

## **A Study on the Multiple UAVs Cooperative Fire Fighting based on Consensus Algorithm**

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### **Abstract**

*As for multi-uninhabited aerial vehicle cooperative fire fighting problem, a multi-uninhabited aerial vehicle cooperative control method was proposed based on consensus algorithm. multi-uninhabited aerial vehicle cooperative control problem was divided into two steps: 1) a multi-uninhabited aerial vehicle arrival simultaneous control strategy under influence of wind field based on consensus algorithm was presented, which achieves the target of multi-uninhabited aerial vehicle arrival simultaneous at fire front and scattered around the fire front, and prepared for cooperative fire fighting; 2) Applying consensus algorithm, cooperative tracking of the designed uninhabited aerial vehicle motional orbit, and fighting fire from outside to inside to avoid recrudescence. In the process of multi-uninhabited aerial vehicle tracking fire front and fire fighting, both step considered the influence of wind, achieved multi-UAV fighting fire simultaneous, shortened fire fighting time to decreased fire loss to minimum. Simulation results demonstrated that the proposed method could achieve target of multi-uninhabited aerial vehicle fire fighting effectively.*

**Keywords:** UAV; consensus algorithm; fire growth model; cooperative control

### **1. Introduction**

Forest fire is one of natural disasters puzzled human security and development, in global forest fire taken place more than twenty thousand times in each year on average, burned forest account for approximately one over one thousand of world's total forest area, its social harm is great.

Forest fire not only burn down forest and other plants on earth surface, made soil bareness, but also reduce forest's function of maintaining water and soil, regulating climate, affects seriously application of animal and plant resource and forest by-products. Forest fire makes the stored energy of forest release abruptly, destroy forest's ecosystem, and cause chaos of biotic factor and ecological factor in ecosystem, which need decades or longer to recover. Forest fire damage all kinds of buildings and production, living materials in forest, even threaten security of villages and other settlements near forest, and

fighting forest fire consume a large amount of manpower, material resources and financial resources. In addition, in forest fire smoke, there exist large amounts of carbon dioxide and ordinary plant toxin, which affect human health seriously.

At present, application of UAV for forest fire fighting has strong advantages. UAV has faster response ability, and strong adaptability to environment and meteorological condition, which is more superior to manned aircraft in this aspect; UAV could withstand hazardous atmosphere which manned aircraft can't bear, such as high temperature; the basic equipment of UAV's configuration is multimedia acquisition system which responsible for visual and audio data collection, and transmit these data to ground receiver through wireless transmission mode, its data transmission distance up to 30 kilometers. Applying UAV for monitoring forest fire area could help fire command department rapidly and effectively organize and deploy fire fighting team, improve fire fighting efficiency and prevent casualties of fire fighting personnel. UAV has certain load capacity, could carry a certain number of equipment or material, such as fire extinguishing dry powder. In addition, applying UAV instead of manned aircraft performing tasks could reduce personnel's harm and the task's cost.

In this paper, consensus algorithm is applied to multi-UAV cooperative fire fighting. In multi-agent system, 'consensus' means concerned information of multi-agent system achieving the same value. In this paper, the task of fire fighting is divided into two parts: 1) multi-UAV cooperative tracking fire front; 2) multi-UAV cooperative fire fighting. Consensus algorithm is applied in both parts, in part 1), multi-UAV arriving at fire front simultaneous by using consensus algorithm, then prepared for fire fighting; in part 2), UAV's motional orbit must be consistent with fire growth model, and cooperative fighting forest fire.

So far, there are lots of research results about consensus algorithm. Ref. 1 presented theoretical proof for Vicsek et al proposed multiple autonomous agents discrete time model based on nearest neighbor rule, moreover, other similarly models' convergence result are also derived. In Ref. 2, a stochastic model for distributed average consensus is considered, in this model, the quality of consensus can be measured by single variable's deviation relative to. In Ref. 3 consensus problem of multi-agent system under time-varying reference state is investigated, proposed consensus algorithm with time-varying reference state, presented sufficient and necessary condition of multi-agent reach consensus under time-varying reference state. In Ref. 4 UAV cooperative control strategy for aerial surveillance is summarized, Ref. 5 presented a theoretical framework for analysis consensus algorithms of multi-agent network system, summarized basic concepts of information consensus of networks and convergence of the algorithms. Its analytical framework is based on matrix theory, algebraic graph theory and control theory, and discussed consensus problem of network dynamic system, such as synchronization of coupled oscillators, formation control and fast consensus in small-world networks.

Multi-agent consensus under fixed and switching topologies are discussed in Ref. 6, including three cases, introduced two consensus protocols for network with and without time-delays and presented convergence analysis for above three cases. Ref. 7 considered multi-agent consensus problems of dynamic switching topology and non-uniform time-varying delays, analysis several consensus problem of different situation, such as non-uniform time-varying delays, intermittent communication and data packet dropout, proposed a new approach based on a tree-type transformation to investigate consensus problems and presented sufficient, necessary condition. Ref. 8 investigated average consensus problem of first order multi-agent network under uncertain communication environment, through applying a distributed stochastic approximation type protocol to weaken communication noise, and obtained the protocol's consensus condition according to probability limit theory and algebraic graph theory, furthermore, presented necessary condition for fixed topology case and sufficient condition for switching topology case.

As for discrete-time consensus problem of multi-agent network with time-varying delays and switching topologies, Ref. 9 proposed an effective consensus protocol based on repeatedly using the same state information at two time-steps. All above-mentioned references are about first order consensus problem; there are also lots of research results about second order and high order consensus problem. In Ref. 10 second order consensus problem of multi-agent system with non-linear dynamics and directed topologies, presented some sufficient conditions for multi-agent system reaching second order consensus based on algebraic graph theory, matrix theory and Lyapunov control approach. Ref. 11 considered fast consensus problem of multi-agent network system, namely optimizing convergence rate problem of consensus protocol with delays, and investigated a consensus protocol with multi-hop relay scheme. High order consensus problem is considered in Ref. 12, and presented sufficient and necessary conditions for information variable and their higher order derivatives converge to common values, proposed the idea of higher-order consensus with a leader, and introduced concept of  $l$ th order consensus problem.

Ref. 13 investigated high order consensus problems of a class of multi-agent system with dynamic switching topologies and time delays, by using nearest neighbor rule to solve communication delays in high order discrete time system. Ref. 14 considered distributed consensus problem of multi-agent system with directed topology and subject to quantized information flow. In Ref. 15 multi-UAV arrival simultaneous problem is researched, proposed a decentralized control method for multi-UAV arriving at same time, including distributed control structure based on local information interaction and distributed control strategy based on consensus algorithm. Ref. 16 considered consensus tracking control problem of first order multi-agent system with switching topology, and proposed new consensus tracking protocol with a constant and time-varying reference state, then analyzed the consensus tracking protocol's stability by using Lyapunov stability theory, and extended the consensus tracking protocol with time-varying reference state to formation control.

Nowadays, research about multi-UAV cooperative control based on consensus algorithm is just beginning, lots of technologies just in exploration, multi-UAV cooperative fire fighting problem based on consensus algorithm have not been solved. Solving multi-UAV cooperative fire fighting problem using consensus theory have special advantages: 1) applying consensus algorithm could make multi-UAV arriving at fire front at same time, and deploy them with regular distance around the fire front, prepare for cooperative fire fighting; 2) applying consensus algorithm could make multi-UAV tracking the designed UAV motional orbit with no deviation, then thoroughly fight forest fire from outside to inside to reduce fire area step by step, and avoid forest fire recrudescence; 3) applying consensus algorithm could achieve multi-UAV fighting forest fire simultaneous, reduce fire fighting time, then decrease fire loss to minimum and fire fighting cost.

In the past few decades, there are also lots of research results on forest fire growth model. Ref. 17 investigated multi-UAV cooperative fire fighting control method, divided the fire fighting task into two parts, 1) cooperative tracking fire front and accurate situational awareness; 2) cooperative fire fighting. The author established two utility functions for each part, designed decentralized control method and analyzed stability of system and its ability for performing task. Ref. 19 introduced fire growth simulation model, FARSITE, and considered its performance. In addition, the author investigated how the simulation was constructed and how the individual fire behavior models perform.

Ref. 20 investigated forest fire growth problem based on Huygens principles under variable weather and fuel conditions, presented a new procedure for the derivation of two-dimensional fire growth model. Ref. 21 summarized the existing fire growth models, and classified these models to physical, semi-physical and empirical according to their nature, and considered each model's advantages and disadvantages, then presented two important

research areas to improving fire growth model: 1) a better understanding of fire-dependent phenomena, 2) development of a 'new generation' of fire spread models.

Ref. 22 presented four forest fire two-dimensional simulation models: cellular automata, semi-empirical, physical and envelop models, introduced basic principle of the envelop model derived by Richards. Ref. 23 analyzed and compared simulation accuracy of FARSITE by using different fuel and meteorological input data in Mediterranean area, the study show that compared with using standard fuel model the model designed for scrubland vegetation provide more realistic values of spread rate.

Ref. 24 proposed a fire spread rate approximation method in two-dimensional with random fuel patterns, through a two-dimensional grid which fire could move forward or laterally calculate mean spread rate of fire. Ref. 25 derived a differential equation system which describes a fire growth model in time for condition of variable fuel, weather and topographical conditions.

From the published literatures, research about fire growth model has become more mature; however, literatures about multi-UAV cooperative fire fighting are rare. Ref. 17 proposed a multi-UAV cooperative fire fighting control method, by using artificial potential field established utility functions, achieved fire front tracking and cooperative fire fighting, and simulation results showed that the method could achieve fire fighting task. However, there are some shortcomings in the method: 1) fire fighting time may be longer, because of UAV's trajectory is not clear, only by attractive force between fire point and UAV and repulsive force between UAVs to control fire fighting, so fire fighting time is hard to control; 2) fire fighting may be incomplete, because fire fighting can't be uniform and comprehensive, may be there are omission fire points that leading to generate new fire. Therefore, if UAV can't fight forest fire completely, it will bring new hidden danger and hardly achieve target of multi-UAV cooperative fire fighting; 3) local extreme problem of artificial potential field method, when fire area is little, each UAV bear attractive force from fire point and repulsive force from other UAVs, then multi-UAV can't access to this fire point and can't fight the fire point.

This paper proposes a multi-UAV cooperative fire fighting control method based on consensus algorithm which could improve above-mentioned shortcomings effectively. After multi-UAV arriving at fire front, they track the designed UAV motional orbit and move to fire center, reduce fire area step by step. At the same time, as UAV's trajectory is known, the time for fire fighting could be calculated. In the fire fighting process, with the fire area decreasing, number of UAVs is reducing at the same time, to save resource and avoid collision of UAVs, and achieving fire fighting target. This paper is organized as follows: firstly, introduce consensus theory and relative theory, including consensus algorithm, UAV motion model, forest fire growth model based on Huygens and its simplified model; secondly, present control strategy of multi-UAV arriving at fire front simultaneous; then, multi-UAV cooperative tracking model and fire fighting process, including UAV tracking model, calculation of trajectory length and angle between trajectory and wind field, finally, present simulation results and conclusion.

## 2.1 Consensus Algorithm

### 2.1.1 Graph Theory

Graph theory is an important tool to analyze consistency problem. Consider a structure composed by multi-agent, and describe the communication relationship between multi-agents by digraph in graph theory is simple and effective.

A topology composed by multi-UAV is represented by using,  $G=(V,E)$  with the set of nodes  $V$ , and set of edges  $E \in V^2$ . When agent  $i$  and agent  $j$  have information exchange, then there is an edge between them, that is  $e_{ij}=(v_i, v_j) \in E$ . If agent

$i$  could receive the information from agent  $j$ , then agent  $j$  is the neighbor of agent  $i$ , and neighbor set of node  $i$  is denoted by  $N_i = \{j \in V : a_{ij} \neq 0\}$ . In digraph, there is an arrow from agent  $i$  to agent  $j$ , or from agent  $j$  to agent  $i$ . In undirected graph, there only a line between agent  $i$  and agent  $j$ , the edge do not have direction, as  $(v_i, v_j) \in E \Leftrightarrow (v_j, v_i) \in E$ .

### 2.1.2 Matrix theory

In order to solve the problem of communication relationship between multi-UAVs, two relative matrixes are introduced:

(1) Adjacent matrix  $A = [a_{ij}]_{n \times n}$

Consider a topology structure composed with  $n$  agents, with the set of nodes  $I = \{1, 2, \dots, n\}$ , the element of adjacent matrix  $A = [a_{ij}]_{n \times n}$  is defined as:

$$a_{ij} = \begin{cases} 1, & (v_j, v_i) \in E \\ 0, & \text{others} \end{cases} \quad (1)$$

(2) Laplacian matrix  $L = [l_{ij}]_{n \times n}$

The element of Laplacian matrix is:

$$l_{ij} = \begin{cases} \sum_{j=1, j \neq i}^n a_{ij}, & i = j \\ -a_{ij}, & i \neq j \end{cases} \quad (2)$$

Continuous consensus model was given by Ref[6] as follows :

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij} (x_j(t) - x_i(t)) \quad (3)$$

Where, the  $a_{ij}$  is the element of the Adjacency matrix,  $N_i$  is the Neighbors set of the agent  $i$ , defined as  $N_i = \{j \in V : a_{ij} \neq 0\}$ .

The agents finally converge to a consistent by exchange their local information, and the continuous consensus model for multi-agent algorithm is as follows:

$$\dot{x} = -Lx$$

Where  $L$  is the Laplacian matrix.

### 2.2 Uav Motion Model

UAV could be seemed as particle in two dimensional planes, the simple motion model [15] is:

$$\begin{cases} \dot{x}_i = v_i \cos \varphi_i \\ \dot{y}_i = v_i \sin \varphi_i, & (i = 1, 2, \dots, n) \\ \dot{\varphi}_i = \omega_i \end{cases} \quad (4)$$

Where  $x_i, y_i$  is the coordination of the UAV  $i$  in plane,  $v_i, \varphi_i$  and  $\omega_i$  means the velocity, course angle and course angular velocity of the  $i$  UAV respectively. The flight control system of UAV have velocity maintaining ability of autopilot, it could follow the given velocity instruction. The velocity maintaining of autopilot could describe approximately by first-order dynamic model:

$$\dot{v}_i = \alpha (v_i^c - v_i)$$

While,  $v_i^c$  is the interval velocity instruction of the  $i$  UAV,  $\alpha$  is a positive constant. In addition, in real environment UAV has flying velocity limitation, namely  $0 < v_{\min} \leq v \leq v_{\max}$ .

### 2.3 Fire Growth Model Based On Huygens Principle

The fire growth model based on Huygens principle is regarding each point on fire front as individual fire source, calculating each fire sources' propagation area on the next time, and the new fire front is the envelop of these fire sources' propagation area. The fire growth model [25] is:

$$\begin{cases} X_t = \frac{a^2 \cos \alpha (x_s \sin \alpha + y_s \cos \alpha) - b^2 \sin \alpha (x_s \cos \alpha - y_s \sin \alpha)}{\sqrt{b^2 (x_s \cos \alpha + y_s \sin \alpha)^2 - a^2 (x_s \sin \alpha - y_s \cos \alpha)^2}} + c \sin \alpha \\ Y_t = \frac{-a^2 \sin \alpha (x_s \sin \alpha + y_s \cos \alpha) - b^2 \cos \alpha (x_s \cos \alpha - y_s \sin \alpha)}{\sqrt{b^2 (x_s \cos \alpha + y_s \sin \alpha)^2 - a^2 (x_s \sin \alpha - y_s \cos \alpha)^2}} + c \cos \alpha \end{cases} \quad (5)$$

While,  $X_t$  and  $Y_t$  are the rate differentials, and angle  $\alpha$  is the wind direction,  $x_s$  and  $y_s$  are the orientation of the vertex on the fire front, the new fire front is obtained by multiplying the rate differentials with the step time. In addition, in equation (5),  $a$ ,  $b$  and  $c$  are the semi-minor axes, semi-major axes and length between focus and central point of propagation ellipse area of fire source respectively.

### 2.4 Simplification Of Fire Growth Model Based On Huygens Principle

As the fire growth model based on Huygens principle is complex, by using curve fitting method, the simplified formula (5) is as follow:

$$\begin{cases} X_t = i * (1.412 * \cos \theta - 2.8278 * \sin \theta - 1.4142) \\ Y_t = i * (1.412 * \cos \theta + 2.8278 * \sin \theta + 1.4142) \end{cases} \quad (6)$$

While, the parameters' value in formula are  $a=0.5$ ,  $b=1$  and  $c=0.5$ , and the new fire front is obtained by multiplying  $X_t$  and  $Y_t$  with step time  $i$ . In figure 1, the change of wind direction  $\theta=0$  and  $\theta=\pi/4$  are shown.

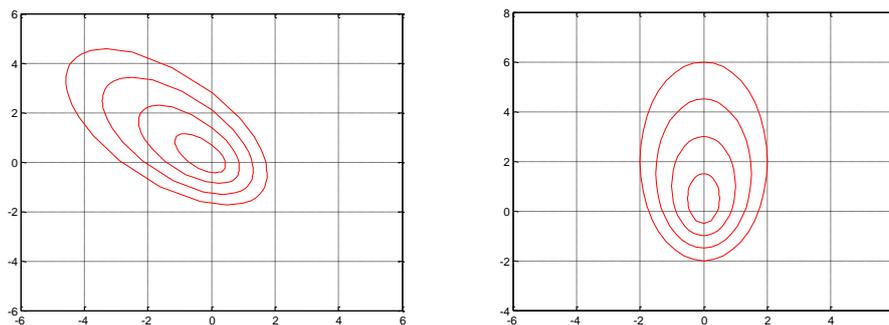


Figure 1: Fire Growth Diagram

## 3 Multi-UAV Arrivals Simultaneous Control Strategy under Wind Field Environment

It needs to consider the effect of wind when investigate multi-UAV arrival simultaneous, the derivation process of multi-UAV arrival simultaneous under wind field environment model as follow:

Suppose that  $L_i$  is remaining path length of UAV  $i$  to its target at time  $t$ , and  $L_i$  is function of  $t$ , differentiate  $L_i$  with respect to  $t$ , then:

$$\dot{L}_i = -v_i \quad (7)$$

In equation (7),  $v_i$  is resultant flying velocity of UAV  $i$  under influence of wind at moment  $t$ . The estimated value of  $L_i$  could be provided by path planning module of UAV according to current position of UAV. Moreover, resultant velocity of UAV subject to follow equation:

$$v_u^2 = v_i^2 + v_f^2 - 2v_i v_f \cos \theta \quad (8)$$

While,  $v_u$  is provided by autopilot of UAV,  $v_f$  is the velocity of wind, and the angle between UAV's trajectory and wind direction is  $\theta$ . Then expression of  $v_i$  is:

$$v_i = v_f \cos \theta + \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2} \quad (9)$$

The expected arrival moment of UAV  $i$  at moment  $t$  could calculate by  $L_i$  and  $v_i$ :

$$\zeta_i = t + \tau_i = t + \frac{L_i}{v_i} \quad (10)$$

While,  $\tau_i = L_i / v_i$  is expected arrival time of UAV  $i$  at time  $t$ . Differentiate equation (10) with respect to  $t$ , and plug equation (4) and (9) into it:

$$\dot{\zeta}_i = -\frac{\tau_i v_u \alpha (v_u^c - v_u)}{(v_f \cos \theta + \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}) \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}} \quad (11)$$

Let  $x_i = \zeta_i$ , then:

$$u_i = -\frac{\tau_i v_u \alpha (v_u^c - v_u)}{(v_f \cos \theta + \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}) \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}} \quad (12)$$

Derive equation (12), then:

$$v_u^c = v_u - \frac{u_i (v_f \cos \theta + \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}) \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}}{\tau_i v_u \alpha} \quad (13)$$

Combine equation (2) and (13), control strategy of multi-UAV arrival simultaneous under wind condition is:

$$\begin{cases} v_u^c = v_u - \frac{u_i (v_f \cos \theta + \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}) \sqrt{v_f^2 \cos^2 \theta - v_f^2 + v_u^2}}{\tau_i v_u \alpha} \\ u_i = -\sum_{j=1}^n a_{ij} (x_j - x_i) \end{cases} \quad (14)$$

According to above equation, each UAV could arrive at same time with velocity instruction.

#### 4 Fire Front Tracking And Cooperative Fire Fighting Process

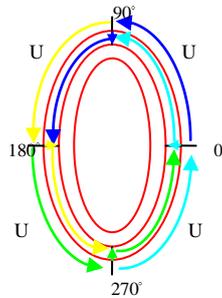
This paper presents UAV tracking model and cooperative fire fighting process by combining forest fire growth simplified model based on Huygens with consensus algorithm:

#### 4.1 UAV Tracking Model

When UAV tracking fire front, fire growth model is equation (6), after multi-UAV arrival at fire front and start to fire fight, their trajectory is equidistant, and all ellipses' center are the same, however center of ellipses of forest fire growth model based on Huygens are not the same, therefore it needs to change ellipse orbit, then motional orbit equation of UAV is:

$$\begin{cases} X_t = i * \left( \frac{a^2 \cos \alpha (x_s \sin \alpha + y_s \cos \alpha) - b^2 \sin \alpha (x_s \cos \alpha - y_s \sin \alpha)}{\sqrt{b^2 (x_s \cos \alpha + y_s \sin \alpha)^2 - a^2 (x_s \sin \alpha - y_s \cos \alpha)^2}} \right) + c \sin \alpha \\ Y_t = i * \left( \frac{-a^2 \sin \alpha (x_s \sin \alpha + y_s \cos \alpha) - b^2 \cos \alpha (x_s \cos \alpha - y_s \sin \alpha)}{\sqrt{b^2 (x_s \cos \alpha + y_s \sin \alpha)^2 - a^2 (x_s \sin \alpha - y_s \cos \alpha)^2}} \right) + c \cos \alpha \end{cases} \quad (15)$$

When multi-UAV arrive at fire front at same time, divides fire front ellipse into several equal parts according to number of UAV, then multi-UAV tracking their motional orbit at certain velocity. After multi-UAV finished fire fight at outer elliptical orbit, they shift to next elliptical orbit inside. At present, new motion elliptical orbit of UAV is equate and each UAV's path length are the same, but they have different angle. For instance, fire fighting scope of UAV 1 is  $0^\circ \sim 90^\circ$ , fire fighting scope of UAV 2 is  $90^\circ \sim 180^\circ$ , fire fighting scope of UAV 3 is  $180^\circ \sim 270^\circ$ , and fire fighting scope of UAV 4 is  $270^\circ \sim 360^\circ$ , then after each UAV finished regulated trajectory, they shift to next orbit, at this moment, fire fighting scope of UAV 1 is  $90^\circ \sim 180^\circ$ , fire fighting scope of UAV 2 is  $180^\circ \sim 270^\circ$ , fire fighting scope of UAV 3 is  $270^\circ \sim 360^\circ$  and fire fighting scope of UAV 4 is  $0^\circ \sim 90^\circ$ , as shown in figure 2:



**Fig. 2: Multi-UAV Cooperative Fire Fighting Motional Orbit Diagram**

While, blue arrow line represent motional orbit of UAV 1, yellow arrow line represent motional orbit of UAV 2, green arrow line represent motional orbit of UAV 3 and cyan arrow line represent motional orbit of UAV 4. In order to explain each UAV's motional orbit, only draw two trajectories as an example.

#### 4.2. Trajectory Length Calculation Of UAV

As task of cooperative fire fighting needs several UAV, then each UAV's fire fighting scope could be designed to same, namely each UAV's trajectory length is same on the same elliptical orbit. Parametric equation of UAV motional orbit is:

$$\begin{cases} X_t = i * (1.412 * \cos \theta - 2.8278 * \sin \theta) - 1.4142 \\ Y_t = i * (1.412 * \cos \theta + 2.8278 * \sin \theta) + 1.4142 \end{cases}$$

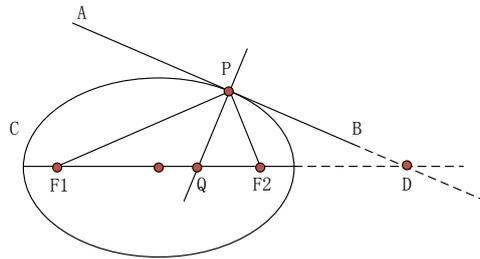
The curvilinear integral of elliptic arc length is:

$$\int_L ds = \int_a^\beta \sqrt{X'^2(t) + Y'^2(t)} dt = \int_a^\beta \sqrt{(-1.412 * \sin \theta - 2.8278 * \cos \theta)^2 + (-1.412 * \sin \theta + 2.8278 * \cos \theta)^2} d\theta \quad (19)$$

While  $\alpha < \beta$ , because of curvilinear integral of elliptic arc length can hardly solved by analytical method, this paper solves it by numerical integration.

### 4.3 Calculation of Angle between Trajectory and Wind Field

Because of influence of wind, it needs to solve the angle between UAV's trajectory and direction of wind for calculating resultant force.



**Fig. 3: Angle between Ellipse's Tangent and Major-Axis Diagram**

In figure 3, ellipse  $c$  is the motional orbit of UAV, suppose current location is at point  $P$ ,  $AB$  is the ellipse's tangent at point  $P$ , namely movement direction of UAV, suppose major axis of ellipse is the direction of wind,  $F1$  and  $F2$  are focus of ellipse,  $PQ$  is the normal of  $AB$ ,  $Q$  is the intersection point of  $PQ$  and major axis, extends tangent  $AB$  and major axis to intersection point  $D$ . When coordinates of point  $P$ ,  $F1$  and  $F2$  are known, then  $\angle F1PF2$  and  $\angle F1F2P$  could be calculated, according to nature of ellipse,  $\angle F1PQ = \angle F2PQ$  and  $\angle APF1 = \angle BPF2$  are found. Because of  $\angle F2PQ$  and  $\angle F1F2P$ ,  $\angle F2QP$  could be calculated, then  $\angle QDP$  could be obtained, namely angle between tangent  $AB$  and major axis, then angle between movement direction of UAV and wind direction is known.

## 5 Simulation Results and Analysis

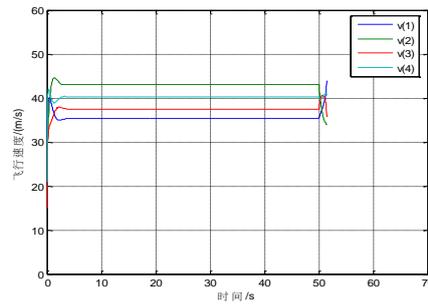
### 5.1 Simulation Results of Multi-UAV Arriving At Fire Front Simultaneous

Consider four UAV cooperative fire fighting under influence of wind, for simple, suppose four UAV located on extension cord of 0, 90, 180 and 270 respectively. Assume that when forest fire is discovered, elliptic parameter equation of fire front is:

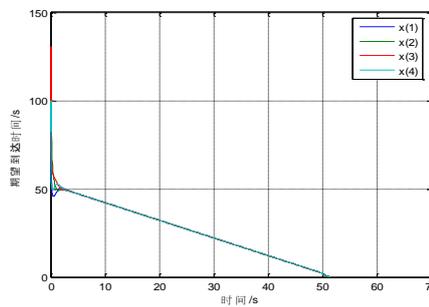
$$\begin{cases} x = 1.412 * \cos \theta - 2.8278 * \sin \theta - 1.4142 \\ y = 1.412 * \cos \theta + 2.8278 * \sin \theta + 1.4142 \end{cases}$$

That is to say, in equation (6)  $i$  take 1, four UAV's initial path lengths to fire front are: 1.846 km, 2.247 km, 1.954 km and 2.099 km, initial velocity of each UAV are: 25 m/s, 24 m/s, 20 m/s and 25.5 m/s, and suppose that velocity limitation is:  $5 \leq v_i \leq 50$  m/s, velocity of wind is 7 m/s, and direction of wind is northwest. Because initial position of UAV are very far away from fire front, and its velocity is larger than fire growth velocity, therefore it doesn't need to gather distance data in regular interval time, instead it needs to estimate the fire growth to adjust each UAV's path length. Apply the decentralized

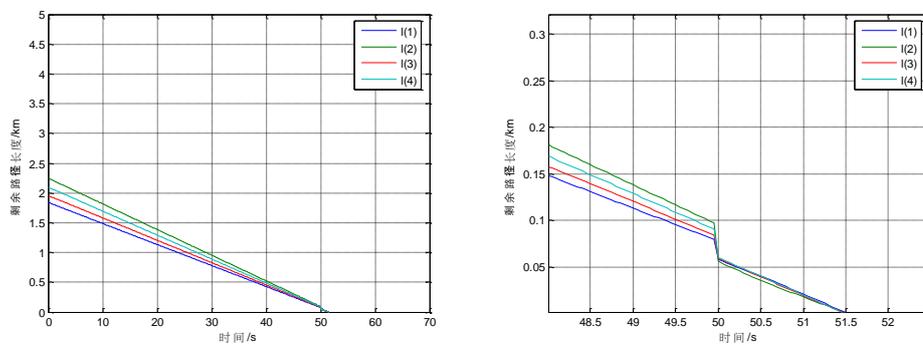
control strategy of multi-UAV arrival simultaneous under the effect of wind, then the simulation results are as follow:



**Fig. 4: Flying Velocity Diagram**

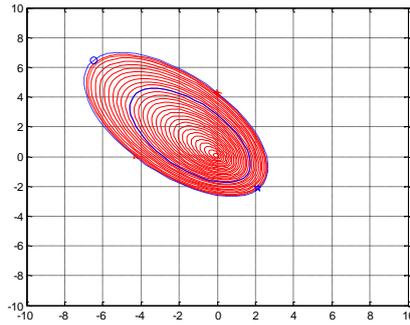


**Fig. 5: Expected Arrival Time Diagram**



**Fig. 6 Remainder Path Length Diagram Of UAV And Its Partial Enlarged Diagram**

It can be seen from figure 4 and 5, the expected arrival time of multi-UAV converges to same soon by using consensus algorithm, and achieved the target of arrival simultaneous. In figure 6, each UAV's path length decrease a few at moment 50 second, because when UAVs are moving, forest fire growing at the same time, and detect fire front at moment 50 second, decrease the length of fire growing from remainder path length, at last multi-UAV arriving at fire front at moment 51.5 second.



**Fig. 7 Multi-UAV Cooperative Tracking Diagram**

At present, the red dotted line represents fire front, and its elliptic parameter equation is:

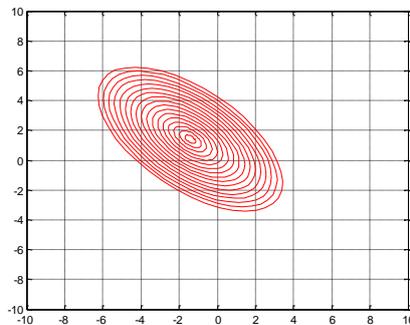
$$\begin{cases} x = 1.48 * (1.412 * \cos \theta - 2.8278 * \sin \theta - 1.4142) \\ y = 1.48 * (1.412 * \cos \theta + 2.8278 * \sin \theta + 1.4142) \end{cases}$$

The blue dotted line represents the elliptical orbit which multi-UAV arriving at, and its elliptic parameter equation is:

$$\begin{cases} x = 1.53 * (1.412 * \cos \theta - 2.8278 * \sin \theta - 1.4142) \\ y = 1.53 * (1.412 * \cos \theta + 2.8278 * \sin \theta + 1.4142) \end{cases}$$

At this time, in figure 7, the elliptical orbit of multi-UAV located is the next orbit of fire front growth, in order to extinguish forest fire completely, it needs to calculate time of multi-UAV complete the desired trajectory and fire growing to the elliptical orbit which multi-UAV located, and guarantee former less than later.

## 5.2 Cooperative Fire Fighting Simulation Result



**Fig. 8 Multi-UAV Cooperative Fire Fighting Motional Orbit**

At present, elliptic equation of multi-UAV motional orbit is (16),  $i$  is step value, and suppose 5 second is a step. And gap between motional orbits of  $0^\circ$  and  $180^\circ$  is  $1.797 m$ , gap between motional orbits of  $90^\circ$  and  $270^\circ$  is  $3.6 m$ . At the beginning of fire fighting, as the fire area is large, it needs lots of UAV, as fire fighting going, fire area is smaller and smaller, at this time, it needs to reduce the number of UAV to reduce the unnecessary waste of resources and improve the efficiency of fire fighting, in addition, suppose UAV could extinguish the fire in a circle of  $2^m$  radius beneath it. According to the analysis of part 3.2 and equation (19), divided ellipse into four parts, calculate UAV's trajectory length on each orbit, and set UAV's velocity of each orbit, as the following table shown:

**Table.1 UAV Trajectory Length and Velocity Value**

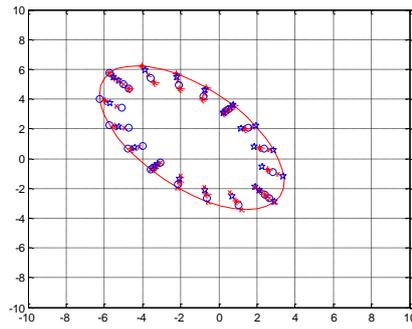
<i>i</i>	1.5	1.44	1.35	1.26	1.17	1.08	0.99	0.9	0.81
	3								
trajectory length (m)	74	69.73	65.37	61.02	56.66	52.3	47.94	43.58	39.22
Velocity( $m/s$ )	18.5	17.4	16.3	15.3	14.2	13.1	12	10.9	9.8
<i>i</i>	0.72	0.63	0.54	0.45	0.36	0.27	0.18		0.09
trajectory length <i>m</i>	34.8	61	52.3	43.6	34.9	26	17.4		8.7
Velocity( $m/s$ )	8.7	15.3	13.4	10.9	8.7	13	8.7		8.7

In table 1, there are two changes in the number of UAV during multiple UAV cooperative fighting fire, the first appeared when  $i=0.63$ , that is because the compound velocity of the UAV is  $8.7 m/s$  at the moment  $i=0.72$ , if there are still four UAVs when they move to the next trajectory, and the compound velocity of the UCAV would less than the wine speed, that is  $v_i < 7m/s$ , but the movement direction of the UAV and wine direction are the same. So  $v_u < 0$ , there will be a contradiction, so the number of UAV declined to two, and declined to one when  $i=0.09$  for the same reason.

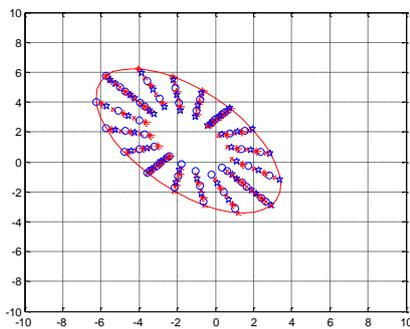
According to the analysis of 3.3, we can easily calculate the speed of the UAV 1 as follows:

label2 the speed value of UAV autopilot and the angle between trajectory and the wind

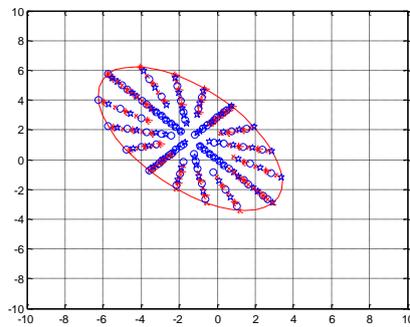
degr ee	0°	3°	6°	9°	12°	15°	18°	21°	24°	27°	30°
Cutti ng angle	0	0.02	0.03	0.05	0.07	0.09	0.1	0.12	0.14	0.16	0.18
Velocit y ( $v_u$ )	11.5	11.5	11.52	11.54	11.56	11.6	11.65	11.7	11.77	11.84	11.93
degree	33°	36°	39°	42°	45°	48°	51°	54°	57°	60°	63°
Cutting angle	0.2	0.22	0.25	0.27	0.3	0.32	0.35	0.38	0.42	0.45	0.49
Velocit y ( $v_u$ )	12.04	12.16	12.29	12.45	12.63	12.83	13.07	13.33	13.63	13.98	14.36
degree	66°	69°	72°	75°	78°	81°	84°	87°	90°	90°	
Cutting angle	0.54	0.58	0.63	0.69	0.74	0.8	0.87	0.93	1.00	2.00	
Velocit y ( $v_u$ )	14.79	15.28	15.81	16.39	17.02	17.69	18.38	19.09	19.78	25.5	



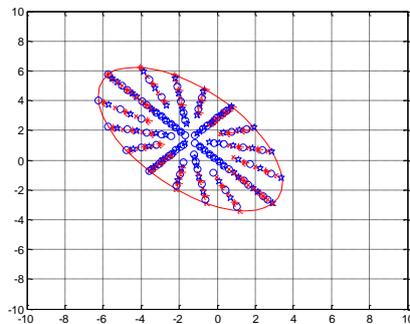
**Figure 9: The UAV Fire Fighting Trajectory At 20s**



**Figure 10: The UAV Fire Fighting Trajectory At 40s**



**Figure 11: The UAV Fire Fighting Trajectory At 52s**



**Figure 12: The UAV Fire Fighting Trajectory At 53s**

Figure 9 and Figure 10 are four UAVs fire fighting figure, the red represents UAV1, the blue circle represents UAV2, the red fork represents UAV3, said, the blue Pentagram represents UAV4. According to circle number, the time for fighting fire can be calculated, UAV finished a circle needed 4s, total completed 10 circle, 40s is needed. Figure 11 for two UAV fire fighting figure, due to reduced of the length, the of time UAV for fighting fire is also reduced, total completed 6 circle, 12s is needed. last figure 12 for a rack UAV oval of long axis movement, because at this time, oval of short half axis length is less than unmanned machine fire range, that 2, eventually after  $40 + 12 + 1 = 53$  will throughout fire district fight, completed the fire fighting task.

## 6 Conclusions

Combined consensus algorithm with the model of fire spread, a fire fighting method for multi-UAV was proposed in this paper. At first, the UAV reach the boundary of fire area at the same time. Using the multi-UAV arrival simultaneous control strategy under influence of wind field, the speed of each UAV was controlled for reaching the boundary at the same time. The second is multi-UAV fire fighting. Using consensus algorithm, the UAV track the tracks designed by this paper for fire fighting. The Multi-UAV cooperative control method has the following advantages: 1) consistency algorithm can achieve complete fire suppression, avoid the risk of fire flame; 2) multi-UAV can be achieved while on fire, reducing fire time, reducing fire losses.

This article is only a preliminary study on multi-UAV cooperative fire fighting, and many conditions is based on assumption, taking into the actual conditions, such as the changing wind field, the fire area is not a flat area, the weather conditions and the different types of fire, a more complete and complex algorithms is need to solve the fire fighting problem.

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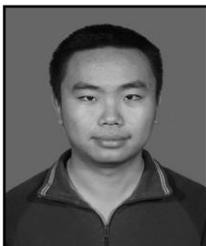
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