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Control System Optimization Design and Simulation for BBT Missile Based on Kriging Meta-model

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Abstract

The backstepping design method has been used to design the control system of the Bank-to-turn (BTT) missile, which depends on the designer's personal experience and a large number of complicated iterative calculation, and needs to consume a lot of time throughout the design process. To solve this problem, a missile control system surrogate model is established by using Kriging meta-model. After establishing the surrogate model, the genetic algorithm is applied to the control system optimal solution. The proposed method can obtain a set of control parameters that minimizes the performance function by searching the given control parameters space. The simulation results show that the method can reduce the amount of calculations and achieve the desired control parameters effectively.

Keywords: *BTT Missile, Backstepping Design, Kriging Metamodel, Genetic Algorithm*

1. Introduction

Along with the air combat environment being increasingly fierce in the future, the mobility of combat aircrafts and missiles would be improved. Bank - To – Turn (BTT)[1] control technology which can meet the challenge is adopted in the new air-to-air missile. The new air-to-air missile presents a complex system with the characteristics of nonlinearity, coupling and uncertainty. Since the classical control methods are unable to solve the problem of the air-to-air missile control system, a new control method is proposed.

Backstepping control method is a new way for nonlinear systems, which is often be used in aircraft control system in recent years. In 1998, Corey Schumacher presented a nonlinear six-degree-of-freedom simulation of a maneuver performed with the dynamic inversion controller [2]. In 2006, Hwang designed a guidance and control incorporation algorithm of pitch channel by using backstepping design method [3].

In this paper, the BTT missile mathematic model is established and the three-channel control system law is designed by using backstepping method. As shown in Fig. 1, the performance indicators can only draw through the numerical simulation analysis, because it can't be showed with math expression. So a large number of complicated iterative calculations are needed for the optimization of this controller, which makes the calculation too time-consuming to realize.

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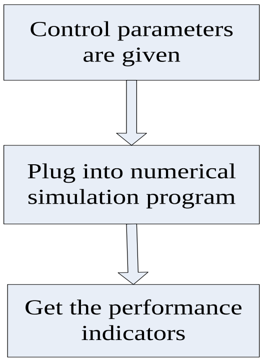

Figure 1: The Procedure of Performance Evaluation

In order to reduce the amount of calculation, and make the optimization results accurate, an optimization method based on surrogate model is proposed. Surrogate model is constructed by using the experimental design method, which is about how to arrange test mathematical methods. The Latin Hypercube design which is based on random numbers is adopted. Many different surrogate models, including polynomial response model [4-5], radial basis function [6,7], and Kriging model[8-10], have been widely used in engineering applications. In this paper, Kriging model is adopted, which is an unbiased estimation model based on least estimation of variance and fits with the original model better in the whole design space.

2. Missile Control System Design

2.1. BTT Missile Modeling

The principle of BTT missile flight control mode: The roll control loop transfers its max lifting surface to the ideal position according to the guidance law rapidly. At the same time, the pitch loop change the size of attack angle according to the acceleration command in order to provide the required overload to realize the aircraft large angle turning. The yaw loop has stability function to the aircraft, and the sideslip angle is small and neglected generally.

System assumption:

1) The sideslip angle β is close to zero.
2) Ignore the gravity, and the thrust is zero.
3) Missile velocity V is along the longitudinal axis of the missile body.
4) For axisymmetric missiles.

Thus $\sin\alpha \approx \alpha, \sin\beta \approx \beta, \cos\beta \approx 1$. The BTT missile system model is established in the missile body coordinate, as follows:

$$\begin{cases} \dot{\omega}_x = a_{11}\omega_x + a_{12}\delta_x \\ \dot{\omega}_y = a_{21}\omega_x\omega_z + a_{22}\beta + a_{23}\omega_y + a_{24}\delta_y \\ \dot{\omega}_z = a_{31}\omega_x\omega_y + a_{32}\alpha + a_{33}\omega_z + a_{34}\delta_z \\ \dot{\gamma} = \omega_x \\ \dot{\alpha} = -\omega_x\beta + \omega_z - a\alpha \\ \dot{\beta} = \omega_x\alpha + \omega_y + b\beta \end{cases} \tag{1}$$

where

a_{11}, a_{12}, a_{21} , etc.: missile coefficients determined by aerodynamic parameters and state of the missile,

$\omega_x, \omega_y, \omega_z$: the rotating angular velocity for x,y,z direction respectively.

γ : missile roll angle.

2.2. Structure of Missile Control System

The BTT missile adopting proportional guidance law and the guidance instructions include longitudinal acceleration a_{yc} ,lateral acceleration a_{zc} ,and roll angle γ_c .

In order to facilitate the design of controller, we convert a_{yc} and a_{zc} to instruction of attack angle α^r and sideslip angle β^r . As follows:

$$\begin{cases} \alpha^r = \dfrac{m a_{yc}}{\left(c_y^{\alpha} - c_y^{\delta_z} \dfrac{m_z^{\alpha}}{m_z^{\delta_z}}\right) qS} \\ \beta^r = \dfrac{m a_{zc}}{\left(c_z^{\beta} - c_z^{\delta_y} \dfrac{m_y^{\beta}}{m_y^{\delta_y}}\right) qS} \end{cases} \tag{2}$$

here $c_y^{\alpha}, c_y^{\delta_z}, m_z^{\alpha}$,etc. are coefficients of missile.

The structure of the missile control system is shown in Fig. 2.

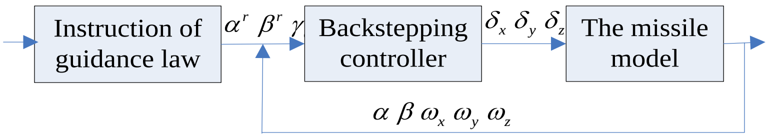

Figure 2: Control System Structure

2.3. The Backstepping Controller Design

1) Controller design in roll channel

First of all, we introduce new variables z_1, z_2 according to the formula 1st and 4th in missile model(1).

$$\begin{cases} z_1 = \gamma - \gamma_c \\ z_2 = \omega_x - \omega_{xc} \end{cases} \tag{3}$$

Then

$$\dot{z}_1 = \dot{\gamma} - \dot{\gamma}_c = \omega_x - \dot{\gamma}_c \tag{4}$$

Constructing Lyapunov equation: $V_1(\gamma) = \dfrac{1}{2} z_1^2$

Derivative of the last equation:

$$\dot{V}_1(\gamma) = z_1 \dot{z}_1 = z_1(\omega_x - \dot{\gamma}_c) = z_1(z_2 + \omega_{xc} - \dot{\gamma}_c) \tag{5}$$

In order to ensure $\dot{V}_1(\gamma) < 0$, make $\omega_{xc} = -k_1 z_1 + \dot{\gamma}_c$.

As a result:

$$\dot{V}_2(\gamma) = -k_1 z_1^2 + z_1 z_2 + z_2(a_{11}\omega_x + a_{12}\delta_x + k_1 \dot{z}_1 - \ddot{\gamma}_c) \tag{6}$$

Further constructing Lyapunov function:

$$V_2(\gamma)=\frac{1}{2}z_1^2+\frac{1}{2}z_2^2 \tag{7}$$

Then:

$$\dot{V}_2(\gamma)=-k_1z_1^2+z_1z_2+z_2(a_{11}\omega_x+a_{12}\delta_x+k_1\dot{z}_1-\ddot{\gamma}_c) \tag{8}$$

According to $\dot{V}_2(\gamma)<0$:

$$\delta_x=\frac{1}{a_{12}}(-a_{11}\omega_x-z_1-k_2z_2-k_1\dot{z}_1+\ddot{\gamma}_c) \tag{9}$$

It also can be written as:

$$\delta_x=\frac{1}{a_{12}}[(-k_1-k_2-a_{11})\omega_x+(-k_1k_2-1)(\gamma-\gamma_c)+(k_1+k_2)\dot{\gamma}_c+\ddot{\gamma}_c] \tag{10}$$

Then Eq. (9) is plugged into Eq. (8):

$$\dot{V}_2(\gamma)=-k_1z_1^2-k_2z_2^2 \tag{11}$$

where k_1,k_2 are tunable control parameters, and make $k_1>0,k_2>0$.

2) Controller design in pitch and yaw channel

Introduce new variables z_3,z_4 and z_5,z_6 to the pitch and yaw channels respectively, similar to the roll channel:

$$\begin{cases} z_3=\alpha-\alpha^r \\ z_4=\omega_z-\omega_{zc}=\omega_z-(\omega_x\beta+a\alpha^r+\dot{\alpha}^r-k_3z_3) \end{cases} \tag{12}$$

$$\begin{cases} z_5=\beta-\beta^r \\ z_6=\omega_y-\omega_{yc}=\omega_y-(\omega_x\alpha-k_5z_5-b\beta^r-\dot{\beta}^r) \end{cases} \tag{13}$$

Construct the Lyapunov function and derivative, respectively:

$$\begin{aligned} \dot{V}(z_3,z_4)&=(a_{32}-\omega_x^2-k_3a-k_3^2-k_3a_{33})(a+k_3)z_3^2 \\ &+z_4[(a_{33}+k_3)z_4+\varphi_1(x)+A+a_{34}\delta_z-a_{12}\beta\delta_x] \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{V}(z_5,z_6)&=(a_{22}+\omega_x^2-k_5b+k_5^2-k_5a_{23})(k_5-b)z_5^2 \\ &+z_6[(a_{23}+k_5)z_6+\varphi_2(x)+B+a_{24}\delta_y-a_{12}\alpha\delta_x] \end{aligned} \tag{15}$$

Design the controller, respectively:

$$\delta_z=\frac{1}{a_{34}}[a_{12}\beta\delta_x-k_4z_4-\varphi_1(x)-A] \tag{16}$$

$$\delta_y=\frac{1}{a_{24}}[a_{12}\alpha\delta_x-k_6z_6-\varphi_2(x)-B] \tag{17}$$

where

$$\begin{cases} \varphi_1(x) = (a_{31}+1)\omega_x\omega_y + (a_{33}+b-a_{11})\omega_x\beta - \omega_x^2\alpha^r \\ A = (a_{33}a + a_{32})\alpha^r + (a_{33}-a)\dot{\alpha}^r - \ddot{\alpha}^r \\ \varphi_2(x) = (a_{21}-1)\omega_x\omega_z - (a_{11}-a-a_{23})\omega_x\alpha + \omega_x^2\beta^r \\ B = (b-a_{23})\dot{\beta}^r + \ddot{\beta}^r + (a_{22}-a_{23}b)\beta^r \end{cases} \tag{18}$$

k_3, k_4, k_5, k_6 are tunable control parameters, and meet the following requirements:

$$\begin{cases} (a_{32}-\omega_x^2-k_3^2-k_3a-k_3a_{33})(k_3+a) < 0 \\ k_4 > a_{33}+k_3 \\ (a_{22}-\omega_x^2-k_5^2+k_5\mathrm{b}-k_5a_{23})(k_5-b) < 0 \\ k_6 > a_{23}+k_5 \end{cases} \tag{19}$$

3. The Missile Control System Optimization

According to the design of control system, there are unknown control parameters, which need to consume a lot of time to determine. This paper introduces the experimental design method (DOE) and approximate surrogate model to establish the prediction model of performance indexes in order to reduce the amount of computation and shorten the design time. After establishing the surrogate model, the genetic algorithm is applied to the control system optimal solution.

3.1. Construct the Kriging Approximation Model

A given set of control parameters can get a corresponding output response according to the original model of control system. The form of the output response is

$$F = w_1 \frac{\left|M - M^*\right|}{s_1} + w_2 \frac{\left|t_s - t_s^*\right|}{s_2} \tag{20}$$

where M, t_s denote the overshoot and settling time of the original model; M^*, t_s^* denote the required overshoot and settling time; w_1, w_2 are the weighting coefficients; s_1, s_2 indicate the scaling factors.

Given a set of n design sites $\mathbf{S}\big|_{n\times m} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_n]^T$ with $s_i \in R^m$ by using latin hypercube sampling, and the responses value of the sampling points indicated by $\mathbf{Y}\big|_{n\times 1} = [y_1, y_2, \cdots, y_n]^T$. The form is

$$\mathbf{Y} = \mathbf{g}(\mathbf{S}) \tag{21}$$

The responses of the Kriging meta-model is

$$\hat{\mathbf{y}}(\mathbf{x}) = \mathbf{F}(\beta, \mathbf{x}) + \mathbf{z}(\mathbf{x}) \tag{22}$$

where $\mathbf{F}(\beta, \mathbf{x}) = \mathbf{f}(\mathbf{x})^T \beta$ is a regression model which is a linear combination of p chosen functions f_i : $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_p(\mathbf{x})]^T$. The coefficients $\beta = [\beta_1, \beta_2, \cdots, \beta_p]^T$ are regression parameters

$$\operatorname{cov}[\mathbf{z}(\mathbf{x}_i)\mathbf{z}(\mathbf{x}_j)^T] = \sigma^2 \mathbf{R}[R(\theta, \mathbf{x}_i, \mathbf{x}_j)] \tag{23}$$

where $\boldsymbol{R}$ denotes correlation matrix of $n_s \times n_s$ order positive definite diagonal, n_s indicates the number of sampling points; R is the correlation function about θ. The correlation function has different types such as Exponential, Generalized exponential, Gaussian, Linear, Spherical, Cubic Spline, etc. The form of the Gaussian correlation function[11] is

$$R(\theta, x_i, x_j) = \prod_{k=1}^{n} \exp(-\theta \left| x_k^i - x_k^j \right|^2) \tag{24}$$

where n denotes the number of design variables, θ is the unknown parameter. Any θ can generate a Kriging meta-model.

Introduce a defined function:

$$\phi = -\frac{n \ln(\hat{\sigma}^2) + \ln\left|R\right|}{2} \tag{25}$$

where $\hat{\sigma}^2$ and $\left|R\right|$ are functions about θ and $\theta > 0$. We can establish an optimal Kriging meta-model by solving the maximum of Eq.(25).

After the surrogate model is established, the model's performance is evaluated by using Error of mean square root. The expression is:

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{26}$$

where y_i is value of the original calculation model, $\hat{y}_i$ is estimate of the surrogate model.

The smaller RMS is, the higher accuracy surrogate model can be obtained.

3.2. Optimal Algorithm

Genetic optimization algorithm is adopted in this paper based on the established surrogate model.

The genetic algorithm (GA) is a kind of probability searching and optimization method which simulates the natural evolution, drawing on Darwin's theory of evolution and Mendelism. Since biological evolution is mainly done through crossover and mutation between chromosomes, the genetic algorithm is based on the principle of survival of the fittest. Each run of genetic algorithm will create a new approximate optimal solution and this process leads to the evolution of the individuals in the population just as the reconstruction of nature. Individuals coded as strings according to a certain program. Binary string is the most commonly used in genetic algorithm. Many scholars have designed many different coding methods to imitating biological genetic characteristics under different environment for different problems. The different kinds of genetic algorithm are formed by different coding methods and genetic operators.

Assuming the control parameters k_3, k_5 are fixed value.

Optimization problem can be expressed as follows:

Given parameter: $\mathbf{P}$

Solution: $\mathbf{k_p} = [k_1, k_2, k_4, k_6]$

To minimize: F

Constraint condition: $\mathbf{k_{pl}} < \mathbf{k_p} < \mathbf{k_{pu}}$

where $\mathbf{P}$ are the given design parameters, $\mathbf{k_p}$ are control variables and $\mathbf{k_{pl}}$, $\mathbf{k_{pu}}$ are their up and down limit, F is the performance function that determined by Eq.(20).

4. The Results of Simulation and Analysis

The given parameters [12] are listed in Table 1.

Table 1: Given Parameters

a_{11}	10.8994	a_{12}	459.6575
a_{21}	0.95789	a_{32}	-470.75
a_{22}	-200.32	a_{33}	7.4914
a_{23}	1.8728	a_{34}	0.65725
a_{24}	0.57549	a	1.8728
a_{31}	-0.95789	b	0.3861

Input instructions of the attack angle α^r and sideslip angle β^r are 1°and 0°respectively. Input instruction of the roll angle γ_c is a sine wave whose amplitude is 1.The control parameters k_3 and k_5 are fixed at 20. The objective function is determined by the overshoot and settling time of the attack angle α. The overshoot amount to 5% and the settling time set to 0.1 s.

The relationship between the control parameters k_4, k_6 and the values of F is built according to the results of surrogate model, as shown in Fig 3.

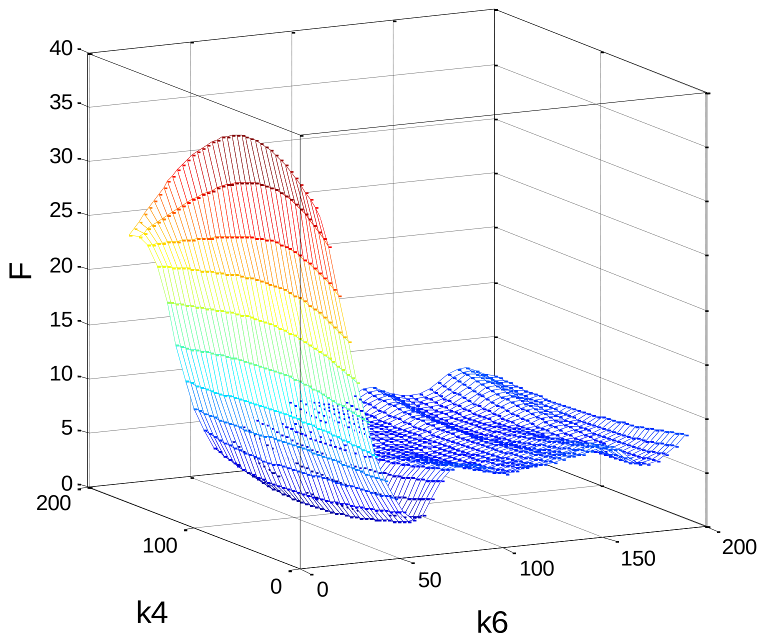

Figure 3: The Relationship Between k_4, k_6 And F

The optimization results based on the surrogate model are shown in Table 2.

Table 2: Optimization Results

k_1	k_2	k_4	k_6
32.4300	88.8543	65.8165	140.0143

The results of the original simulation model based on the optimization parameters are showed in Fig.4, Fig.5 and Fig.6.

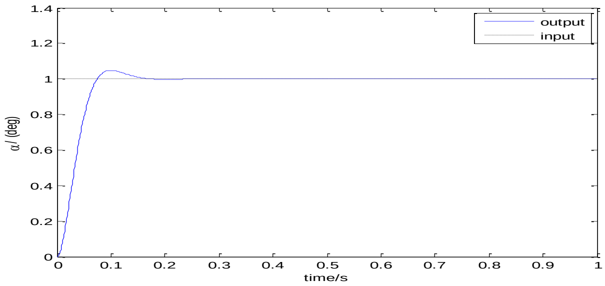

Figure 4: The Step Response Curve of Attack Angle α

It can be seen from Fig. 4 that the response curve of attack angle α is conform to the requirements. As shown in Fig. 5, the sideslip angle β is steady at near zero. In Fig. 6, the output of roll angle γ can track the changes of the input command rapidly and smoothly.

The results of simulation verify the efficiency of the method.

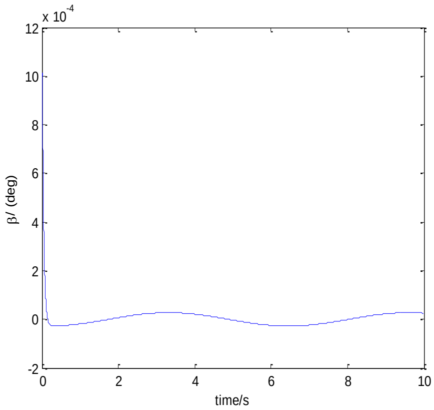

Figure 5:The Response Curve Of Sideslip Angle β

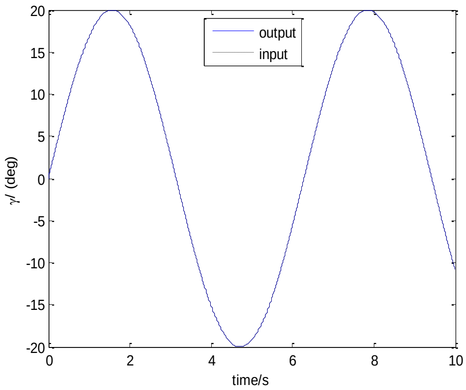

Figure 6: Tracking Curve Of Roll Angle γ

5. Conclusion

Due to the experimental design method is adopted in the process of Kriging meta-modeling, a high-precision predictive model as close as possible to the original model of control system can be constructed with fewer sample points. Then the optimization parameters based on the predictive model were used in the original numerical simulation model, and the results verified the efficiency of the method. The method can quickly provide the corresponding control parameters for different control requirements, and the simulation results are reliable. This research results can provide a reference for optimization of backstepping control method.

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References

[1] R. T. Reichert. Homing Performance Comparison of Selected Airframe Configurations Using Skid –to –Turn Steering Policies[R]. National Aeronautics and Space Administration. **(1981)**:723-725

[2] Corey Schumacher, Pramod P. Khargonekar. Stability Analysis of a Missile Control System with a Dynamic Inversion Controller[J]. Journal of Guidance, Control, and Dynamics, **(1998)**,21(3):508-515

[3] T. Hwang, M.Tahk.T. Integrated Backstepping Design of Missile Guidance and Control with Robust Disturbance Observer[C]. SICE-ICASE International Joint Conference. Busan, Korea, **(2006)**:4911-4915

[4] Knill D L, Giunta A A, Baker C A. Response surface models combining linear and euler aerodynamics for supersonic transport design[J]. Journal of Aircraft. **(1999)** 36(1): 75-86

[5] Vadde K. K., AI Et. Optimizing protocol interaction using response surface methodology[J]. IEEE Transactions on Mobile Computing **(2006)**,5:627-637

[6] Jin R, Chen W, Simpson T W. Comparative studies of metamodeling techniques under multiple modeling criteria[J]. Journal of Structural and Multidisciplinary Optimization. **(2001)**, 23(1): 1-13

[7] F Hussain M. , Rr Barton, Sb Joshi. Metamodeling: radial basis functions, versus polynomials [J]. European Journal of Operational Research. **(2006)**, 138(142-154)

[8] Yajun Yang, Ying Liao, and Xingxing He. Phased Array Antenna Design Based on Kriging Meta-model[C]. Asia Simulation Conference 2012. Shanghai, China, October **(2012)**, Proceedings, Part 3:312-322

[9] Wang G G, Shan S. Review of metamodeling techniques in support of engineering design optimization[J]. Transactions of the ASME. **(2007)**, 129: 370-380

[10] Kleijnen Jack P C. Kriging metamodeling in simulation:A review[J]. European Journal of Operational Research. **(2009)**, 192(3): 707-716

[11] J. Sacks, W.J. Welch, T.J. Mitchell, H.P. Wynn, Design and Analysis of Computer Experiments, Statistical Science, vol. 4, no. 4, pp. 409-435, **(1989)**

[12] Shuqiang Zheng. Research on BTT Controller Design and Engineering Realization Based on Backstepping Method for Small Smart Bomb[D]. Nanjing University of Science and Technology. **(2013)**. (In China)