nussbaum gain function as $N(k) = k^2 \sin k$. Obviously, choose that design the gain of the system can be bounded, and the saturation value of gain can be chosen according to the system energy demand or actual physical limitation or engineering experience [17-19].

And now we only have to prove the system can keep stable with bounded virtual control even under the gain limitation situation. And also the system should has the ability to adapt to the disturbance of changing of control directions.

At first, assume l is unbounded, and consider the case that $l \to -\infty$, the above equation divided by l on both side then it holds:

$$\frac{k_l V(t) - k_l V(0) + l(0)}{l} = \frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl - 1 \tag{13}$$

And it can be arranged as:

$$\frac{k_l V(t)}{l} = \frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl - 1 \tag{14}$$

Since $-bN(k)$ is bounded, then there exists a positive constant ε_1 and ε_2 such as:

$$-\varepsilon_1 \le \frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl - 1 \le \varepsilon_2 \tag{15}$$

Obviously if $k_l > 0$, it holds:

$$\frac{l}{k_l}\varepsilon_2 \le V(t) \le -\frac{l}{k_l}\varepsilon_1 \tag{16}$$

if $k_l < 0$, it holds:

$$\frac{l}{k_l}\varepsilon_1 \le V(t) \le -\frac{l}{k_l}\varepsilon_2 \quad (17)$$

Assume the limitation of above integration exists, then

$$\lim_{l\to\infty}\frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl = \varepsilon + 1 \quad (18)$$

So whether it is positive or negative, it always has

$$V(t) = \varepsilon k_l l \quad (19)$$

Obviously, exists $k_l > 0$ or $k_l < 0$ such that $V(t) \le 0$ is contradictory.

If the limitation of above integration is not exists, then it will has oscillation, and there exists t such that

$$\lim_{l\to\infty}\frac{1}{l}\int_{l(0)}^{l(t)} -bN(k)dl = \varepsilon_3 \quad (20)$$

where

$$-\varepsilon_1 \le \varepsilon_3 \le \varepsilon_2 \quad (21)$$

So whether ε_3 is positive or negative, there exists $k_l > 0$ or $k_l < 0$ such that $V(t) \le 0$, the the contradiction is appeared[20-23].

Then $l(t)$ is proved to be bounded, and then $V(t)$ is bounded, $z(t)$ and u^d can also be proved to be bounded. Since the Lyapunov is positive and it has integration of error, so it is easy to prove that $z(t) \to 0$.

5. Numerical Simulation

By using above limit input Nussbaum gain method, the numerical simulation is done with a kind of simple one order system with unknown control direction and its model can be written as

$$\dot{x} = 3x\sin x + bu \quad (22)$$

Where the unknown control direction b is set as follows

$$b = \begin{cases} 1 & 0 < t < 3 \\ -2 & 3 < t < 6 \\ 2 & t > 6 \end{cases} \quad (23)$$

Choose the initial value of Nussbaum gain is $l(0) = 1$ and initial value of state is $x(0) = 0$, the expected value is 2, set the saturation of input is $u_{sat} = 20$, choose simulation step as 0.001s and use the simple Euler simulation method, the simulation result is as bellows.

Consider choose the initial value of Nussbaum gain as $x(0) = 0$, the simulation result is as follows:

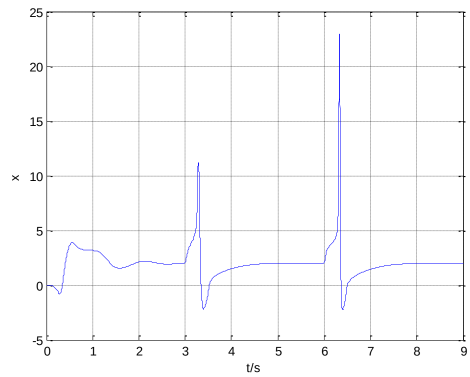

Figure 1. Curve of State x

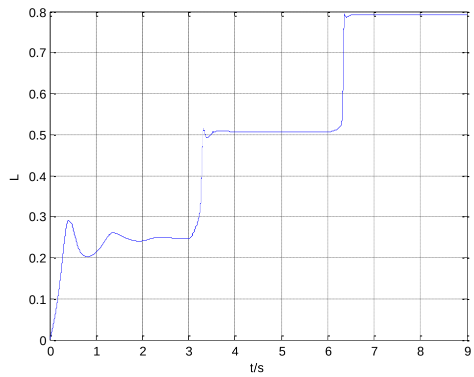

Figure 2. Curve of Gain L

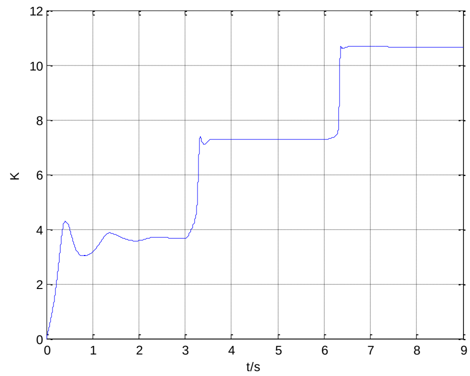

Figure 3. Curve of Gain K

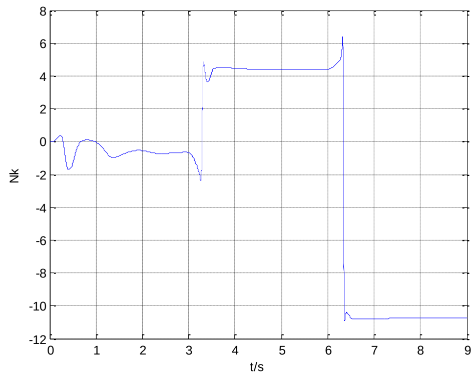

Figure 4. Curve of Gain Nk

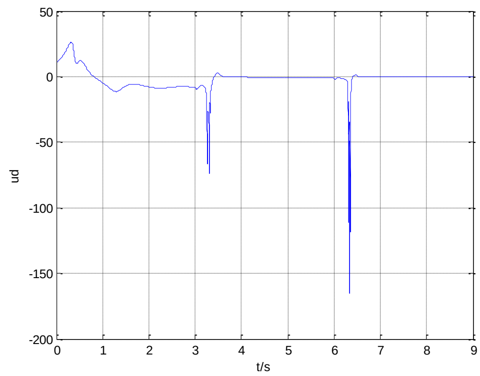

Figure 5. Ideal Control Ud

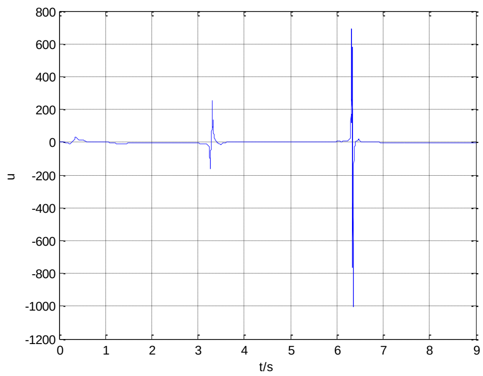

Figure 6. Curve of Control U

So the above simulation result testifies that the limit Nussbaum control method can make the system stable. The below simulation is done to testify the influence under the disturbance of initial Nussbaum gain. Choose initial Nussbaum gain as $l(0) = 10$, the simulation result is as follows.

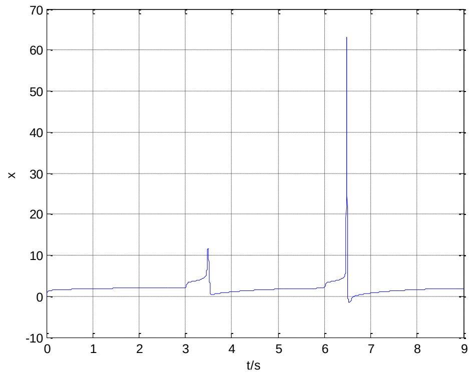

Figure 7. Curve of State x

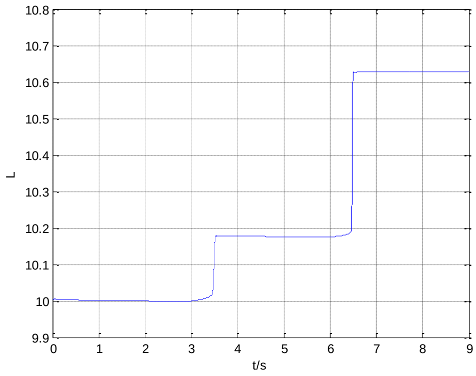

Figure 8. Curve of Gain L

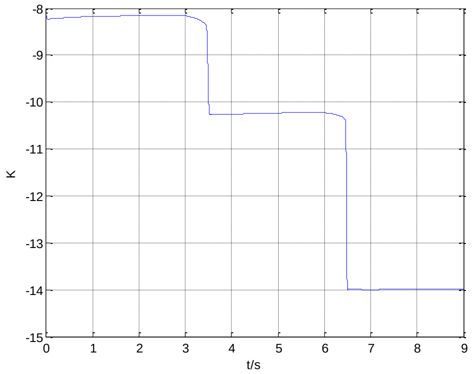

Figure 9. Curve of Gain K

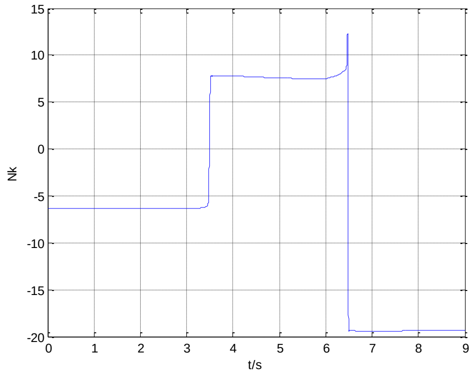

Figure 10. Curve of Gain Nk

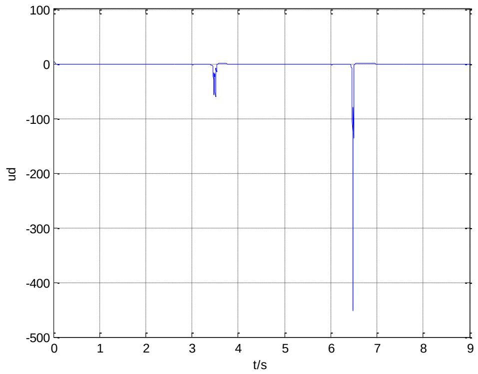

Figure 11. Ideal Control Ud

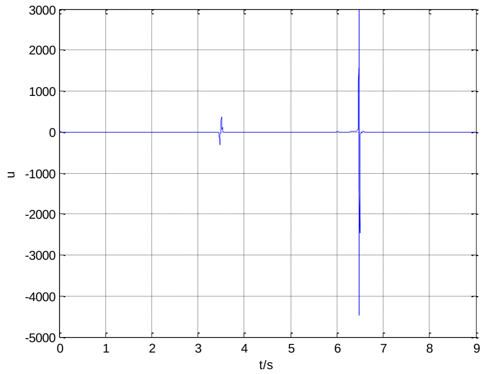

Figure 12. Curve of Control U

So the above simulation result shows that the limit input Nussbaum gain control can make the system stable with initial gain value $l(0)=10$ and below is the simulation result for the situation that the initial value of Nussbaum gain is $l(0) = -10$.

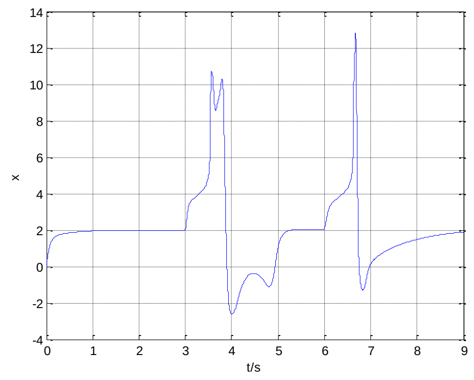

Figure 13. Curve of State x

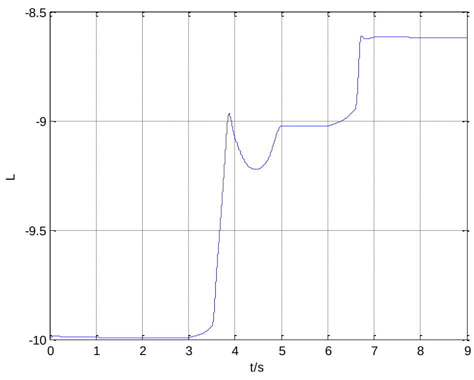

Figure 14. Curve of Gain L

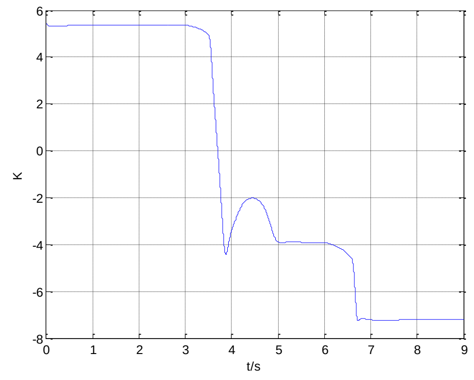

Figure 15. Curve of Gain K

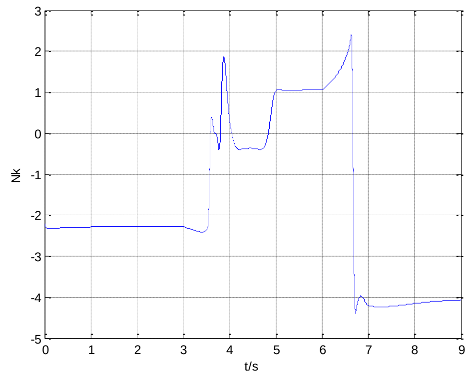

Figure 16. Curve of Gain Nk

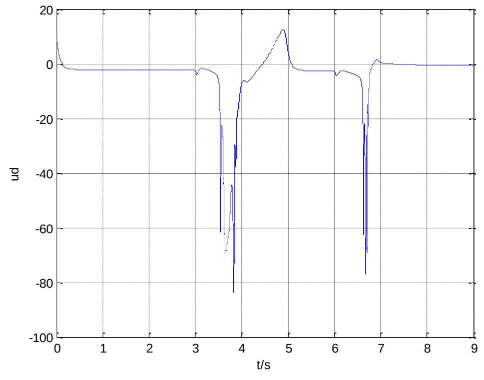

Figure 17. Ideal Control Ud

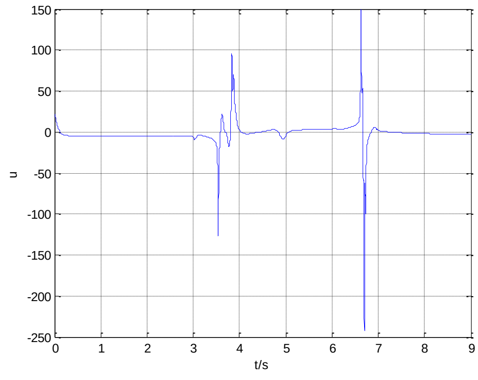

Figure 18. Curve of Control U

So the above simulation result shows that the limit input Nussbaum gain control strategy has good robustness for the disturbance of initial gain value. Choose different initial state below to testify its robustness with disturbance of initial state value, first set initial value of state is $x(0) = -2$, the simulation result is as follow:

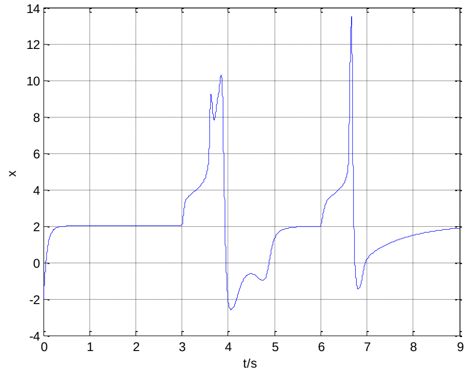

Figure 19. Curve of State x

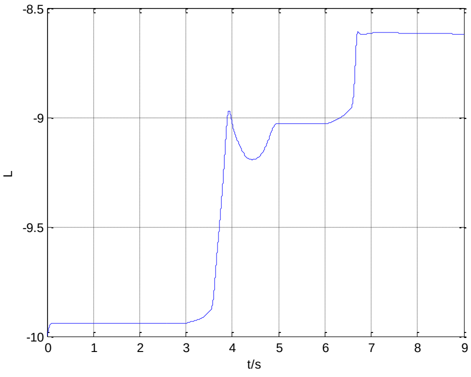

Figure 20. Curve of Gain L

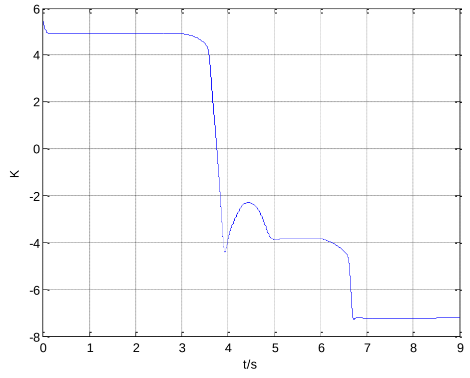

Figure 21. Curve of Gain K

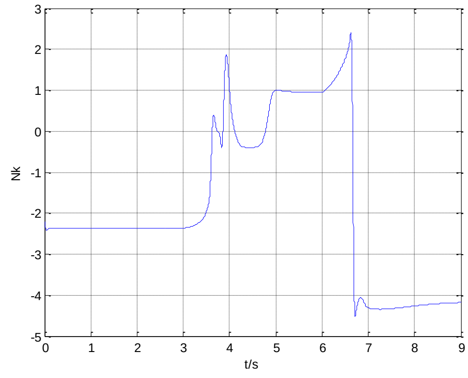

Figure 22. Curve of Gain Nk

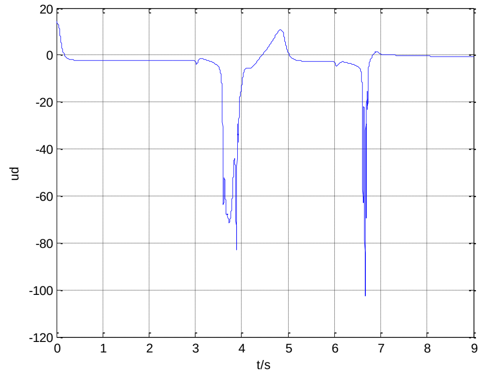

Figure 23. Ideal Control Ud

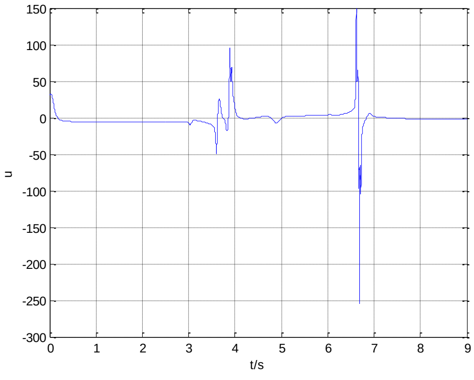

Figure 24. Curve of Control U

Choose the initial state value as $x(0) = 8$, the simulation result is as follows:

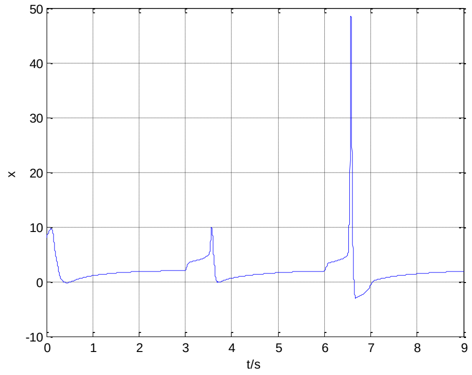

Figure 25. Curve of State x

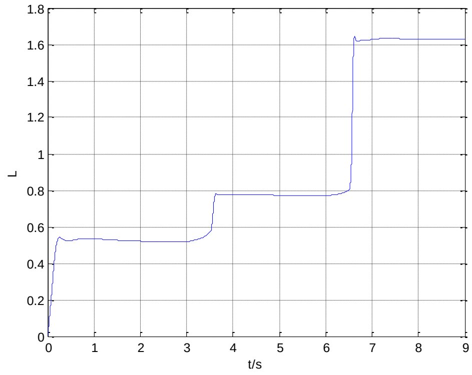

Figure 26. Curve of Gain L

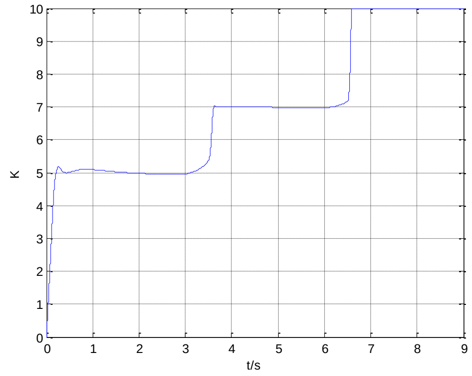

Figure 27. Curve of Gain K

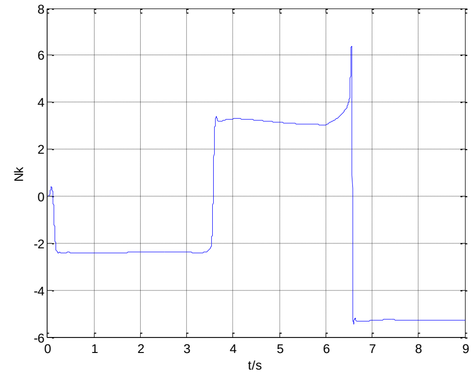

Figure 28. Curve of Gain Nk

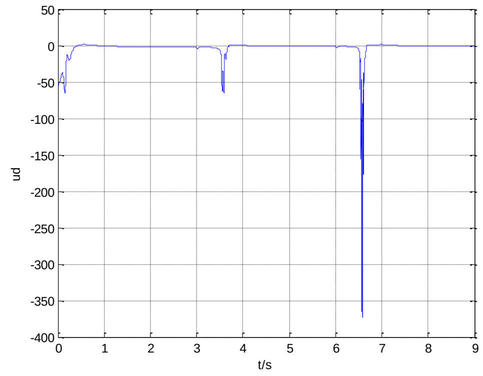

Figure 29. Ideal Control Ud

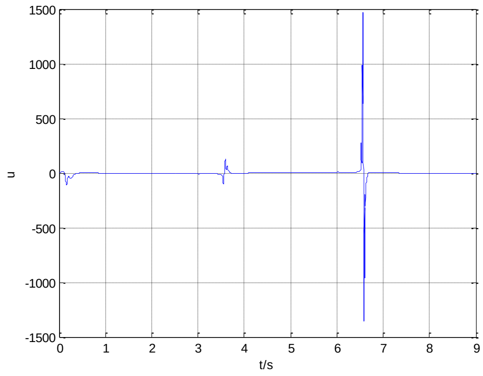

Figure 30. Curve of Control U

Choose the initial state value as $x(0) = -20$, expected value as $x^d = 12$, the simulation result is as follow:

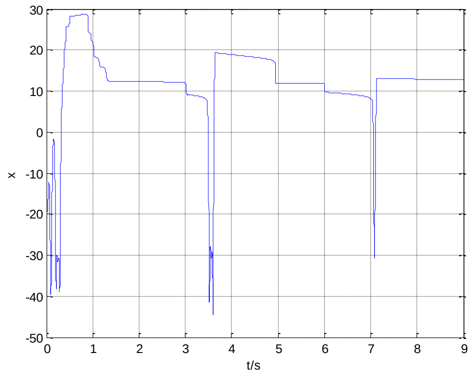

Figure 31. Curve of State x

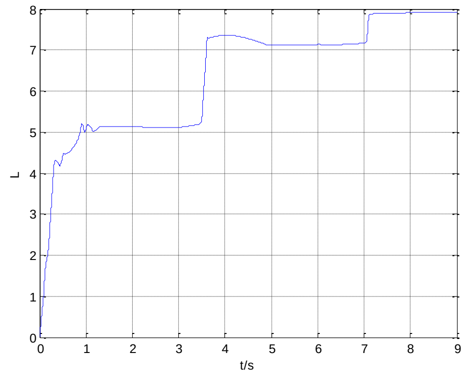

Figure 32. Curve of Gain L

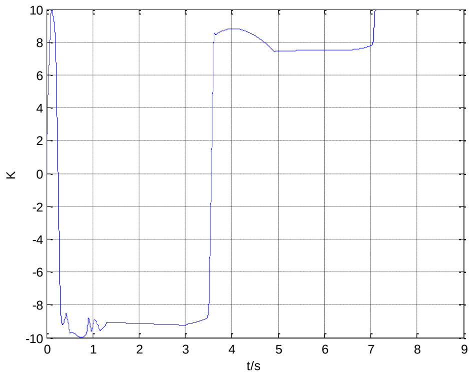

Figure 33. Curve of Gain K

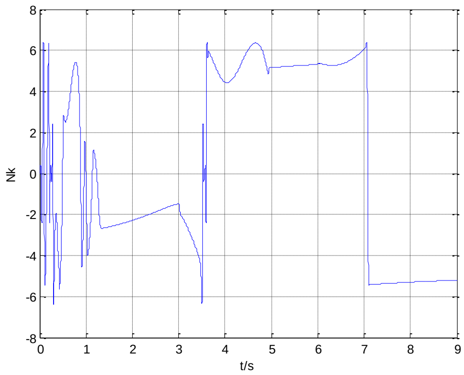

Figure 34. Curve of Gain Nk

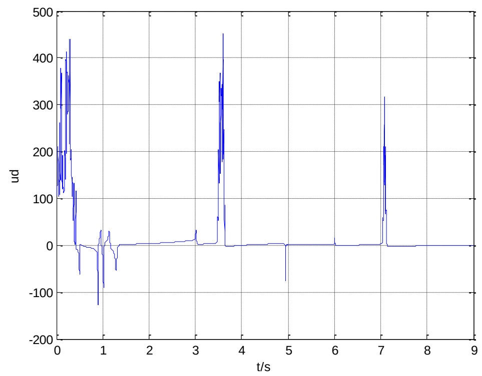

Figure 35. Ideal Control Ud

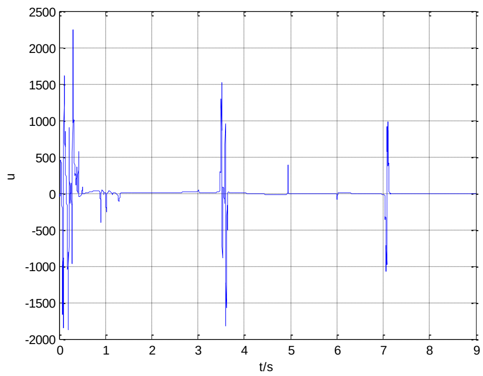

Figure 36. Curve of Control U

So the conclusion can be made as follows: the system can be stable with a initial state belongs to the interval $[-6,6]$ if the limit input Nussbaum gain method is not adopted, and the stable interval for initial value can be improved to be $[-25,25]$ if the limit input Nussbaum gain method is used. So the proposed method can improve the stability of nonlinear system control and make it depended on the initial value less and less.

6. Conclusion

On the basis of limited gain, the problem of actuator saturation is considered in this paper. This problem is a inherent in the actual physical systems and must be solved when Nussbaum gain control experiences from academic research to practical application.

Nussbaum gain control strategy with limited input is proposed in this paper. It can solve the above problem better. At last, detailed numerical simulation is done to testify the rightness of the proposed method in case of a first order system.

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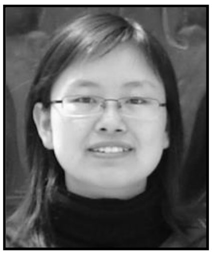
Authors

Junwei Lei, he was born on 1981 at Chibi of Hubei province of China and received his Doctor degree in Guidance, Navigation and Control in 2010 from Naval Aeronautical and Astronautical University, Yantai of China. Her present interests are control theory, chaotic system control, aircraft control and adaptive control.

He was promoted to be a lecture of NAAU in 2010. His typical book named Nussbaum gain control technology of supersonic missiles was published in 2013 in China.

Di Liu, he was born on 1983 at Shuangyashan of Heilongjiang province of China and received his master degree in control theory and control engineering in Heilongjiang University in 2008, Haerbin of China. Now he works at Naval Aeronautical and Astronautical University, and became a lecturer in 2010. Her interest is in control theory.



Lingling Wang, she was born on 1984 at Tongling of Anhui province of China and received her master degree in control theory and control engineering in Hehai University in 2009, Nanjing of China. Now she works at Naval Aeronautical and Astronautical University, and became a lecturer in 2011. Her interest is in automatic testing.

