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Finite-Time Chaotic Control of Unified Hyperchaotic Systems with Multiple Parameters

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Abstract

This paper is concerned with finite-time chaos control of unified hyperchaotic systems with multiple parameters. Based on the finite-time stability theory in the cascade-connected systems, a nonlinear control law is presented to achieve finite-time chaos control. The controller is simple and easy to be constructed. Simulation results for Lorenz hyperchaotic system, L ̈u hyperchaotic system, Chen hyperchaotic system are provided to illustrate the effectiveness of the proposed scheme.

Keywords: *Finite-time chaos control, Unified hyperchaotic system, Cascade-connected system*

1. Introduction

Chaotic system has attracted much attention because of its powerful application in the areas of secure communication, information processing, biological engineering, and chemical processing [1-9]. As a new subject in 1980s, chaos almost covers all the fields of science. It is known that chaos is an interesting nonlinear phenomenon which may lead to irregularity and unpredictability in the dynamic system, and it has been intensively studied in the last three decades. Since Pecora and Carroll proposed the PC method to synchronize two chaotic systems in 1990 [10, 11], the study of synchronization of chaotic systems has been widely investigated due to their potential applications in various fields, for instance, chemical reactions, biologicals systems, and secure communication. Over the past decades, a variety of control approaches such as adaptive control [12], linear feedback control [13], active control [14], and back stepping control [15] have been proposed for various types of synchronization, which include complete synchronization [16], projective synchronization [17-18], general synchronization [19], lag synchronization [20], and novel compound synchronization [21].

Notice that the mentioned literatures mainly investigated the asymptotic synchronization of chaotic systems. However, in the view of practical application, optimizing the synchronization time is more important than achieving synchronization asymptotically [22-26]. Recently, based on the step-by-step control method, Wang et al.

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realized the finite-time synchronization of two chaotic systems by designing a proper controller [22]. The method has the ability to achieve global stability in finite time. In addition, the step-by-step technique has the advantage of reducing controller complexity.

In this paper, we deal with the so-called unified hyperchaotic systems presented in [27], which is a generalized form of the hyperchaotic Lü system, hyperchaotic Chen system, and hyperchaotic Lorenz system. Based on the finite-time stability theory and stability theory of cascade-connected system, we present a new continuous controller to realize finite-time chaos control for the unified hyperchaotic systems.

2. Preliminary Definitions and Lemmas

Finite-time stability means that the state of the dynamic system converges to a desired target (in general referring to the origin) within a finite time. First, let us introduce some necessary definitions and lemmas.

Lemma 1 (see [22]) For the system,

$$\dot{x} = f(x). \tag{1}$$

Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -cV^{\eta}(t), \quad \forall t \ge t_0, \quad V(t_0) \ge 0,$$

where $c > 0$ and $0 < \eta < 1$ are constants. Then, for any initial time t_0, $V(t)$ satisfies

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), \quad t_0 \le t \le t_1,$$

$$V(t) \equiv 0, \quad \forall t \ge t_1,$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}.$$

Thus, for any initial value $V(t_0)$, the system (1) has $V(t) = 0$ in $t_1 = t_0 + V^{1-\eta}(t_0)/c(1-\eta)$, that is, the system can achieve global stability in finite time.

Lemma 2 Let $0 < c < 1$. Then for positive real numbers a and b, the following inequality holds

$$(a+b)^c \le a^c + b^c.$$

This result is quite straight forward and the proof is omitted here.

2.1. Main Results

It is known that a chaotic system has sensitive dependence on its initial conditions and parameters. Before 1990, most researchers thought that the chaotic systems cannot be controlled to a desired target. However, in 1990, the pioneering work of Ott, Grebogi, and Yorke [28] denied completely the viewpoint. In this section, our aim is to design a control law that realizes finite-time chaos control for the unified hyperchaotic systems. We start with considering the finite-time stable problem for the case of certain parameters. Then we turn the problem to the system with uncertain parameters.

The unified hyperchaotic system can be described as

$$\begin{cases} \dot{x}_1 = (25a+20)(x_2 - x_1) + x_4, \\ \dot{x}_2 = (28-35a)x_1 + (29a-1)x_2 - x_1x_3, \\ \dot{x}_3 = -\dfrac{8+a}{3}x_3 + x_1x_2, \\ \dot{x}_4 = bx_4 + cx_2x_3, \end{cases} \tag{2}$$

where $[x_1, x_2, x_3, x_4]^T$ is the state variables group of the unified hyperchaotic system, and $a \in [0,1]$ is a parameter [27].

Because the system (2) is belong to the generalized Lorenz hyperchaotic system for a = 0, b =−1, c = −1, it belongs to Lü hyperchaotic system for a = 0.8, b = 0.3, c = 0.1 and belongs to generalized Chen hyperchaotic system for a = 1, b = 0.2, c = 0.1, so we consider the system (2) as unified hyperchaotic system. The hyperchaotic attractor are shown in Figure1-3, respectively.

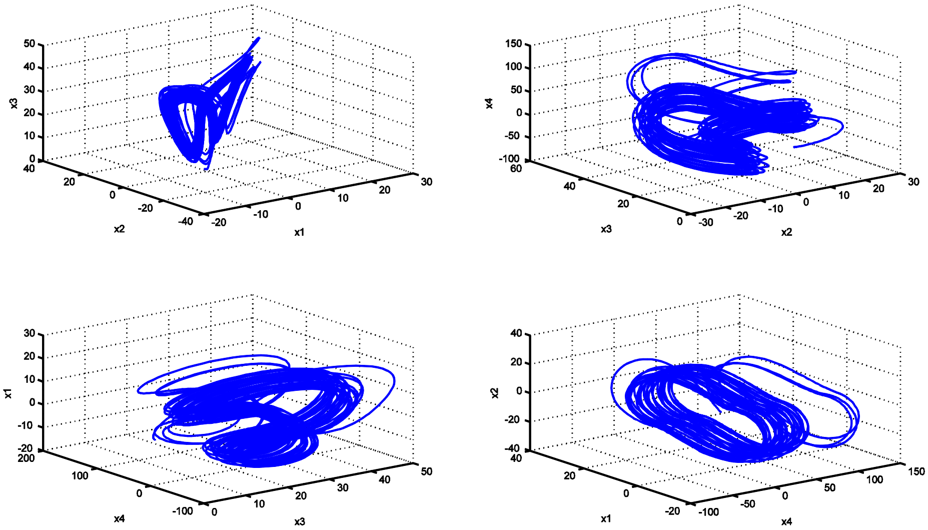

Figure 1. Phase Planes of Lorenz Hyperchaotic System

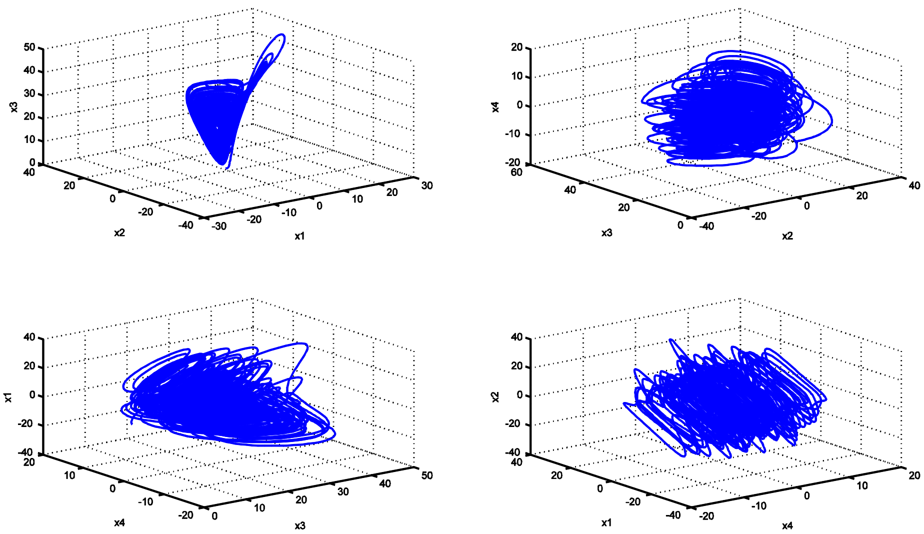

Figure 2. Phase Planes of Lü Hyperchaotic System

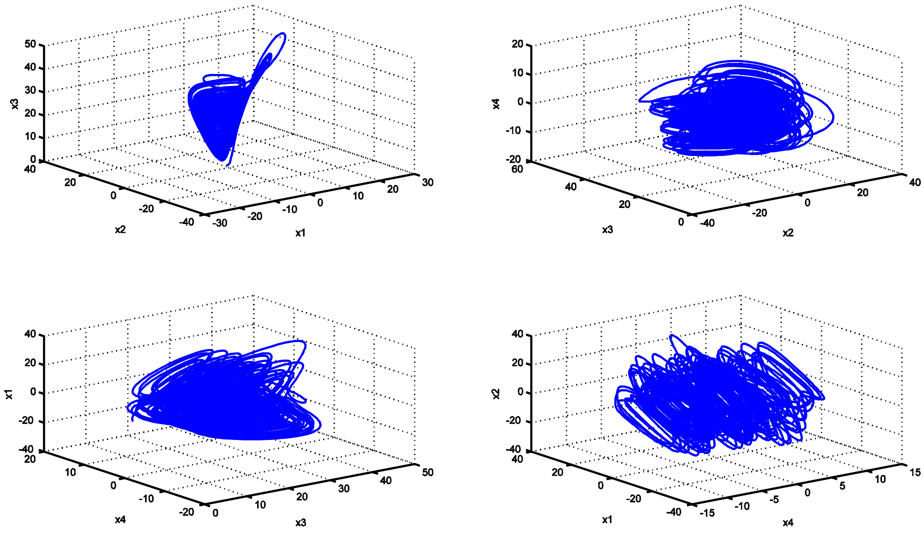

Figure 3. Phase Planes of Chen Hyperchaotic System

In this section, we consider the unified chaotic system with certain parameters, and design a controller to globally stabilize the unstable equilibrium o = $[0, 0, 0, 0]^T$ in a finite time. As to other equilibria, we can also adopt the technique presented below to realize the chaos control.

The controlled unified chaotic systems can be described by

$$\begin{cases} \dot{x}_1 = (25a+20)(x_2 - x_1) + x_4 + u_1, \\ \dot{x}_2 = (28-35a)x_1 + (29a-1)x_2 - x_1x_3 + u_2, \\ \dot{x}_3 = -\dfrac{8+a}{3}x_3 + x_1x_2 + u_3, \\ \dot{x}_4 = bx_4 + cx_2x_3 + u_4 \end{cases} \tag{3}$$

where u_1, u_2, u_3 and u_4 are controllers. The design procedure is divided into three steps.

Step 1 Let $u_1 = -(25a+20)x_2 - x_4 - x_1^{\beta}$, and where are positive odd integers. By the controller, the first equation of (3) is

$$\dot{x}_1 = -(25a+20)x_1 - x_1^{\beta}. \tag{4}$$

Consider the candidate Lyapunov function $V_1 = \frac{1}{2}x_1^2$. The time derivative of V_1 along the trajectory of (4) is

$$\begin{aligned}\dot{V}_1 &= x_1\left(-(25a+20)x_1 - x_1^{\beta}\right)\\ &= -(25a+20)x_1^2 - x_1^{\beta+1}\\ &\le -x_1^{\beta+1}\\ &= -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\frac{1}{2}x_1^2\right)^{\frac{\beta+1}{2}}\\ &= -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(V_1\right)^{\frac{\beta+1}{2}}.\end{aligned} \tag{5}$$

Since $0<\beta<1$, then $0<\dfrac{\beta+1}{2}<1$. From Lemma 1, the state variable x_1 will reach $x_1=0$ at a finite time $T_1=\dfrac{x_1(0)}{1-\beta}$.

Step 2 Let $u_2=-L_1x_2-x_2^{\beta}$, where $L_1\ge 29a-1$, and $u_3=-x_3^{\beta}$. If $t>T_1$, then $x_1\equiv 0$.

Substituting $x_1=0$ into the second and the third equations of system (3), it yields

$$\begin{cases}\dot{x}_2=(29a-1)x_2-L_1x_2-x_2^{\beta},\\ \dot{x}_3=-\dfrac{8+a}{3}x_3-x_3^{\beta}.\end{cases} \tag{6}$$

Choose Lyapunov function for (6) as follows

$$V_2=\frac{1}{2}x_2^2+\frac{1}{2}x_3^2. \tag{7}$$

The derivative of V_2 along the trajectories of (6) is

$$\begin{aligned}\dot{V}_2 &= x_2\left((29a-1)x_2-L_1x_2-x_2^{\beta}\right)-x_3\left(\frac{8+a}{3}x_3+x_3^{\beta}\right)\\ &= -(L_1-(29a-1))x_2^2-x_2^{\beta+1}-\frac{8+a}{3}x_3^2-x_3^{\beta+1}\\ &\le -x_2^{\beta+1}-x_3^{\beta+1}\\ &= -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\frac{1}{2}x_2^2\right)^{\frac{\beta+1}{2}}-\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\frac{1}{2}x_3^2\right)^{\frac{\beta+1}{2}}\\ &= -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left((\frac{1}{2}x_2^2)^{\frac{\beta+1}{2}}+(\frac{1}{2}x_3^2)^{\frac{\beta+1}{2}}\right)\\ &\le -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\frac{1}{2}x_2^2+\frac{1}{2}x_3^2\right)^{\frac{\beta+1}{2}}\\ &= -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}V_2^{\frac{\beta+1}{2}}.\end{aligned} \tag{8}$$

It should be pointed out that we have used Lemma 2 to deduce the above result. Since $0<\beta<1$, $0<\frac{\beta+1}{2}<1$. Form Lemma 1, the state variables x_2 and x_3 will converge to $x_2=0$, $x_3=0$ at a finite time $T_2=T_1+\frac{x_3T_1}{1-\beta}$.

Step 3 Let $u_4=-L_2x_4-x_4^{\beta}$, where $L_2\geq b$. If $t>T_1+T_2$, then $x_1\equiv x_2\equiv x_3=0$. Substituting $x_2=0$ into the fourth equations of system (3), it yields

$$\dot{x}_4=-(L_2-b)x_4-x_4^{\beta}. \tag{9}$$

Consider the Lyapunov function

$$V_3=\frac{1}{2}x_4^2. \tag{10}$$

The time derivative of V3 along the trajectories of (9) is

$$\begin{aligned}\dot{V}_3&=x_4\left(-(L_2-b)x_4-x_4^{\beta}\right)\\&=-(L_2-b)x_4^2-x_4^{\beta+1}\\&\leq -x_4^{\beta+1}\\&=-\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\frac{1}{2}x_4^2\right)^{\frac{\beta+1}{2}}\\&=-\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}(V_3)^{\frac{\beta+1}{2}}.\end{aligned}$$

Since $0<\beta<1$, then $0<\frac{\beta+1}{2}<1$. Form Lemma 1, the state variable x_4 will reach $x_4=0$ at a finite time $T_3=T_2+\frac{x_4T_2}{1-\beta}$.

Then after T_3, the state of the system (3) will stay at $x_1=0$, $x_2=0$, $x_3=0$, $x_4=0$. This means the controlled unified hyperchaotic systems (3) are finite-timely stabilized by the controller

$$\begin{cases}u_1=-(25a+20)x_2-x_4-x_1^{\beta},\\u_2=L_1x_2-x_2^{\beta},\\u_3=-x_3^{\beta},\\u_4=-L_2x_4-x_4^{\beta},\end{cases}$$

where $L_1\geq 29a-1$, $L_2\geq b$.

2.2 Simulation Results

In this section, to demonstrate the effectiveness of the proposed method, we present the simulation results for the unified hyperchaotic systems. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. The initial condition of the chaotic system is always adopted as

$[x_1(0), x_2(0), x_3(0), x_4(0)]^T = [-0.5, 0.5, 1, -1]^T$. Figures (4)-(6) show the simulation results for the Lorenz, Lü and Chen hyperchaotic systems, respectively. The controller gains are chosen as $L_1 = 29a + 1$, $\beta = \dfrac{2}{3}$. From the figures, we can see the state reaches the origin in a finite time.

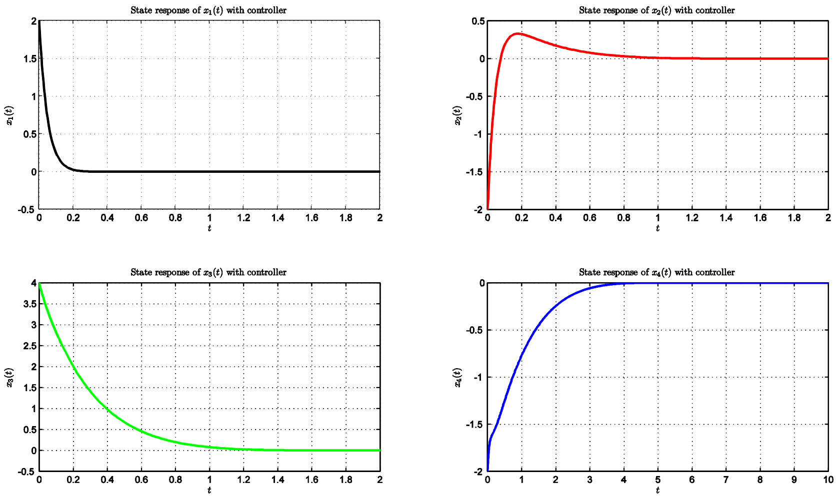

Figure 4. Time Response of Lorenz Hyperchaotic System

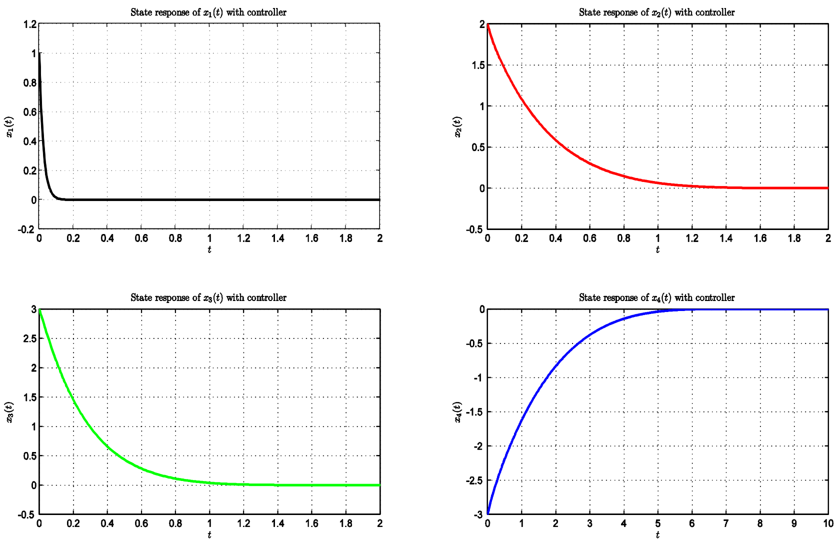

Figure 5. Time Response of Lü Hyperchaotic System

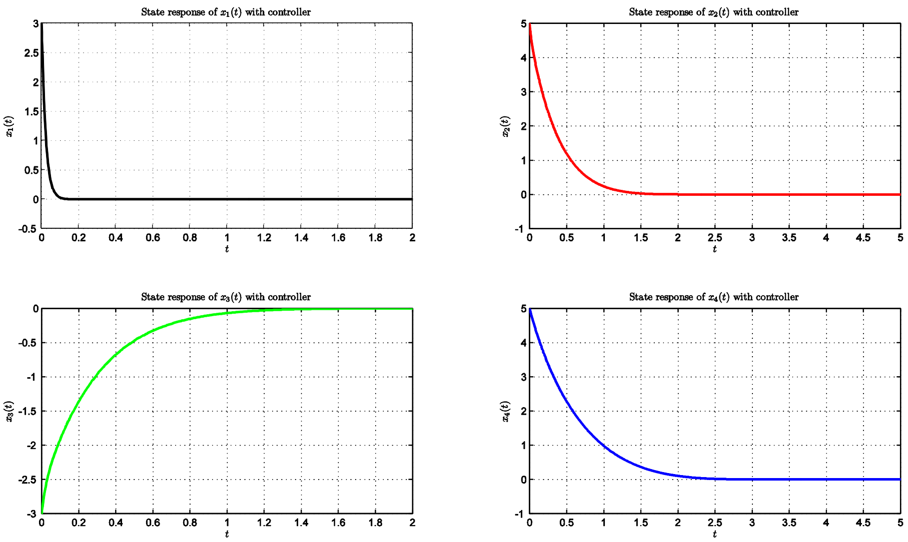

Figure 6. Time Response of Chen Hyperchaotic System

3. Conclusion

In this paper, the problem of finite-time chaos control for the unified hyperchaotic systems was investigated. Based on the finite-time stability theory, the step-by-step control and nonlinear control approach, a suitable controller was introduced. The simulation results demonstrated that the proposed controller works well for synchronizing three hyperchaotic systems in finite dime. From the proofs, we can see that this method can be extended to other chaotic systems.

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