

## Study on Gas-solid Second-order Interaction Model in Fluidized Bed Reactors

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### Abstract

Considering the fluctuating energy transfer between the SGS (Sub-Grid Scale, SGS) turbulent kinetic energy of gas and the second-order moment of particles, the gas-solid interaction model is present. The second-order term is studied, and the simulation results of second-order interaction are analyzed and compared with the simulation results by Koch. The effects of second-order interaction on concentration, velocity and second-order moment of particles and the SGS turbulent kinetic energy of gas are all researched.

**Keywords:** gas-solid second-order interaction; second-order moment of particles; gas-solid two-phase flow; turbulent kinetic energy of gas; fluidized bed reactor

### 1. Introduction

The study of gas-solid two-phase flow has been concerned closely as the complicated process including the turbulence of gas phase, the collision between particles, and the multi-directional coupling effects between gas and solid two phases. Unlike the single-phase flow, the gas-solid interaction will have a great impact on the flow process. The gas-solid interaction includes two parts, gas-solid two-phase force and gas-solid two-phase fluctuating energy. The gas-solid two-phase force is the embodiment of the first-order interaction, with which many scholars has done a lot of work. The gas-solid two-phase fluctuating energy is the embodiment of the second-order interaction, while which has been rarely researched.

The gas-solid second-order interaction is reflected as the gas-solid second-order correlation term, which need be closed to establish model. Sangani *et al.* [1] derived the gas-solid interaction model from the isotropic Maxwell distribution for sparse flow. On the basis, Koch *et al.* [2] established the model considering the anisotropy fluctuating velocity of particles for horizontal and vertical orientation respectively, and the simulation results had a good agreement with experimental data under low Reynolds number. Sun Dan *et al.* [3] considered the anisotropy fluctuating velocity of particles and studied the fluctuating characteristics of particles using Koch model. But for the algebraic form, it just applied to the sparse gas-solid flow under large Stokes number and low Reynolds number with Koch model. It will cause a certain deviation for the simulation of dense gas-solid two-phase flow in circulating fluidized beds with Koch model. In addition, although it considered the influence of anisotropic fluctuating velocity of particles, the influence of fluctuating velocity of gas has been not considered in Koch model. Simonin [4] presented gas-solid second-order interaction model from another perspective, taking into account the fluctuating energy of both gas and particles. With Simonin model, Riber *et al.* [5]

predicted the characteristics of solid circulation flow, Benyahia *et al.* [6] studied the features of the sparse flow in pipe, and Hosseini *et al.* [7] analyzed the diffusion and cycle characteristics of particles in gas-solid fluidized bed.

In this paper, the framework is Large eddy simulation of gas-Second order moment of particles model (LES-SOM model) based on two-fluid method. Analogizing Simonin model, the revised gas-solid second-order interaction model is presented for LES-SOM model. The characteristics of gas-solid two-phase flow are studied with both revised model and Koch model. The various factors are analyzed for gas-solid second-order interaction.

## 2. Mathematical Model

### 2.1. Governing Equations

Using cartridge filter, the filter equations of mass and momentum for gas are as follows [8]:

$$\frac{\partial}{\partial t}(\bar{\alpha}_g \rho_g) + \frac{\partial}{\partial x_i}(\bar{\alpha}_g \rho_g \tilde{u}_{gi}) = 0 \quad (1)$$

$$\frac{\partial}{\partial t}(\bar{\alpha}_g \rho_g \tilde{u}_{gi}) + \frac{\partial}{\partial x_j}(\bar{\alpha}_g \rho_g \tilde{u}_{gi} \tilde{u}_{gj}) = -\bar{\alpha}_g \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\tau}_{gl}}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \bar{\alpha}_g \rho_g (\tilde{u}_{gi} \tilde{u}_{gj} - u_{gi} u_{gj}) \right] + \bar{\alpha}_g \rho_g g_i - \beta_{gs} (\tilde{u}_{gi} - u_{si}) \quad (2)$$

Where  $\bar{\alpha}_g \rho_g (\tilde{u}_{gi} \tilde{u}_{gj} - u_{gi} u_{gj})$  is the sub-grid stress term of gas, and need be closed by SGS turbulent kinetic energy model [9].

Considering the anisotropy fluctuating velocity of particles, the governing equations of particles are obtained using second-order moment model [10]:

$$\frac{\partial}{\partial t}(\alpha_s \rho_s) + \frac{\partial}{\partial x_i}(\alpha_s \rho_s u_{si}) = 0 \quad (4)$$

$$\frac{\partial}{\partial t}(\alpha_s \rho_s u_{si}) + \frac{\partial}{\partial x_j}(\alpha_s \rho_s u_{si} u_{sj}) = -\frac{\partial}{\partial x_j} \Xi_{ij} - \alpha_s \frac{\partial \bar{p}}{\partial x_i} + \alpha_s \rho_s g_i + \beta_{gs} (\tilde{u}_{gi} - u_{si}) \quad (5)$$

$$\frac{\partial}{\partial t}(\alpha_s \rho_s M_{ij}) + \frac{\partial}{\partial x_k}(\alpha_s \rho_s u_{sk} M_{ij}) = \chi (m C_i C_j) - \frac{\partial}{\partial x_k} \Sigma_{kij} - \Xi_{ik} \frac{\partial u_{sj}}{\partial x_k} - \Xi_{jk} \frac{\partial u_{si}}{\partial x_k} + \beta_{gs} (\langle C_i C_{gj} \rangle + \langle C_{gi} C_j \rangle - 2M_{ij}) \quad (6)$$

Where  $C_{gi}$  is the fluctuating velocity of gas. The term  $\beta_{gs} (\langle C_i C_{gj} \rangle + \langle C_{gi} C_j \rangle)$  represents the gas-solid second-order interaction, and which needs to be closed.

### 2.2. Second-order Interaction Model

The gas-solid second-order interaction model with considering the anisotropic characteristics was first presented by Koch *et al.* [2], which can be expressed:

$$\beta_{gs} \langle C_{gi} C_j \rangle = \frac{162 \alpha_s \mu_g^2}{\rho_s d_s^3 \sqrt{\Theta}} \left[ S_{\perp} |\tilde{u}_g - u_s|^2 \delta_{ij} + (S_{\square} - S_{\perp}) (\tilde{u}_{gi} - u_{si}) (\tilde{u}_{gj} - u_{sj}) \right] \quad (7)$$

$$S_{\perp} = \frac{3 R_s R_{drag}^2}{16 \sqrt{\pi}} \sqrt{\frac{\Theta}{M_{\square}}} \left[ \frac{1}{2} (C_{\Theta}^3 - C_{\Theta}^5) \ln \left( \frac{C_{\Theta} + 1}{C_{\Theta} - 1} \right) - \frac{2}{3} C_{\Theta}^2 + C_{\Theta}^4 \right] \quad (8)$$

$$S_{\square} = \frac{3 R_s R_{drag}^2}{16 \sqrt{\pi}} \sqrt{\frac{\Theta}{M_{\square}}} \left[ (C_{\Theta}^5 + C_{\Theta}) \ln \left( \frac{C_{\Theta} + 1}{C_{\Theta} - 1} \right) - \frac{2}{3} C_{\Theta}^2 - 2 C_{\Theta}^4 \right] \quad (9)$$

$$R_{drag} = \begin{cases} \frac{1+3\sqrt{\varepsilon_s/2}+(135/64)\varepsilon_s \ln \varepsilon_s+17.14\varepsilon_s}{1+0.681\varepsilon_s-8.48\varepsilon_s^2+8.16\varepsilon_s^3} & \varepsilon_s < 0.4 \\ \frac{10\varepsilon_s}{(1-\varepsilon_s)^3} & \varepsilon_s \geq 0.4 \end{cases}, R_s = \frac{1}{g_0(1+3.5\sqrt{\varepsilon_s}+5.9\varepsilon_s)},$$

$$C_\Theta^2 = \frac{M_\square}{M_\square - M_\perp}.$$

Where  $M_\square$  and  $M_\perp$  are the vertical and parallel direction of second-order moment of particles respectively, and satisfy  $\Theta = M_\square + 2M_\perp$ .

However, it only considered the effect of particles fluctuating to gas by Koch model, and not considered the effect of gas to particles. Simonin *et al.* [4] considered both the fluctuating effect of gas to solid and solid to gas by Simonin model, which was consistent with the results of single flow when particles collision time was much longer than particles relaxation time. Analogy Simonin model, the revised model is established for LES-SOM model considering the transfer between SGS turbulent kinetic energy of gas and second-order moment of particles in this paper.

$$\beta_{gs} \langle C_{gi} C_j \rangle = \beta_{gs} \frac{\eta}{1+\eta(1+\alpha_s \rho_s / \alpha_g \rho_g)} \left( 2k_{sgs} + 3 \frac{\alpha_s \rho_s}{\alpha_g \rho_g} \Theta \right) \quad (10)$$

Where  $\eta$  is the ratio of particles collision time to particles relaxation time,  $\eta = \tau_{gs}^t / \tau_{gs}^x$ .

Crowe believed that when particles collision time was far longer than particles relaxation time, particles had enough time to respond to the role exerted by flow behavior. Thus, the particles motion was mainly dominated by flow behavior, and the role of particles collisions can be ignored. The gas-solid flow belonged to thin flow. On the contrary, the gas-solid flow belonged to dense flow.  $\eta$  was an important indicator for measuring the degree of particles collisions and flow behavior. Particles collision time  $\tau_{gs}^t$  means

particles to particles collision time in direct contact,  $\tau_{gs}^t = \frac{\tau_g^t}{\sqrt{1+C_\beta \xi_r^2}}$ ,  $\tau_g^t$  is the time scale of

turbulent eddies and can be expressed as  $\tau_g^t = \frac{3}{2} C_\mu \frac{k_{sgs}}{\varepsilon_{sgs}}$ , where  $C_\mu$  is model constants,

$C_\mu=0.09$ .  $k_{sgs}$  and  $\varepsilon_{sgs}$  are the SGS turbulent kinetic energy and SGS dissipation. There is a relative velocity between particles and the surrounding fluid, so the direction-dependent

parameter  $C_\beta$  should be calculated,  $C_\beta = 1.8 - 1.35 \left[ \frac{(\tilde{u}_g - u_s) u_s}{|\tilde{u}_g - u_s| |u_s|} \right]^2$ , where  $\xi_r^2 = \frac{3|\tilde{u}_g - u_s|^2}{2k_{sgs}}$ .

Particles relaxation time  $\tau_{gs}^x$  means the time of particles effect on its surrounding fluid

through the fluid dynamics factors, and expressed as  $\tau_{gs}^x = \frac{\alpha_s \rho_s}{\beta_{gs}}$ .

### 3. Calculate Results and Discussions

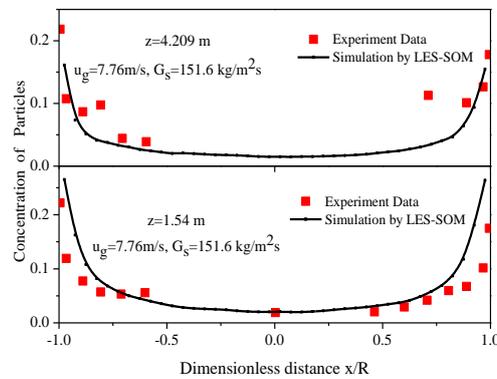
#### 3.1. Model Validation

The experimental study was done for the riser with 8.5m height and 0.411m width by Herbert *et al.* [11]. There is an uniform air distribution at the bottom with gas superficial velocity of 7.76m/s from inlet. The gas and solid inlets are located in two

sides of the wall at the bottom of 0.342m with the width of 0.411m. There is standard atmospheric pressure outlet. The experimental bed materials are made of the uniform spherical glass beads. The simulation region is divided by non-uniform meshes, and the simulations are based on LES-SOM-FIX by self-programming. The simulation time lasts 60s, and last 30s is used. The main parameters are shown in Table 1.

**Table 1. Parameters for Herbert Experiments and Simulations**

Significance	Experiment	data	Unit
Riser height	8.5	8.5	m
Riser diameter	0.411	0.411	m
Superficial gas velocity	7.76	7.76	m/s
Gas dynamic viscosity	--	$1.789 \times 10^{-5}$	kg/m.s
Gas density	--	1.225	kg/m <sup>3</sup>
Particle density	2500	2500	kg/m <sup>3</sup>
Particle diameter	300	300	m
The restitution coefficient	--	0.9	--

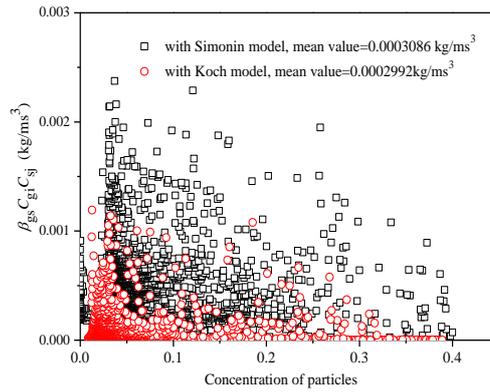


**Figure 1. Comparison of Concentration of Particles with Herbert *et al.* Experimental Data**

Figure 1 shows the distribution of average particles concentration along radial direction at  $z=1.54\text{m}$  and  $z=4.209\text{m}$ . As can be seen from the figure, both simulation and experiment show high concentrations of particles near the wall and low values in the center. The particles concentration is decreased gradually along the riser. The predictions are agrees with experiments.

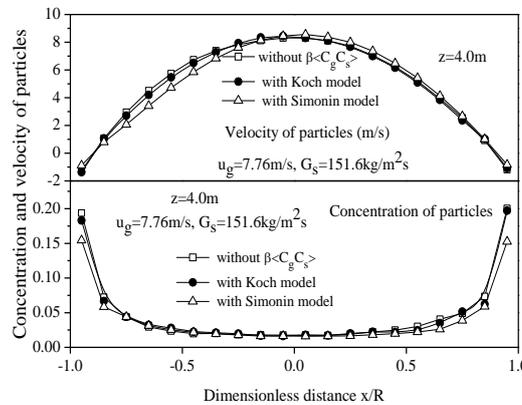
### 3.2. Analysis of Gas-solid Second-order Interaction

Figure 2 shows the distribution of second-order term  $\beta_{gs} \langle C_{gi} C_j \rangle$  associated with particles concentration using both Koch model and revised Simonin model. As it can be seen from the figure, the trends are consistent with different models, which the value of the second-order term increases with particles concentration increasing rapidly, and reaches maximum when the particles concentration is 0.05, then decreases gradually with particles concentration increasing. The mean value  $\beta_{gs} \langle C_{gi} C_j \rangle$  with revised Simonin model is higher than with Koch model.



**Figure 2. Second Order Interaction Items as a Function of Solid Volume Fraction**

### 3.3. Effect of Second-order Interaction on Concentration and Velocity of Particles



**Figure 3. Distribution of Concentration and Velocity of Particles**

Figure 3 shows the effect of gas-solid second-order interaction on the concentration and velocity of particles at  $z = 4.0 \text{ m}$ . The simulation results are calculated by no using second-order interaction model, using Koch model, and using revised Simonin model respectively. From the figure, the concentration of particles by revised Simonin model is slightly higher in the center, and lower near the wall. The simulation results of concentration and velocity of particles by different models have close distribution, which means the effect of gas-solid second-order interaction on concentration and velocity of particles is small.

## 4. Conclusions

The simulation results by the revised Simonin model have a good agreement with experimental data. The value of the second-order term  $\beta_{gs} \langle C_{gi} C_{sj} \rangle$  increases with particles concentration increasing rapidly, and reaches maximum, then decreases gradually with particles concentration increasing. The mean value  $\beta_{gs} \langle C_{gi} C_{sj} \rangle$  with revised Simonin model is higher than with Koch model. The effect of gas-solid second-order interaction on second-order fluctuation is not ignore, but on concentration and velocity of particles is small.

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