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Simplified Sliding Mode of a Novel Class of Four-dimensional Fractional-order Chaos

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Abstract

We study the simplified control for a novel class of four-dimensional (4-D) fractional-order chaos in this paper. Firstly, a novel class of 4-D fractional-order chaos is introduced, which can express many actual projects. Secondly, a new simplified controller based on sliding mode theory which needs only one single control input is designed for the control of the proposed class systems. Furthermore, the controller can stabilize the systems with uncertainty and external disturbance. Finally, numerical simulations including 4-D fractional-order hyperchaos, 4-D Lorenz-Stenflo chaos as a special case and three-dimensional (3-D) simplified fractional-order Lorenz chaos as another special case are employed to demonstrate the universality and effectiveness of the sliding mode. The proposed scheme can be easily generalized to similar fractional-order chaos.

Keywords: *chaos control; fractional-order chaos; simplified sliding mode; uncertainty*

1. Introduction

With the emergence of integer calculus, fractional calculus also appeared. However, for a long time, fractional calculus has not attracted much attention because of the difficulty for solving mathematical equations. In recent years, as the fast development of computer processing, scholars paid more attention to the fractional calculus [1, 2]. And it was found that, fractional calculus is more universal in actual project. Especially for systems with memory and hereditary factors, fractional calculus could describe these systems better such as electromagnetism [3], memristor [4], finance system [5], power system [6].

Chaos has been widely studied for the great potential to secure communication and signal processing. Many new integer-order chaos have been put forward, for instance, Chen system [7], Lorenz-like chaos [8], a 4-D simplified Lorenz system [9], a new hyperchaotic system [10]. By introducing fractional calculus to chaotic systems, people have proposed a lot of fractional-order chaos, for example, fractional-order Chen chaos [11], the complex T chaos [12], a new chaos without equilibrium points [13], the unified system [14].

Chaotic vibration is harmful to many nonlinear actual projects. So how to eliminate and control chaos become attractive. There already are many results for control or synchronization of integer-order chaos [15-17]. However, as we all know, fractional-order chaos has different controllability region with chaos of integer-order. So people have paid much attention to fractional-order chaos control. For example, the controller design for a fractional-order Lipschitz system is investigated in [18]. In [19], a fuzzy control scheme is designed to stabilize a representative fractional-order financial chaotic system. In [20], a sliding mode strategy is presented for adaptive control of a novel fractional-order chaos. A new double-wing fractional-order chaos is proposed, then chaos control of the system is completed by a novel sliding mode in [21]. In [22], control of a stochastic fractional-order

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chaos with random and uncertain parameters considered is investigated. In [23], synchronization of a class of integer-order and fractional-order chaos is finished. The dynamic behavior of fractional-order complex Lorenz chaos is analyzed and corresponding control scheme is designed in [24]. However, most of the existing research results are for specific systems which have poor general applicability. Designing of control scheme for general class of fractional-order chaos is quite few, and there is almost no literature about chaos control of a class of 4-D fractional-order chaos which is considered in our paper.

As we all know, sliding mode is an effective and robust control strategy. People have applied it to control chaos because it can drive the state which is not on the sliding surface to the steady state in limited time [25, 26]. However, there are almost no relevant outcomes about sliding mode control for general class of 4-D fractional-order chaos. Can sliding mode be applied to the proposed general class of 4-D fractional-order chaos control? If the hypothesis is true, what are the specific mathematical derivation and application conditions? There are no relevant results. It is worthy of studying. Besides, the uncertainty and external disturbance often exist in actual projects. Therefore, it is necessary and meaningful to ensure the applicability of the designed controller for practical systems.

Motivated by the above analysis, some advantages of our research are drawn. Firstly, a general class of 4-D fractional-order chaos is introduced, which is a general form for many practical systems. Furthermore, a new simplified sliding mode control scheme which needs only one single control input is designed for the stabilization of the proposed 4-D fractional-order chaos even the system with external interference, which is of great convenience for the lowering of controller complexity. Finally, numerical simulations including a 4-D fractional-order system, a 4-D integer-order chaos as a special case and a 3-D fractional-order chaos as another special case are presented to demonstrate the universality and effectiveness.

The contents of our paper are given as: The general class of 4-D fractional-order chaos is given in Section 2. In Section 3, with Lyapunov stability theory and sliding mode method, a new simplified sliding mode control scheme for the proposed system is introduced. In Section 4, numerical simulations are implemented. Section 5 draw the conclusions.

2. System Description

A novel class of 4-D fractional-order chaos is presented as:

$$\begin{cases} \dfrac{\mathrm{d}^{q_1} x}{\mathrm{d}t^{q_1}} = f(x,y,z,w) - rx \\ \dfrac{\mathrm{d}^{q_2} y}{\mathrm{d}t^{q_2}} = x \cdot g(x,y,z,w) - ay \\ \dfrac{\mathrm{d}^{q_3} z}{\mathrm{d}t^{q_3}} = x \cdot h(x,y,z,w) - bz \\ \dfrac{\mathrm{d}^{q_4} w}{\mathrm{d}t^{q_4}} = x \cdot p(x,y,z,w) - cw \end{cases}, \tag{1}$$

Table 1. List of Fractional-Order Chaos Published

Name	Mode	$f(\cdot)$	$g(\cdot)$	$h(\cdot)$	$p(\cdot)$
Hyperchaotic Lorenz system	$\begin{cases} \frac{d^{\beta_1}x}{dt^{\beta_1}} = a(y-x)+w \\ \frac{d^{\beta_2}y}{dt^{\beta_2}} = -xz+cx-y \\ \frac{d^{\beta_3}z}{dt^{\beta_3}} = xy-bz \\ \frac{d^{\beta_4}w}{dt^{\beta_4}} = -xz-rw \end{cases}$	$ay+w$	$c-z$	y	$-z$
Hyperchaotic Liu system	$\begin{cases} \frac{d^{\alpha_1}x}{dt^{\alpha_1}} = 10(y-x)+w \\ \frac{d^{\alpha_2}y}{dt^{\alpha_2}} = 2.5x \\ \frac{d^{\alpha_3}z}{dt^{\alpha_3}} = -2.5z-4x^2 \\ \frac{d^{\alpha_4}w}{dt^{\alpha_4}} = 40x+xz-w \end{cases}$	$10y+w$	2.5	$-4x$	$40+z$
Commensurate fractional-order hyperchaotic system	$\begin{cases} \frac{d^{\alpha}x}{dt^{\alpha}} = 10(y-x)+w \\ \frac{d^{\alpha}y}{dt^{\alpha}} = -xz+28x-y \\ \frac{d^{\alpha}z}{dt^{\alpha}} = xy-\frac{8}{3}z \\ \frac{d^{\alpha}w}{dt^{\alpha}} = -xz-w \end{cases}$	$10y+w$	$28-z$	y	$-z$
Simplified Lorenz system	$\begin{cases} \frac{d^{q_1}x}{dt^{q_1}} = 10(y-x) \\ \frac{d^{q_2}y}{dt^{q_2}} = (24-4c)x-xz-cy \\ \frac{d^{q_3}z}{dt^{q_3}} = xy-(8/3)z \end{cases}$	$10y$	$24\text{-}4c-z$	y	0
Lorenz-Stenflo system	$\begin{cases} \frac{dx}{dt} = a(y-x)+dw+u(t) \\ \frac{dy}{dt} = rx-xz-y \\ \frac{dz}{dt} = xy-bz \\ \frac{dw}{dt} = -x-aw \end{cases}$	$ay+dw$	$r-z$	y	-1

where $q_i\,(i=1,2,3,4)$ are the orders of the system, which satisfy $0<q_i\le 1$; The state variables are x,y,z,w; The smooth functions $f(\cdot), g(\cdot), h(\cdot)$ and $p(\cdot)$ are in $R^4 \to R$ space respectively, and a,b,c,r are known constants which are non-negative.

Remark 2.1 Note that the proposed 4-D fractional-order nonlinear chaotic systems (1) can describe a lot of fractional-order nonlinear chaos which have been performed in the Numerical simulations of part 4 and listed in Table 1.

3. Simplified Sliding Mode Controller Design

Adding controller *u*(t) to the introduced fractional-order system (1). Then we can get the controlled system as follows:

$$\begin{cases} \frac{\mathrm{d}^{q_1} x}{\mathrm{d}t^{q_1}} = f(x,y,z,w) - rx + u(t) \\ \frac{\mathrm{d}^{q_2} y}{\mathrm{d}t^{q_2}} = x \cdot g(x,y,z,w) - ay \\ \frac{\mathrm{d}^{q_3} z}{\mathrm{d}t^{q_3}} = x \cdot h(x,y,z,w) - bz \\ \frac{\mathrm{d}^{q_4} w}{\mathrm{d}t^{q_4}} = x \cdot p(x,y,z,w) - cw \end{cases}, \tag{2}$$

Our aim is to achieve the chaos control of system (2), there are two steps for designing the sliding mode controller. Firstly, a switching surface should be formed to ensure the state trajectories can be controlled to the sliding motion. Then, the sliding mode *S*=0 should be completed, and the state trajectories which are not on the sliding surface can be controlled to the steady state.

We select the sliding surface as:

$$S(t) = D^{q_1-1}x(t) + D^{-1}\varphi(t) = D^{q_1-1}x(t) + \int_0^t \varphi(\tau)\mathrm{d}\tau, \tag{3}$$

where

$$\varphi(t) = y \cdot g(x,y,z,w) + z \cdot h(x,y,z,w) + w \cdot p(x,y,z,w) + rx, \tag{4}$$

When the system is operating on the sliding mode, one can be got:

$$S(t) = D^{q_1-1}x(t) + D^{-1}\varphi(t) = 0, \tag{5}$$

Differentiating (5), one gets

$$\begin{aligned} \dot{S}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}S(t) \\ &= \frac{\mathrm{d}}{\mathrm{d}t}[D^{q_1-1}x(t) + D^{-1}\varphi(t)] \\ &= \frac{\mathrm{d}}{\mathrm{d}t}[D^{q_1-1}x(t) + \int_0^t \varphi(\tau)\mathrm{d}\tau] \\ &= \frac{\mathrm{d}}{\mathrm{d}t}D^{q_1-1}x(t) + \frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \varphi(\tau)\mathrm{d}\tau \\ &= \mathrm{D}^{q_1}x(t) + \varphi(t) = 0 \end{aligned}, \tag{6}$$

$$\mathrm{D}^{q_1}x(t) = -\phi(t) = -y \cdot g(x,y,z,w) - z \cdot h(x,y,z,w) - w \cdot p(x,y,z,w) - rx, \tag{7}$$

From (7), we can get the dynamics of the system on sliding mode:

$$\begin{cases} \frac{\mathrm{d}^{q_1} x}{\mathrm{d}t^{q_1}} = -y \cdot g(x,y,z,w) - z \cdot h(x,y,z,w) - w \cdot p(x,y,z,w) - rx \\ \frac{\mathrm{d}^{q_2} y}{\mathrm{d}t^{q_2}} = x \cdot g(x,y,z,w) - ay \\ \frac{\mathrm{d}^{q_3} z}{\mathrm{d}t^{q_3}} = x \cdot h(x,y,z,w) - bz \\ \frac{\mathrm{d}^{q_4} w}{\mathrm{d}t^{q_4}} = x \cdot p(x,y,z,w) - cw \end{cases}, \quad (8)$$

From (2) and (8), we can get the equivalent sub-controller based on sliding mode theory:

$$\begin{aligned} u_{\mathrm{eq}}(t) &= \frac{\mathrm{d}^{q_1} x}{\mathrm{d}t^{q_1}} - f(x,y,z,w) + rx \\ &= -y \cdot g(x,y,z,w) - z \cdot h(x,y,z,w) - w \cdot p(x,y,z,w) - rx\text{-}f(x,y,z,w) + rx \\ &= -y \cdot g(x,y,z,w) - z \cdot h(x,y,z,w) - w \cdot p(x,y,z,w) - f(x,y,z,w) \end{aligned} \quad (9)$$

To ensure the state trajectories which are not on the sliding surface can be controlled to the steady state, the discontinuous reaching control law is designed as:

$$u_d(t) = K \cdot sign(s), \quad (10)$$

where

$$sign(s) = \begin{cases} -1, s < 0 \\ 0, s = 0 \\ +1, s > 0 \end{cases} \quad (11)$$

and K is the controller gain.

Now, by combining the equivalent control law (9) and discontinuous reaching control law (10), the total controller could be presented as follows:

$$\begin{aligned} u(t) &= u_{\mathrm{eq}}(t) + u_{\mathrm{d}}(t) \\ &= -y \cdot g(x,y,z,w) - z \cdot h(x,y,z,w) - w \cdot p(x,y,z,w) - f(x,y,z,w) + K \cdot sign(S) \end{aligned}, \quad (12)$$

Theorem 1 As to the general four-dimensional fractional-order nonlinear chaos (2), if the controller gain $K < 0$ is satisfied, the controller (12) can drive the state trajectories to the sliding mode S=0 in limited time.

Proof We construct the Lyapunov function as:

$$V = \frac{1}{2} S^2, \quad (13)$$

One has

$$
\begin{aligned}
\dot{V} = S\dot{S} &= S[\mathrm{D}^{q_1}x(t)+\varphi(t)] \\
&= S[f(x,y,z,w)-rx+u(t)+y\cdot g(x,y,z,w)+ \\
&\quad z\cdot h(x,y,z,w)+w\cdot p(x,y,z,w)+rx] \\
&= S[f(x,y,z,w)-rx-y\cdot g(x,y,z,w)-z\cdot h(x,y,z,w)- \\
&\quad w\cdot p(x,y,z,w)-f(x,y,z,w)+K\cdot \mathrm{sign}(S)+y\cdot g(x,y,z,w)+ \\
&\quad z\cdot h(x,y,z,w)+w\cdot p(x,y,z,w)+rx] \\
&= S[K\cdot \mathrm{sign}(S)] = K\cdot |\mathrm{S}| < 0
\end{aligned} \tag{14}
$$

According to Lyapunov stability theorem, the conditions (V>0, $\dot{V}$ <0) is satisfied and this completes the proof. Thus, the proposed class of fractional-order chaos (2) can be controlled to the sliding surface S=0 with the controller (12) in a limited time.

Remark 3.1 For practical systems which can be modeled by the class system (2), when the uncertainty and disturbance is considered, system (2) can be presented as:

$$
\begin{cases}
\dfrac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}} = f(x,y,z,w)-rx+\Delta g(x,y,z,w)+\xi(t)+u(t) \\
\dfrac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}} = x\cdot g(x,y,z,w)-ay \\
\dfrac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}} = x\cdot h(x,y,z,w)-bz \\
\dfrac{\mathrm{d}^{q_4}w}{\mathrm{d}t^{q_4}} = x\cdot p(x,y,z,w)-cw
\end{cases}, \tag{15}
$$

where $\Delta g(x,y,z,w)$ is the uncertainty which is bounded as $|\Delta g(x,y,z,w)| < d_1$, and $\xi(t)$ is external disturbance which is bounded as $|\xi(t)| < d_2$. When the controller gain $K < -(d_1+d_2)$, the controller (12) can drive the state trajectories to the sliding mode S=0 in limited time.

Proof We select (13) as the Lyapunov function, one gets

$$
\begin{aligned}
\dot{V} = S\dot{S} &= S[\mathrm{D}^{q_1}x(t)+\varphi(t)] \\
&= S[f(x,y,z,w)-rx+\Delta g(x,y,z,w)+\xi(t)+u(t)+y\cdot g(x,y,z,w)+ \\
&\quad z\cdot h(x,y,z,w)+w\cdot p(x,y,z,w)+rx] \\
&= S[f(x,y,z,w)-rx+\Delta g(x,y,z,w)+\xi(t)-y\cdot g(x,y,z,w)- \\
&\quad z\cdot h(x,y,z,w)-w\cdot p(x,y,z,w)-f(x,y,z,w)+K\cdot \mathrm{sign}(S)+ \\
&\quad y\cdot g(x,y,z,w)+z\cdot h(x,y,z,w)+w\cdot p(x,y,z,w)+rx] \\
&= S[\Delta g(x,y,z,w)+\xi(t)+K\cdot \mathrm{sign}(S)] \\
&\le |S|\cdot(d_1+d_2+K) < 0
\end{aligned}, \tag{16}
$$

Therefore, with the input uncertainty and external disturbance considered, when the controller gain $K < -(d_1+d_2)$, the controller (12) can make the system state to the sliding mode S=0 in limited time.

Remark 3.2 If the fractional orders of the fractional-order system (2) are $q_1 = q_2 = q_3 = q_4 = 1$, then the control of an integer-order system can be achieved by the controller (12).

4. Numerical Simulations

To evaluate the effectiveness of the designed controller, we perform three representative examples in this section. The simulation results are modeled in MATLAB software using the fractional predictor-corrector algorithm [27].

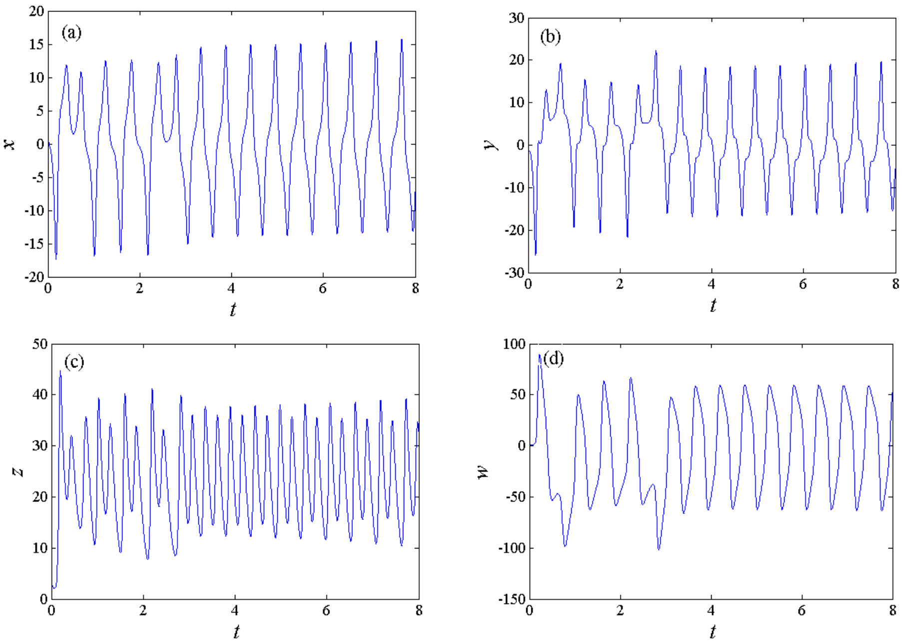


Figure 1. State Time Domain of Fractional-Order Hyperchaos (17)
(a) $x-t$;(b) $y-t$;(c) $z-t$;(d) $w-t$.

***Case 1**: Non-commensurate 4-D fractional-order hyperchaos*

Fractional-order hyperchaotic Lorenz system is considered as [28]:

$$\begin{cases} \frac{d^{\beta_1} x}{dt^{\beta_1}} = a(y-x)+w+u(t) \\ \frac{d^{\beta_2} y}{dt^{\beta_2}} = -xz+cx-y \\ \frac{d^{\beta_3} z}{dt^{\beta_3}} = xy-bz \\ \frac{d^{\beta_4} w}{dt^{\beta_4}} = -xz-rw \end{cases} , \tag{17}$$

where the fractional orders are: β_1 =0.99, β_2 =0.98, β_3 =0.97, β_4 =0.98, r=1, [a, b, c]=[10, 8/3, 28]. Figure 1 shows the state time domain without controller which exhibits chaotic vibration.

Considering (3) and (12), we can get the sliding surface $S(t)$ and the corresponding $u(t)$:

$$\begin{aligned} S(t) &= D^{q_1-1}x(t)+\int_0^t \varphi(\tau)d\tau \\ &= D^{q_1-1}x(t)+\int_0^t [y(\tau)-w(\tau)\cdot z(\tau)+r\cdot x(\tau)]d\tau \end{aligned} , \tag{18}$$

$$u(t) = -c\cdot y(t)+z(t)\cdot w(t)+K\cdot \text{sign}(S), \tag{19}$$

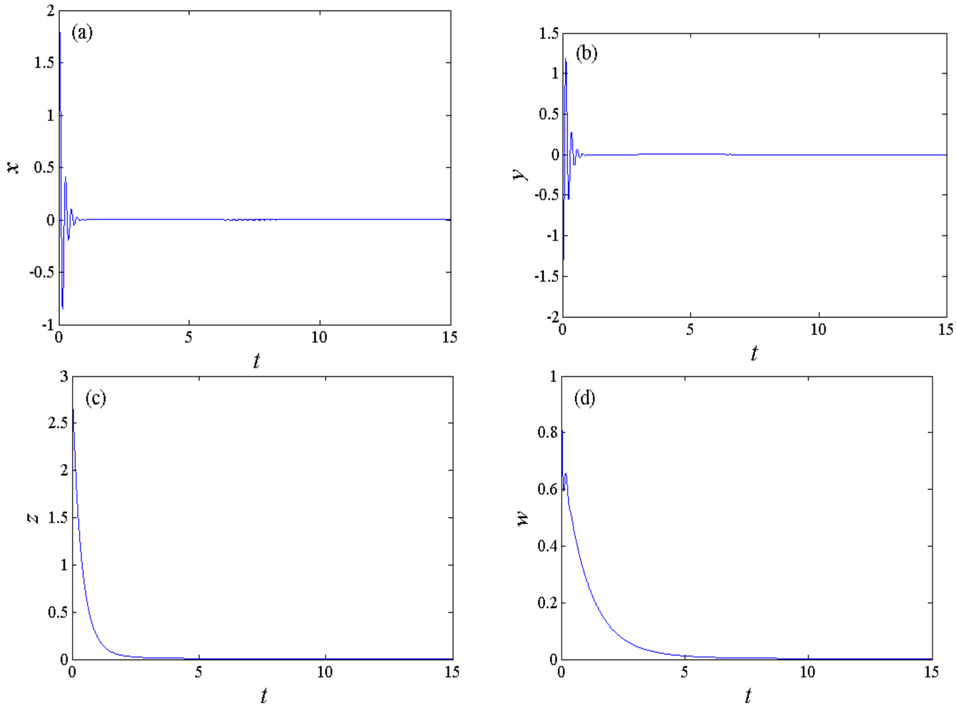
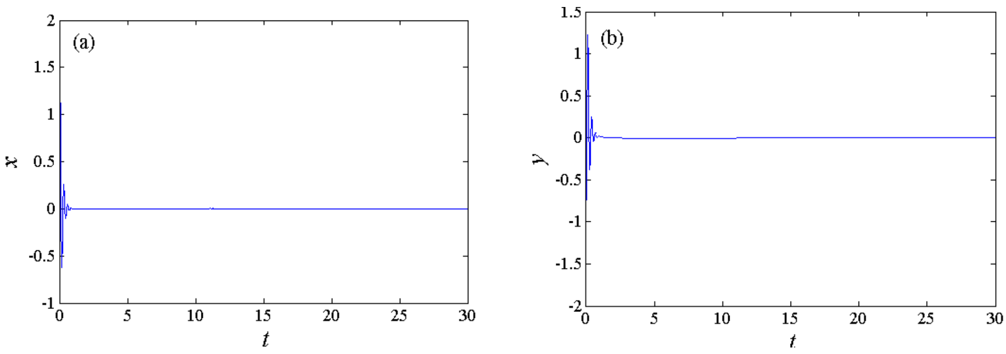


Figure 2. State Time Domain of Controlled Fractional-order hyperchaos (17)
$(a)x-t;(b)y-t;(c)z-t;(d)w-t.$

When the controller gain $K=-0.1$, Figure 2 present the simulation results with initial value [1, -2, 3, 1]. The state trajectories of system (17) under the controller (19) are presented in Figure 2.

Now an uncertainty term $\Delta g(\square)=0.1\sin(\pi x)\cdot\cos(\pi y)\cdot\sin(2\pi z)$ and an external disturbance $\xi(t)=0.01\sin(\pi t)$ are considered, where $|\Delta g(x,y,z,w)|\le d_1=0.1$ and $|\xi(t)|\le d_2=0.01$. Corresponding simulation results of the state responses under the designed controller (19) are shown in Figure 3, which show the effectiveness of the proposed scheme.



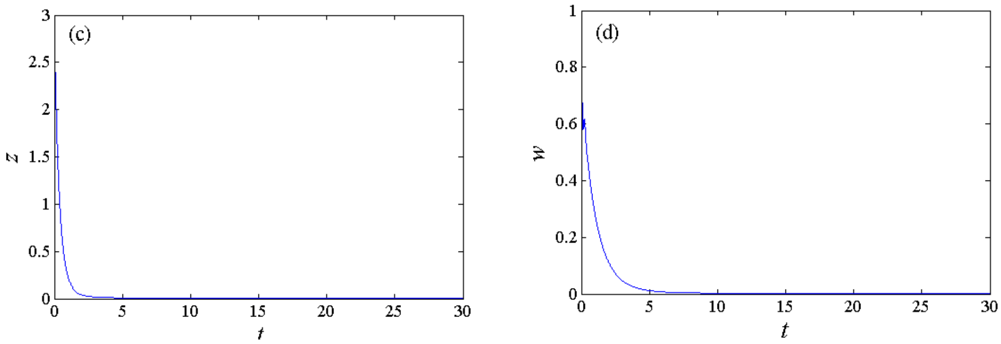


Figure 3. State Time Domain of Controlled Fractional-order Hyperchaos (17) with Uncertainty

$(\mathrm{a})x-t;(\mathrm{b})y-t;(\mathrm{c})z-t;(\mathrm{d})w-t.$

***Case 2**: 4-D integer-order chaos as a special case*

The 4-D Lorenz-Stenflo chaos is presented as [29]:

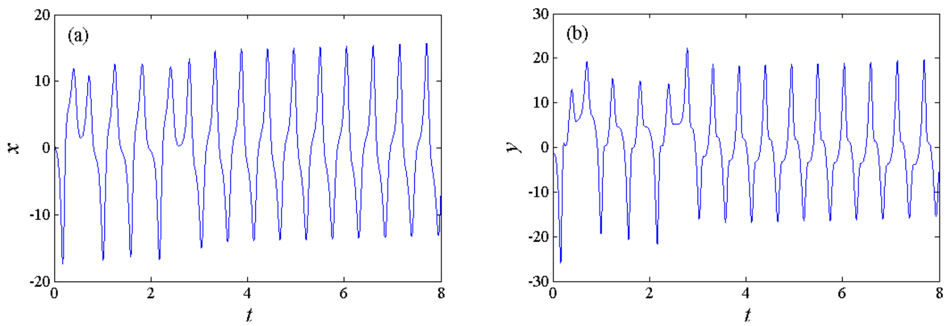
$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t}=a(y-x)+dw+u(t) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t}=rx-xz-y \\ \dfrac{\mathrm{d}z}{\mathrm{d}t}=xy-bz \\ \dfrac{\mathrm{d}w}{\mathrm{d}t}=-x-aw \end{cases}, \quad (20)$$

When a=1, b=0.7, c=1, d=26, system (20) is chaotic. Figure 4 shows the chaotic state of system (20) when the controller is not added.

Considering (3) and (12), we can get the sliding surface $S(t)$ and the corresponding controller:

$$\begin{aligned} S(t) &= \mathrm{D}^{1\text{-}1}x(t)+\int_0^t \varphi(\tau)\mathrm{d}\tau \\ &= \mathrm{D}^{1\text{-}1}x(t)+\int_0^t [y(\tau)\cdot(\mathrm{d}-z(\tau))+y(\tau)\cdot z(\tau)-c\cdot w(\tau)+r\cdot x(\tau)]\mathrm{d}\tau \end{aligned}, \quad (21)$$

$$u(t)=-\mathrm{a}\cdot y(t)-r\cdot w(t)\text{-}y(t)\cdot(\mathrm{d}-z(t))-z(t)\cdot y(t)+w(t)\cdot \mathrm{c}+K\cdot \mathrm{sign}(S), \quad (22)$$



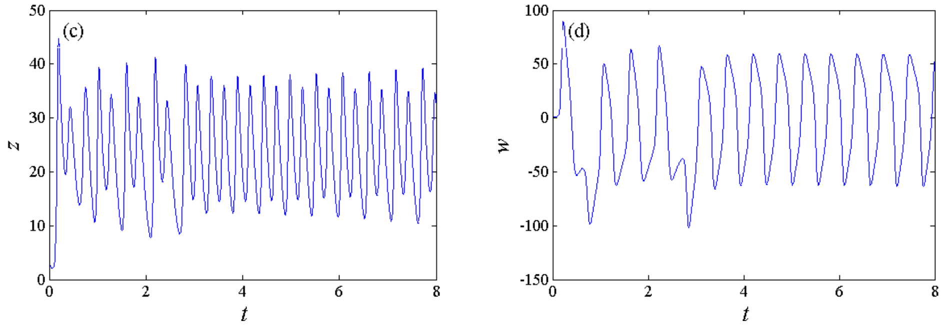


Figure 4. State Trajectories of Lorenz–Stenflo System (20)

$$(\mathrm{a})x-t;(\mathrm{b})y-t;(\mathrm{c})z-t;(\mathrm{d})w-t.$$

We set the initial value x=1, y=-1, z=3, w=-2 and $K=-0.1$. The state responses of system (20) with the controller (22) are shown in Figure 5. The state can be stabilized in a limited time, which shows the applicability for special case of integer-order chaos.

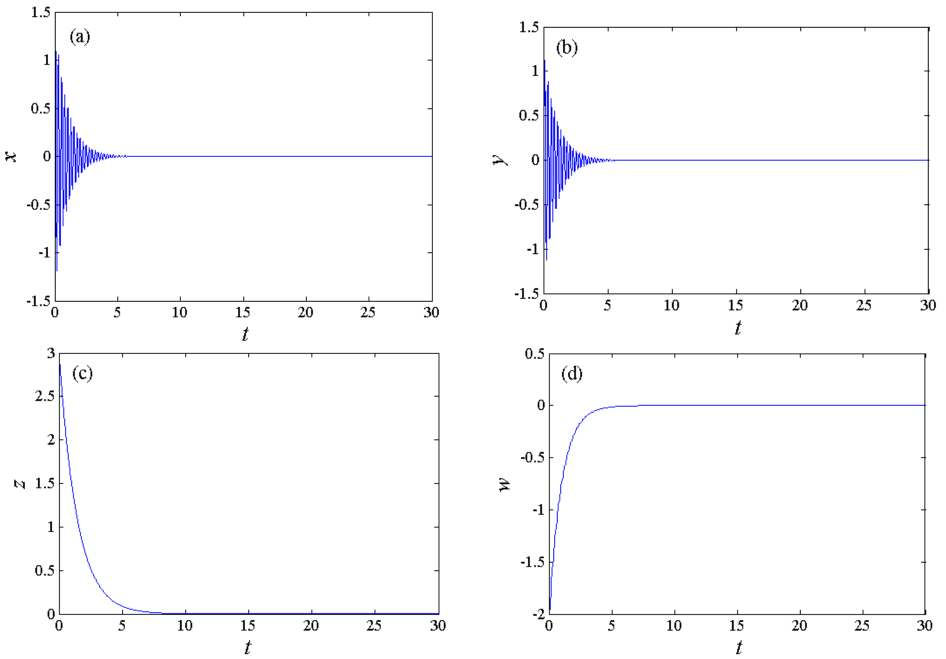


Figure 5. State Responses of Controlled Lorenz-Stenflo System (20)

$$(\mathrm{a})x-t;(\mathrm{b})y-t;(\mathrm{c})z-t;(\mathrm{d})w-t.$$

***Case 3**: 3-D fractional-order chaos as another special case*

We select fractional-order simplified Lorenz chaos given as [30]:

$$\begin{cases}\dfrac{\mathrm{d}^{q_1}x}{\mathrm{d}t^{q_1}}=10(y-x)\\ \dfrac{\mathrm{d}^{q_2}y}{\mathrm{d}t^{q_2}}=(24-4c)x-xz-cy\,,\\ \dfrac{\mathrm{d}^{q_3}z}{\mathrm{d}t^{q_3}}=xy-(8/3)z\end{cases} \tag{23}$$

where $q_1 = q_2 = q_3 = 0.995$ are the orders of the system, $c = 1$. Figure 6 shows the state time domain of system (23) which performs a chaotic behavior.

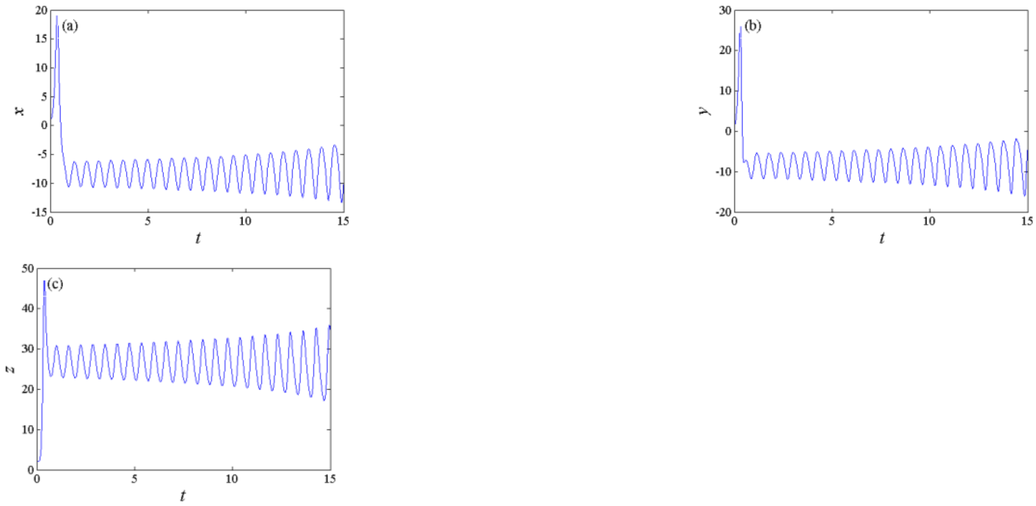


Figure 6. State Time Domain of Fractional-order Simplified Lorenz Chaos (23)

$$(a)x-t;(b)y-t;(c)z-t.$$

Considering (3) and (12), the sliding surface $S(t)$ and the corresponding controller $u(t)$ could be given as:

$$S(t) = D^{q_1-1}x(t) + \int_0^t \varphi(\tau)d\tau = D^{q_1-1}x(t) + \int_0^t [y(\tau)(24-4c) + r\cdot x(\tau)]d\tau\,, \quad (24)$$

$$u(t) = -y(t)\cdot(24-4c) - 10y(t) + K\cdot \text{sign}(S)\,, \quad (25)$$

The simulation results of system (23) under the controller (25) are illustrated in Figure 7, when the controller gain $K = -0.01$, and initial value [1, 1.2, 2.3]. It shows the validity for the special case of 3-D fractional-order chaos.

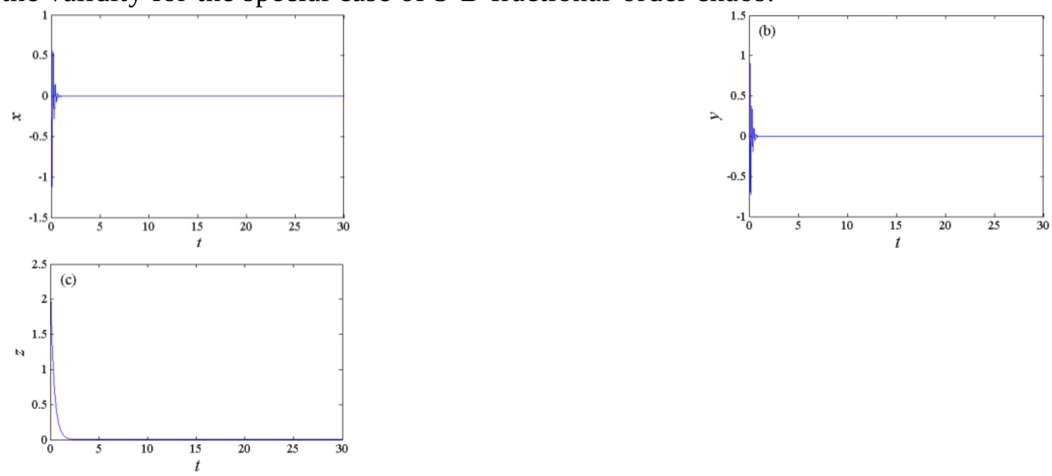


Figure 7. State Time Domain of Controlled Fractional-order Simplified Lorenz Chaos (23)

$$(a)x-t;(b)y-t;(c)z-t.$$

From the above simulations of case 1, case 2 and case 3, we can clear see when the controller was added, state responses of the system can be stabilized in limited time, which shows the effectiveness and robustness of the proposed method.

5. Conclusions

A new simplified sliding mode control method was designed for the control of a novel 4-D fractional-order chaos. Three typical examples were given and corresponding results were simulated to prove the effectiveness and generalization of the proposed scheme. Case 1 presents the control of non-commensurate 4-D fractional-order chaos. Case 2 shows the control of 4-D integer-order Lorenz-Stenflo system, which shows the integer chaos can be regarded as a special case. Maybe it brought some new perspective to cognize the relationship between integer and fractional chaos. Case 3 shows that the control of 3-D fractional-order chaos can be implemented by this method, which shows the general applicability. In practical systems, with the uncertainty considered, the proposed control method is of great significance because of its good robustness and simple realization. The approach is easy to implement and also can be applied to other relevant systems.

In the future, we will continue studying the advanced control scheme for fractional-order chaos.

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