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Approximate BER Performance Analysis of Coherent Amplify-and-Forward Cooperative Relay System with BPSK over Nakagami-*m* Fading Channels

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Abstract

In this paper, we derive an accurate and concise closed-form approximation of bit error rate (BER) for coherent amplify-and-forward (AF) cooperative relay system employing binary phase shift keying (BPSK), especially in Nakagami-m fading Channels. By exploiting the derived closed-form formula, we can easily verify the error rate performance for various system configurations and fading environments without time-consuming computer simulation. Extensive numerical results are provided to validate our theoretical analysis, where it is obviously demonstrated that the average BER versus signal-to-noise ratio (SNR) curves obtained from the derived closed-form expression are very close to those from the existing integral-form expression.

Keywords: *amplify-and-forward (AF), cooperative relay system, Nakagami-m fading.*

1. Introduction

Recently, a cooperative relay technique has emerged as a promising solution for the high data rate and the reliable transmission in wireless communication systems [1-8]. Basically, in addition to the direct link between the source and the destination (*i.e.*, receiver), another link exists, where an additional data transmission is carried out through a relay or multiple relays to the destination. To realize the cooperative rely system and thus to achieve cooperative diversity gains, several cooperative transmission schemes have been proposed [3-5], such as amplify-and-forward (AF), decode-and-forward (DF), etc. In particular, in the AF protocol, the relay amplifies the received signal sent from the source and forwards it to the destination, whereas in the DF protocol the relay decodes the received signal through the adopted decoding method and then encodes and forwards it to the destination. Hence, comparing with DF protocol, AF protocol allows us to significantly decrease the cost and complexity of relaying.

There have been a lot of research efforts in analyzing the performance achieved by the cooperative relay systems (*e.g.*, [1-8] and references therein). In [1] and [2], it was clearly shown that the cooperative relay scheme can lead to higher data rates and robustness against severe effects of fading, compared with the non-cooperative one, which is due to the cooperative diversity gain through an additional link with the relay as well as a direct link between the source and the destination nodes. On the other hand, the end-to-end performance of dual-hop single-link communication schemes has been analyzed without considering the relay diversity in [3]. For various relay protocols, such as coherent/noncoherent AF and DF, a comprehensive performance evaluation was provided over Rayleigh fading channels in [4]. And, it was extended to the case of Nakagami-*m* fading channels in [5], where a unified bit error rate (BER) analysis was presented for binary phase shift keying (BPSK) modulation. Thus, in this paper, our aim is to derive a simple but accurate closed-form approximate formula of BER for a coherent AF

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cooperative relay system over Nakagami-*m* fading channels, which starts from the conventional computationally intensive integral-form expression given in [5].

The rest of is this paper is organized as follows. The system and channel models are described in Section 2. Section 3 presents the derivation of closed-form BER approximation in both Nakagami-*m* and Rayleigh fading channels, followed by some numerical results in Section 4. Finally, Section 5 concludes this paper.

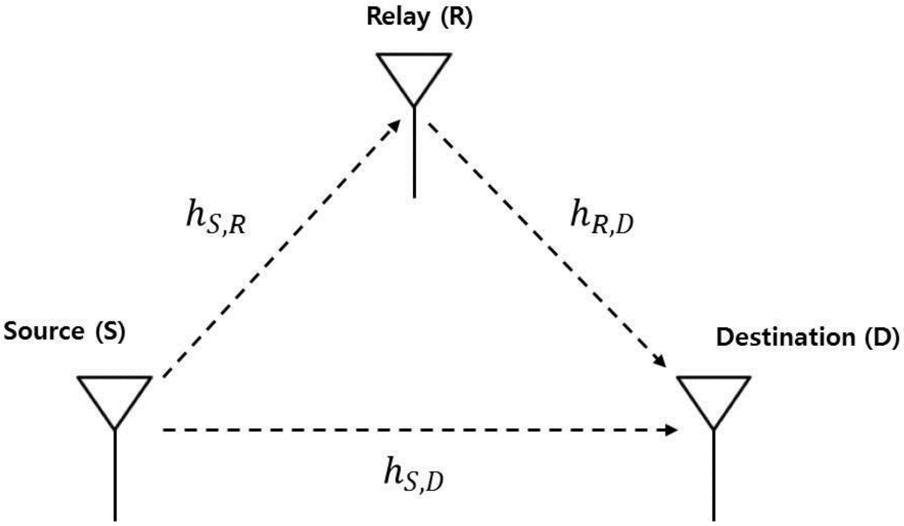

Figure 1. Cooperative Wireless Relay System

2. System and Channel Models

Throughout this paper, we consider a wireless cooperative relay system illustrated in Figure 1, which consists of source (S), relay (R), and destination (D) nodes. To avoid the adverse effect of mutual interference, a half-duplex transmission mode is assumed. That is, we assume that time-division multiplexing is adopted, which involves two-phase transmission. More specifically, during phase I, S transmits a sequence of information symbols, whereas R and D receive it simultaneously. Then, the received signals at R and D are given as [4, 5]

$$x_R = h_{S,R} s + v_R , \tag{1}$$

$$x_D = h_{S,D} s + v_D , \tag{2}$$

where x_R and x_D are the received signal at R and D, respectively, $s \in \{-1,1\}$ denotes the transmitted binary symbol (*i.e.*, BPSK) from S, $h_{S,R}$ and $h_{S,D}$ stand for the fading coefficients of S-R and S-D links, and v_R and v_D are the noises at R and D, respectively.

In phase II, R amplifies the received signal into s_R and retransmits it to D as [4][5]

$$y_D = h_{R,D} s_R + v_D , \tag{3}$$

where y_D , $h_{R,D}$, and v_D are the received signal at D, the fading coefficient of R-D link, and the noise at D, respectively. It is noted that v_R and v_D are distributed as $CN(0, N_0)$.

For Nakagami-*m* fading channels, the instantaneous signal-to-noise ratio (SNR) of ***i-j*** link with $(i, j) \in \{(S,R),(S,D),(R,D)\}$, which is denoted by $\gamma_{i,j} = |h_{i,j}|^2 / N_0$, and has a gamma distribution with the probability density function (PDF) of [11]

$$f_{\gamma_{i,j}}(x) = \frac{1}{\Gamma(m_{i,j})}\left(\frac{m_{i,j}}{\rho_{i,j}}\right)^{m_{i,j}} x^{m_{i,j}-1}\exp\left(-\frac{m_{i,j}x}{\rho_{i,j}}\right), \tag{4}$$

where $m_{i,j}$ is the Nakagami fading parameter, $\rho_{i,j} = E[\gamma_{i,j}] = E[|h_{i,j}|^2]/N_0$ denotes the average SNR of *i-j* link and $E[\cdot]$ stands for the expectation operator. Then, the moment generating function (MGF) of $\gamma_{i,j}$ is given as [5][11]

$$\mathcal{MGF}_{\gamma_{i,j}}(s) = E\left[\exp(-\gamma_{i,j}s)\right] = \left(\frac{m_{i,j}}{m_{i,j} + \rho_{i,j}s}\right)^{m_{i,j}}. \tag{5}$$

For the coherent AF method under consideration in this paper, the amplified and forwarded signal s_R at R can be expressed by [4][5]

$$s_R = \frac{x_R}{\sqrt{|h_{S,R}|^2 + N_0}}, \tag{6}$$

which is to meet a unit average power constraint.

At the receiver (*i.e.*, at D), the received signals x_D and y_D are optimally combined by the maximal ratio combining (MRC) technique. Then, the combined signal can be given by [4, 5].

$$z = \frac{h_{S,D}^{*} x_D}{N_0} + \left(\frac{h_{S,R}h_{R,D}}{\sqrt{|h_{S,R}|^2 + N_0}}\right)^{*} \frac{y_D}{|h_{R,D}|^2\left(|h_{S,R}|^2 + N_0\right)^{-1} N_0 + N_0}, \tag{7}$$

where $(\cdot)^*$ stands for the complex conjugate. Here, it is assumed that the channel state information (CSI) of $h_{S,R}$, $h_{S,D}$, and $h_{R,D}$ is known to D and R knows the CSI of $h_{S,R}$.

Thus, the overall instantaneous SNR at D can be written as

$$\gamma_{Overall} = \gamma_{S,D} + \gamma_{S,R,D}, \tag{8}$$

where the equivalent instantaneous SNR of R, $\gamma_{S,R,D}$, is

$$\gamma_{S,R,D} = \frac{\gamma_{S,R}\gamma_{R,D}}{\gamma_{S,R} + \gamma_{R,D} + 1}. \tag{9}$$

3. Error Rate Analysis of Coherent AF

3.1. BER Evaluation over Nakagami-*m* Fading

Due to the instantaneous SNR of the MRC given in Eq. (8), the conditional BER for the coherent detection of BPSK modulation is given by [5]

$$\begin{aligned} P_b^{Nakagami-m}(\gamma_{CAF}) &= Q\left(\sqrt{2\gamma_{Overall}}\right) \\ &= Q\left(\sqrt{2\left(\gamma_{S,D}+\gamma_{S,R,D}\right)}\right) \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{\gamma_{S,D}+\gamma_{S,R,D}}{\sin^2\phi}\right)d\phi \end{aligned}, \tag{10}$$

where $Q(\cdot)$ denotes the Q-function [11]. Then, by utilizing the MGF-based approach and thus averaging out the channel statistics, the average BER can be obtained as [5]

$$\begin{aligned} \overline{P}_b^{Nakagami-m} &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{MGF}_{\gamma_{Overall}}\left(-\frac{1}{\sin^2\phi}\right)d\phi \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{MGF}_{\gamma_{S,D}}\left(-\frac{1}{\sin^2\phi}\right)\mathcal{MGF}_{\gamma_{S,R,D}}\left(-\frac{1}{\sin^2\phi}\right)d\phi \end{aligned}, \tag{11}$$

with

$$\mathcal{MGF}_{\gamma_{S,D}}(s) = \left(\frac{m_{S,D}}{m_{S,D}+\rho_{S,D}s}\right)^{m_{S,D}}, \tag{12}$$

$$\mathcal{MGF}_{\gamma_{S,R,D}}(s) \approx {}_2F_1\left(m_{S,R}, 2m_{S,R}; m_{S,R}+\frac{1}{2}; -\frac{\rho_{S,R}}{4m_{S,R}}s\right), \tag{13}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [12] and Eq. (13) is due to the approximation of Eq. (9) as $\gamma_{S,R,D} \approx \gamma_{S,R}\gamma_{R,D}/(\gamma_{S,R}+\gamma_{R,D})$ [5]. Additionally, we also note that Eq. (11) is from the fact that the S-D and S-R-D links are statistically independent. As mentioned in [5], the calculation of Eq. (11) can be carried out only through the numerical integration by using well-known mathematical software packages, such as MATLAB, MATHEMATICA, etc. or a numerical technique, such as the Gaussian quadrature method.

It is, however, fortunate that a quite accurate approximation to the calculation of Eq. (11) is presented in [9, 10], where the integration over ϕ is not involved, which is derived from the following approximation as

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{\alpha}{\sin^2\phi}\right)d\phi \approx \frac{1}{12}\exp(-\alpha)+\frac{1}{4}\exp\left(-\frac{4}{3}\alpha\right). \tag{14}$$

Thus, applying Eq. (14) to Eq. (11) leads to the closed-form expression of the average BER for the coherent AF cooperative system with BPSK over Nakagami-*m* fading channels as

$$\begin{aligned}\overline{P}_b^{Nakagami-m} &\approx \frac{1}{12}\mathcal{M}\mathcal{G}\mathcal{F}_{\gamma_{Overall}}\left(1\right)+\frac{1}{4}\mathcal{M}\mathcal{G}\mathcal{F}_{\gamma_{Overall}}\left(\frac{4}{3}\right)\\ &=\frac{1}{12}\left(\frac{m_{S,D}}{m_{S,D}+\rho_{S,D}}\right)^{m_{S,D}}{}_2F_1\left(m_{S,R},2m_{S,R};m_{S,R}+\frac{1}{2};-\frac{\rho_{S,R}}{4m_{S,R}}\right)\\ &+\frac{1}{4}\left(\frac{m_{S,D}}{m_{S,D}+\frac{4}{3}\rho_{S,D}}\right)^{m_{S,D}}{}_2F_1\left(m_{S,R},2m_{S,R};m_{S,R}+\frac{1}{2};-\frac{\rho_{S,R}}{3m_{S,R}}\right)\end{aligned} \quad . \tag{15}$$

3.2. BER Evaluation over Rayleigh Fading

Since the Nakagami-*m* distribution with $m=1$ reduces to the Rayleigh distribution, we can easily derive the closed-form approximate expression of BER for the coherent AF cooperative system employing BPSK in Rayleigh fading channels, by setting $m_{S,D}=m_{S,R}=1$ in Eq. (15), as

$$\overline{P}_b^{Rayleigh} \approx \frac{1}{12}\left(\frac{1}{1+\rho_{S,D}}\right){}_2F_1\left(1,2;\frac{3}{2};-\frac{\rho_{S,R}}{4}\right)+\frac{1}{4}\left(\frac{1}{1+\frac{4}{3}\rho_{S,D}}\right){}_2F_1\left(1,2;\frac{3}{2};-\frac{\rho_{S,R}}{3}\right). \tag{16}$$

Furthermore, using the following relation as [3]

$${}_2F_1\left(1,2;\frac{3}{2};-z\right)=\frac{1}{2\left(1+z\right)}+\frac{\sinh^{-1}\left(\sqrt{z}\right)}{2\sqrt{z}\left(1+z\right)^{\frac{3}{2}}}, \tag{17}$$

Eq. (16) can be further simplified, without the calculation of hypergeometric function, into

$$\begin{aligned}\overline{P}_b^{Rayleigh} &\approx \frac{1}{12}\left(\frac{1}{1+\rho_{S,D}}\right)\left[\frac{1}{\left(2+\frac{\rho_{S,R}}{2}\right)}+\frac{\sinh^{-1}\left(\frac{\sqrt{\rho_{S,R}}}{2}\right)}{\sqrt{\rho_{S,R}}\left(1+\frac{\rho_{S,R}}{4}\right)^{\frac{3}{2}}}\right]\\ &+\frac{1}{4}\left(\frac{1}{1+\frac{4}{3}\rho_{S,D}}\right)\left[\frac{1}{2\left(1+\frac{\rho_{S,R}}{3}\right)}+\frac{\sinh^{-1}\left(\sqrt{\frac{\rho_{S,R}}{3}}\right)}{2\sqrt{\frac{\rho_{S,R}}{3}}\left(1+\frac{\rho_{S,R}}{3}\right)^{\frac{3}{2}}}\right].\end{aligned} \tag{18}$$

In addition, generally, the error rate can be asymptotically approximated as [11]

$$\overline{P}_b^{\infty} \approx \left(G_c\cdot\rho\right)^{-d}, \tag{19}$$

where G_c and d denote the asymptotic coding gain and diversity order, respectively. Thus, from Eq. (18) and the series expansion as ${}_2F_1\left(1,2;3/2;-z\right)\approx 1/2z$ at $z=\infty$, the asymptotic form of closed-form approximation of BER can be derived as

$$\begin{aligned}\bar{P}_b^{Rayleigh,\infty} &\approx \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{1}{\rho_{S,D}\sin^{-2}\phi}\right)\left(\frac{1}{\frac{1}{2}\rho_{R,D}\sin^{-2}\phi}\right)d\phi \\ &= \frac{2}{\pi}\left[\int_0^{\frac{\pi}{2}}\sin^4\phi\, d\phi\right]\frac{1}{\rho_{S,D}\rho_{R,D}} \qquad , \qquad (20) \\ &= \frac{3}{8}\frac{1}{\rho_{S,D}\rho_{R,D}}\end{aligned}$$

which clearly indicates that the asymptotic diversity order achieved by the coherent AF cooperative system in Rayleigh fading channels is $d^{Rayleigh} = 2$.

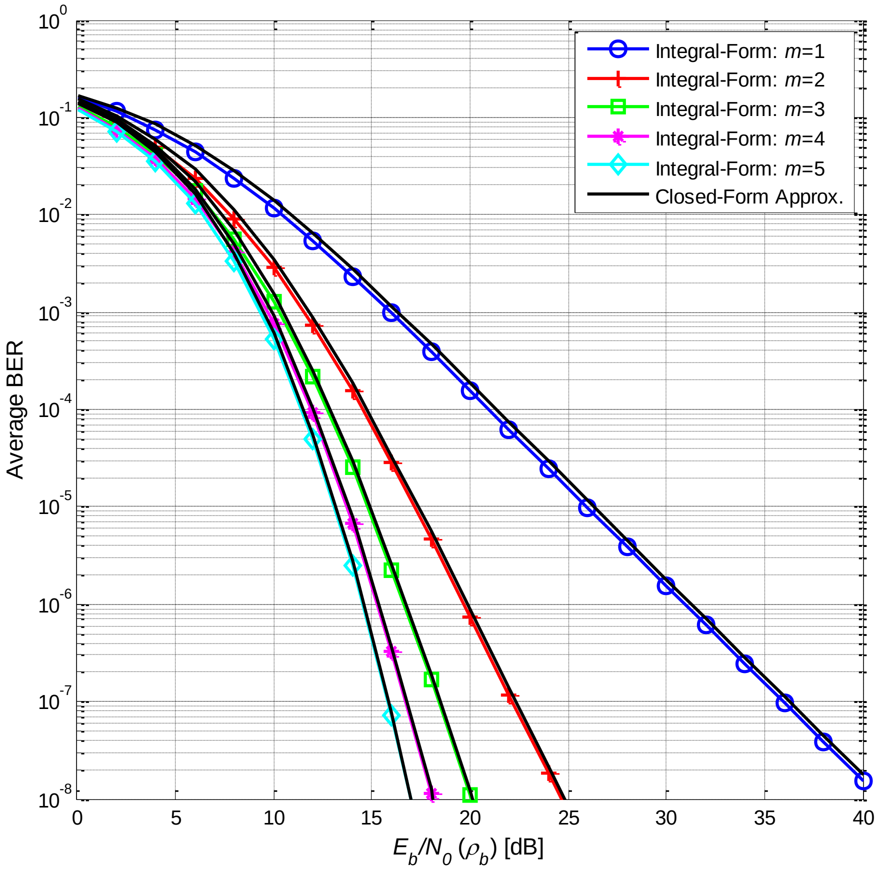

Figure 2. Average BER vs. SNR of Coherent AF Cooperative System with BPSK over Nakagami-*m* Fading Channels in a Symmetric Scenario

4. Numerical Results

In this section, we present some numerical results to demonstrate the error rate performance analysis of the coherent AF cooperative wireless system with BPSK modulation over Nakagami-*m* fading environments, as shown in Figure 1. In addition, we consider two system configuration scenarios. That is, for a symmetric scenario, it is assumed that $\rho_{S,D} = \rho_{S,R} = \rho_{R,D} = \alpha E_b / N_0 = \alpha\rho_b$, where $\rho_b = E_b / N_0$ denotes the average

SNR of the transmit signal. On the other hand, for an asymmetric case, we assume that $\rho_{S,D} = \alpha E_b / N_0$ and $\rho_{S,R} = \rho_{R,D} = \beta E_b / N_0$ for $\alpha \neq \beta$.

Figure 2 presents the average BER performance of coherent AF cooperative wireless system with BPSK in Nakagami-*m* fading channels with various values of $m_{S,D} = m_{S,R} = m_{R,D} = m = \{1,\ldots,5\}$ and $\rho_{S,D} = \rho_{S,R} = \rho_{R,D} = 0.5 E_b / N_0$ for a symmetric scenario. For comparison, we depict the BER curves obtained from both the existing BER formula given in [5] and our derived BER approximate expression in Eq. (15). From the figure, it is obvious that the analytical results from the derived Eq. (15) are very close to those from the existing integral-form expression with an SNR gap of less than 0.4 dB for $m = 1$ in the high SNR regime and the SNR gap becomes indistinguishable with the increase of the value of m . For example, it is less than 0.05 dB for $m = 5$ at high SNR.

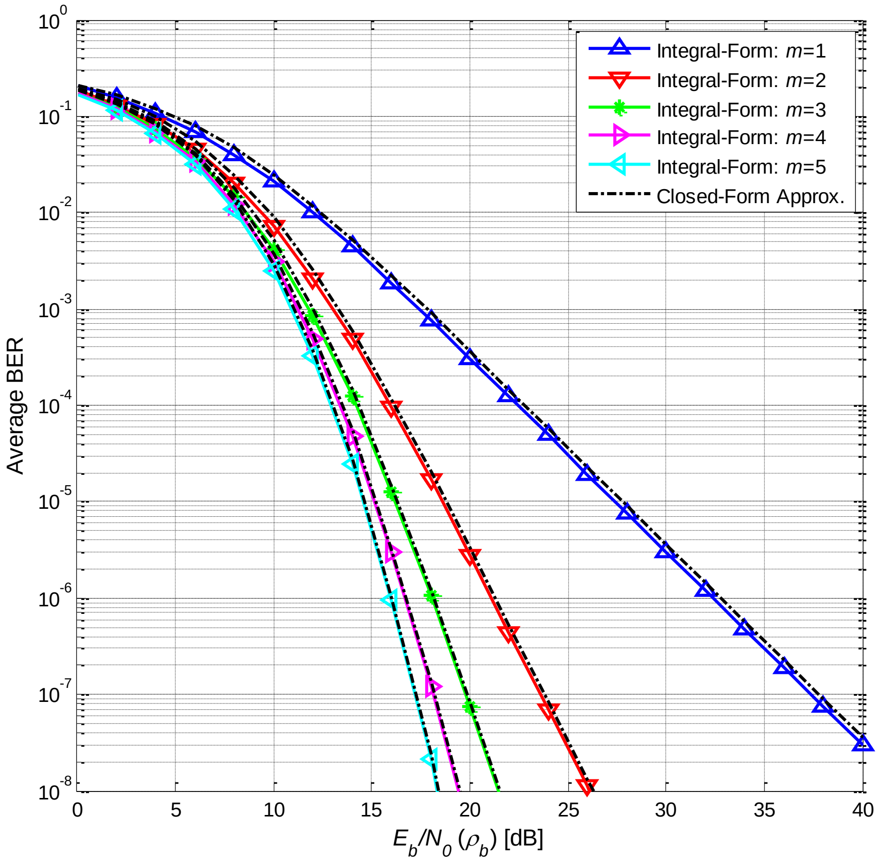

Figure 3. Average BER vs. SNR of Coherent AF Cooperative System with BPSK over Nakagami-*m* Fading Channels in an Asymmetric Scenario

For an asymmetric scenario, Figure 3 shows the SER performance over Nakagami-*m* fading channels with $m = \{1,\ldots,5\}$, where we set $\rho_{S,D} = 0.25 \rho_b$ and $\rho_{S,R} = \rho_{R,D} = 0.5 \rho_b$. As in the case of symmetric scenario, the average BER curves obtained from the derived closed-form approximation are seen to be close to those from the conventional integral-form formula with very small SNR gap in the high SNR regime. Comparing the asymmetric scenario with the symmetric one, we can observe an SNR loss of 2-5 dB for the same channel conditions.

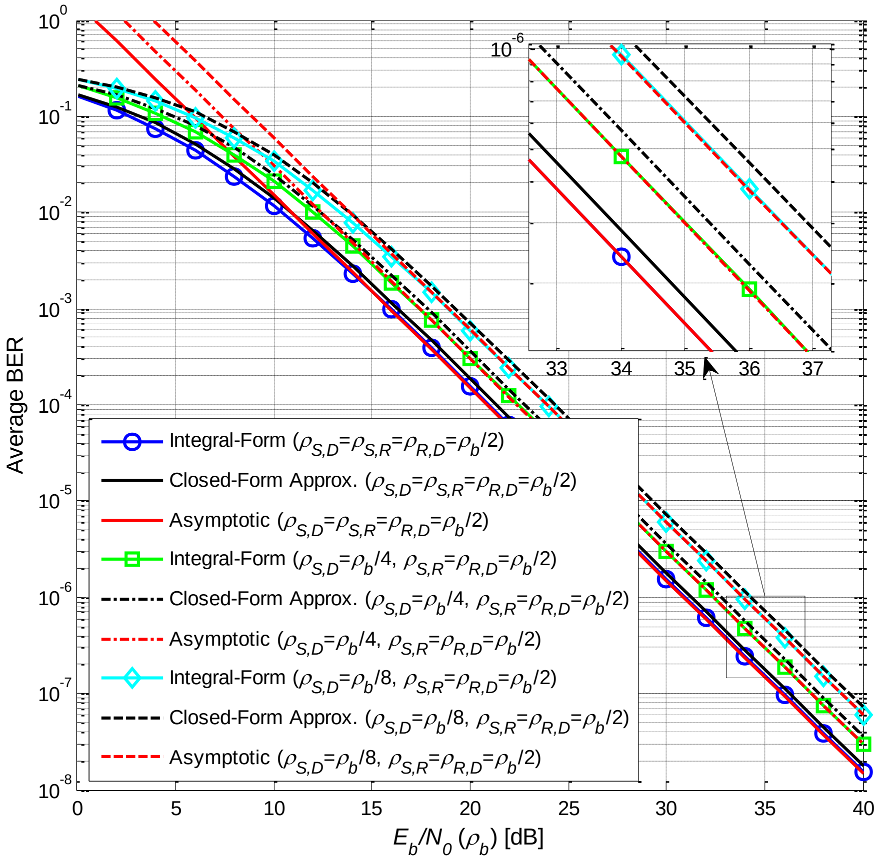

Figure 4. Average BER vs. SNR of Coherent AF Cooperative System with BPSK over Rayleigh Fading Channels in Symmetric/Asymmetric Scenarios

Finally, in Figure 4, average SER curves are depicted especially over Rayleigh fading channels (*i.e.*, $m=1$) for both symmetric scenario with $\rho_{S,D}=\rho_{S,R}=\rho_{R,D}=0.5\rho_b$ and asymmetric scenario with ($\rho_{S,D}=0.25\rho_b$, $\rho_{S,R}=\rho_{R,D}=0.5\rho_b$) and ($\rho_{S,D}=0.125\rho_b$, $\rho_{S,R}=\rho_{R,D}=0.5\rho_b$). It is noted that we can more clearly observe from the zoomed figure that the average BER curves from the proposed asymptotic formula in Eq. (20) asymptotically converge to those from the conventional integral-form expression with $m=1$ in Eq. (11) for various symmetric and asymmetric scenarios, whereas there exist SNR gaps between the BER curves from Eq. (11) and the approximate formula given in Eq. (18), which can be explicitly explained from the approximation technique adopted in Eq. (14). From the fact that the diversity order can be conventionally defined as the negative asymptotic slope of error rate versus SNR on a log-log scale, on the other hand, we can easily observe that the coherent AF cooperative relay system in Rayleigh fading channels can achieve the asymptotic diversity order of 2, which coincides with the analytical result in Eq. (20).

5. Conclusions

In this paper, we have derived a concise but accurate closed-form approximate formula of the average BER for the coherent AF cooperative relay system with BPSK signaling over Nakagami-*m* fading channels. In contrast to the conventional computationally intensive integral-form expression, the derived closed-form formula can be further easily used to effectively evaluate the error rate performance of the AF cooperative system, maintaining the relatively exactness of the performance assessment. For various system configurations with different channel conditions (*i.e.*, $m = \{1, \ldots, 5\}$) and symmetric/asymmetric scenarios with the different average SNRs of each hop, we have demonstrated the advantages (*i.e.*, diversity gain) achieved by the AF cooperative relay system, especially in Nakagami-*m* fading environments. Moreover, as a future research, our efforts on deriving simple closed-form approximations for the considered system can be extended to more general modulation schemes, such as *M*-ary phase shift keying (*M*-PSK), *M*-ary quadrature amplitude modulation (*M*-QAM), *etc*.

Acknowledgments

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