International Journal of Control and Automation
Vol. 8, No. 9 (2015), pp. 365-370
http://dx.doi.org/10.14257/ijca.2015.8.9.35

A New Method to Design a Control the Linear Singular Systems by Chebyshev Wavelets

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Abstract

In this paper, we propose a new method to design an observer and control the linear singular systems described by Chebyshev wavelets. The idea of the proposed approach is based on solving the generalized Sylvester equations. An example is also given to illustrate the procedure.

Keywords*: Chebyshev wavelets; Singular linear system; Operational Matrices; Controllability*

1. Introduction

Singular systems, also commonly called generalized or descriptor systems in the literature, appear in many practical situations including engineering systems, economic systems, network analysis, and biological systems. In fact, many systems in the real life are singular essentially. They are usually simplified or approximated by nonsingular models because there is still lacking of efficient tools to tackle problems related to such systems. The structural analysis of linear singular systems, using either algebraic or geometric approach, has attracted considerable attention from many researchers during the last three decades (see *e.g.* [1-3]).

Beginning from 1991, wavelet technique has been applied to solve integral equations [4-7]. Wavelets, as very well-localized functions, are considerably useful for solving integral equations and provide accurate solutions. Also, the wavelet technique allows the creation of very fast algorithms when compared with the algorithms ordinarily used.

In this paper, we propose a new method to design an observer and control the linear singular systems described by Chebyshev wavelets. The operational matrix of Chebyshev wavelets is introduced and utilized to reduce the solution of linear singular systems to the solution of algebraic equations. Finally, we obtain the interrelations between solution problems for the linear matrix equations of Sylvester with suitable controllability and observability conditions.

ISSN: 2005-4297 IJCA

2. Properties of Chebyshev Wavelets

Wavelets are mathematical functions that are constructed using dilation and translation of a single function called the mother wavelet denoted by $\psi(t)$ and satisfied certain requirements.

If the dilation parameter is a and translation parameter is b, then we have the following family of wavelets:

$$\psi_{a,b}(t)=|a|^{1/2}\psi\left(\frac{t-b}{a}\right) \text{ with } a,b\in R, a\neq 0. \qquad (2.1)$$

Chebyshev wavelets $\psi_{n,m}(t)=\psi(k,n,m,t)$ has four arguments, $n=1,2,3,\cdots,2^{k-1}$, k is assumed to be any positive integer, m is the order for Chebyshev polynomials and t is the normalized time. They are defined on the interval $[0,1]$ as

$$\psi_{n,m}=\begin{cases} 2^{\frac{k+1}{2}}P_m\left(2^{k+1}t-2n-1\right) & for\ \frac{n}{2^k}\le t\le\frac{n+1}{2^k} \\ 0 & otherwise \end{cases} \qquad (2.2)$$

Where

$$P_m=\begin{cases}\frac{1}{\sqrt{\pi}} & m=0\\ \sqrt{\frac{2}{\pi}}T_m(t) & m\ge 1\end{cases}$$

with $m=0,1,\cdots,M$ and $n=0,1,\cdots,2^{k-1}-1$. Here, $P_m(t)$ are the well-known Chebyshev polynomials of order m with respect to the weight function $\omega(t)=\frac{1}{\sqrt{1-t^2}}$, which are defined on the interval $[-1,1]$,and can be determined with the aid of the following recurrence formula[8-10]

$$P_0(t)=1, P_1(t)=t, P_{m+1}(t)=2tP_m(t)-P_{m-1}(t), m=1,2,3\cdots. \qquad (2.3)$$

We should note that in dealing with Chebyshev wavelets the weight function $\omega(t)$ have to be dilated and translated as $\omega_n(t)=\omega\left(2^k t-2n+1\right)$.

A function $f(t)$ defined over $[0,1]$ can be expanded in the terms of Chebyshev wavelets as

$$f(t)=\sum_{n=0}^{\infty}\sum_{m\in Z}c_{n,m}\psi_{n,m}(t)$$

$$\approx\sum_{n=0}^{2^k-1}\sum_{m=0}^{M}c_{n,m}\psi_{n,m}(t)=C^T\psi(t) \qquad (2.4)$$

where

$$c_{n,m}=(f(t),\psi_{n,m}(t))=\int_0^1 f(t)\psi_{n,m}(t)dt \qquad (2.5)$$

C and $\psi(t)$ are $2^k(2M+1)\times 1$ vectors given by

$$C=[c_{0,0},c_{0,1},\ldots,c_{0,M},\ldots c_{2,M},\ldots,c_{2^k-1,0},\ldots,c_{(2^k-1),M}]^T, \qquad (2.6)$$

$$\psi(t)=[\psi_{100},\psi_{0,1},\ldots,\psi_{0,M},\ldots\psi_{2,M},\ldots,\psi_{(2^k-1),0},\ldots,\psi_{(2^k-1),M}]^T. \qquad (2.7)$$

In the following section, we introduce a new Chebyshev wavelets operational matrix of derivative defined by[10]

Theorem 1. The derivative of the Chebyshev wavelets vector $\psi(t)$ can be expressed dy

$$\frac{d\psi(t)}{dt} = D\psi(t) \qquad (2.8)$$

where D is the $2^k(M+1)$ operational matrix of derivative defined as follows

$$D = \begin{pmatrix} F & 0 & \cdots & 0 \\ 0 & F & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F \end{pmatrix}.$$

Here F is $(M+1)(M+1)$ matrix and its (r,s) th element is defined as follows

$$F_{r,s} = \begin{cases} 2^{k+2} m \sqrt{\dfrac{c_{r-1}}{c_{s-1}}} & r = 2,\cdots,(M+1),\ s = 1,\cdots r-1 \ and\ (r+s)\,odd \\ 0 & otherwise \end{cases}.$$

By using Eq. (2.8) the operational matrix for nth derivative can be derived as

$$\frac{d^n\psi(x)}{dx^n} = D^n\psi(x),$$

where D^n is the nth power of matrix D .

3. Properties of the Sylvester Equation

Definition1. The Kronecker product of $M_{r\times s}$ and $H_{m\times n}$ is denoted by $M \otimes H$ and that is the $rm \times sn$ matrix

$$M \otimes H = \begin{bmatrix} m_{11}H & m_{12}H & \cdots & m_{1s}H \\ m_{21}H & m_{22}H & \cdots & m_{2s}H \\ \vdots & \vdots & \vdots & \vdots \\ m_{r1}H & m_{r2}H & \cdots & m_{rs}H \end{bmatrix}.$$

Consider the Sylvester equation

$$AX - XB = \Phi_1\Phi_2, \qquad (3.1)$$

where $A \in C^{n\times n}, B \in C^{m\times m}, X \in C^{n\times m}, \Phi_1 \in C^{n\times k}, \Phi_2 \in C^{k\times m}$.

Some recent developments in the theory of rational matrix functions and linear systems leading to Theorems, we can refer to [11-15].

Definition2. . Identify the least positive integer k for which a solution X, Φ_1, Φ_2 ,with (A, Φ_1) is controllable and (Φ_2, B) is observable

$$k(A,B) = \max_{\lambda \in C}\left\{\dim Ker(\lambda I - A) + \dim Ker(\lambda I - B)\right\}. \qquad (3.2)$$

Lemma 1.Let $A \in C^{n\times n}$ be given. There exists $\Phi_1 \in C^{n\times k}$ such that (A, Φ_1) is controllable, if and only if $k \geq \max_{\lambda\in C} \dim Ker(\lambda I - A)$.

Theorem 1. Given $A \in C^{n\times n}, B \in C^{m\times m}$ .The minimal integer k for which there exist $X \in C^{n\times m}, \Phi_1 \in C^{n\times r}, \Phi_2 \in C^{r\times m}$ satisfying (3.1) and such that (A, Φ_1) is controllable and (Φ_2, B) is observable, which is equal to $k(A,B)$.

4. Model Described

A singular linear system can be described as:

$$K\dot{X}(t) = EX(t) + FU(t), \qquad (4.1)$$

where $X \in R^{n}, U(t) \in R^{m}, K \in R^{n\times n}, E \in R^{n\times n}, F \in R^{n\times m}$.

5. Solution of Singular Linear System

In this section, the Chebyshev wavelet method is used for solving the singular linear system by approximated functions. Assume that $U(t)$ is square integrable in the interval $[0,1)$. In the matrix forms:

$$U(t) = G\psi(t), \qquad (5.1)$$

where G is a $p\times m$ matrix. G can be obtained by the method described in Section 2. Avoiding impulse functions, we expand $\dot{X}(t)$ instead of $X(t)$ itself into Chebyshev wavelet and $X(t)$ is given by $X(t) = C^{T}\psi(t)$, so we get

$$\dot{X}(t) = C^{T}D\psi(t). \qquad (5.2)$$

Plugging (5.1) and (5.2) into (4.1), we have

$$KC^{T}D\psi(t) = EC^{T}\psi(t) + FG\psi(t), \qquad (5.2)$$

from which, we obtain

$$\left(KC^{T}D - EC^{T}\right)\psi(t) = FG\psi(t). \qquad (5.3)$$

Then (5.3) can be written as

$$KC^{T}D - EC^{T} = FG. \qquad (5.4)$$

Thus Eq. (5.4) is solved by using Kronecker product as

$$C^{T} = -\left[E\otimes I - K\otimes D\right]^{-1}(FG)^{T}. \qquad (5.5)$$

So $X(t) = C^{T}\psi(t)$

6. Some Theorems

For $KC^{T}D - EC^{T} = FG$, we apply a pseudo-inverse of matrix to K, then $K^{-1}EC^{T} - C^{T}D = -K^{-1}FG$;let $\tilde{A} = K^{-1}E$, $\tilde{B} = D$, $\tilde{\Phi}_{1} = -K^{-1}$, $\tilde{\Phi}_{2} = FG$, $S = C^{T}$,we obtain $\tilde{A}S - S\tilde{B} = \tilde{\Phi}_{1}\tilde{\Phi}_{2}$.

Accordingly , we propose the following some results

Lemma 2.Let $\tilde{A} \in R^{n\times n}$ be given. There exists $\tilde{\Phi}_{1} \in R^{n\times k}$ such that $(\tilde{A}, \tilde{\Phi}_{1})$ is controllable, if and only if $k \geq \max_{\lambda\in C} \dim Ker\left(\lambda I - \tilde{A}\right)$.

Theorem 2. Given $\tilde{A} \in R^{n\times n}, \tilde{B} \in R^{m\times m}$ .The minimal integer k for which there exist $S \in R^{n\times m}, \tilde{\Phi}_{1} \in R^{n\times r}, \tilde{\Phi}_{2} \in R^{r\times m}$ satisfying (3.1) and such that $(\tilde{A}, \tilde{\Phi}_{1})$ is controllable and $(\tilde{\Phi}_{2}, \tilde{B})$ is observable, which is equal to $k(\tilde{A}, \tilde{B})$.

7. Example

Consider a simple circuit network as shown in Figure 7.1, where voltage source $V_{S}(t)$ is the driver (control input), R, L and C stand for the resistor, inductor and capacity, respectively, as well as their quantities, and their voltages are denoted by $V_{R}(t), V_{L}(t), V_{C}(t)$, respectively.

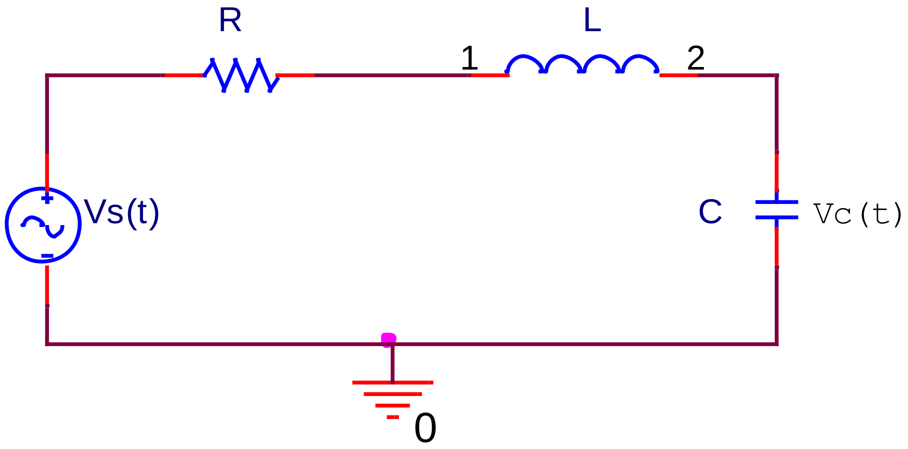

Figure 7.1

Then from Girchoff's laws, we have the following circuit equation:

$$\begin{bmatrix} L & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{I}(t) \\ \dot{V}_L(t) \\ \dot{V}_C(t) \\ \dot{V}_R(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/C & 0 & 0 & 0 \\ -R & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I(t) \\ V_L(t) \\ V_C(t) \\ V_R(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} V_S(t) \quad (7.1)$$

Let $x(t) = \begin{bmatrix} I(t) & V_L(t) & V_C(t) & V_R(t) \end{bmatrix}'$

We choose $L = C = R = 1$ and a measure equation: $y(t) = V_C(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x(t)$,

$V_S = 1$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} V_S(t) \quad (7.2)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x(t)$$

For Chebyshev Wavelets, let $M = 3, k = 2$, We solve Eq. (7.2) using the algorithm described in Section 6 for the case corresponds to Theorem 2.So we can get Eq. (7.2) that is controllable and observable.

8. Conclusions

By the analysis of the above, we find that the method proposed in the paper is efficient in tackling the singular linear systems. Furthermore, the method is also applied for solving the interrelations between solution problems for singular linear systems and the linear matrix equations of Sylvester with suitable controllability and observability conditions. The design example is good enough to illustrate our idea.

Acknowledgements

This work was supported by the National Natural Science Funds of China under Grant No. 51577046, The National Defense Advanced Research Project Grant

No.C1120110004, 9140A27020211DZ5102, the Key Grant Project of Chinese Ministry of Education under Grant No.313018, Anhui Provincial Science and Technology Foundation of China under Grant No. 1301022036.

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