

## Joint Torque Control of Hydraulic Quadruped Robot

Junpeng Shao, Xiaoning Mu, Guitao Sun and Weiyu Yang

*Harbin University of science and technology  
muxiaoning1989@163.com*

### **Abstract**

*For the influence of time varying model parameters on servo accuracy of hydraulic quadruped robot joint torque, the equivalent model of electro-hydraulic force servo system was derived, the compound control strategy including flow compensator and velocity feed forward was proposed, the principle of compound control strategy was given. The flow of electro-hydraulic servo valve is linear with the input signal which realized by the flow compensator, the time varying parameters of inertia and elastic load were inhibited by velocity feed forward, and then the high accuracy joint torque control was realized. The composite strategy was verified on Co-simulation of MATLAB &AMESim platform, the simulation results show that the compound control strategy can effectively inhibit the parameter perturbation and the Amplitude attenuation of the tracking signal is less than 10%, and verify efficiency of the proposed control strategy.*

**Keywords:** *hydraulic quadruped robot; torque control; flow compensator; velocity feed forward*

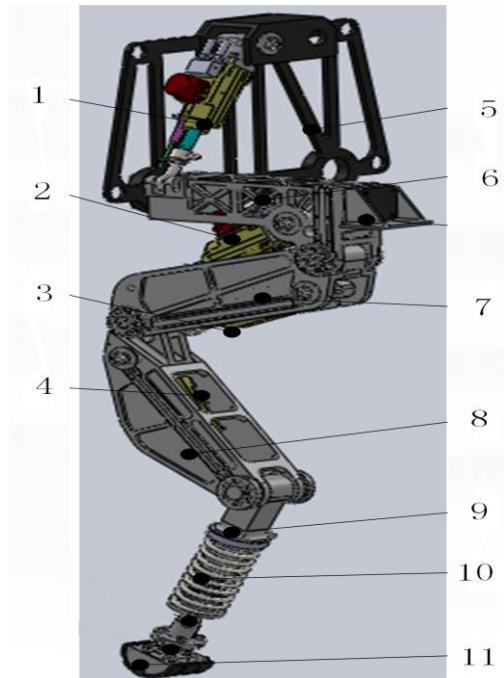
### **1. Introduction**

The electro-hydraulic servo actuator acted as actuator of hydraulic quadruped robot, including servo valve, hydraulic cylinder, displacement sensor, force sensor and so on, there was parameter time varying during the motion of joint which was realized by up and down motion of piston. America, Italy, Korea and China had rolled out their own hydraulic quadruped robot, they mainly researched on the gait of robot, but there were less research on the force control of electro-hydraulic servo actuator, although there were a lot of research results about force control of electro-hydraulic servo actuator, such as predictive control [1], quantitative feedback control [2], μrobust control [3], fuzzy sliding mode control [4] and so on, but they were too complex and particular about the processor.

Therefore, the compound control strategy including flow compensator and velocity feed forward was proposed, the output flow of servo valve had no relation with the load force with compensator, and then the flow characteristics of servo valve was kept constant, the velocity feed forward controller can inhibit the influence which was caused by the parameters time varying on system performance, finally, the higher force servo accuracy can be got.

### **2. Electro-hydraulic Servo Actuator Modeling**

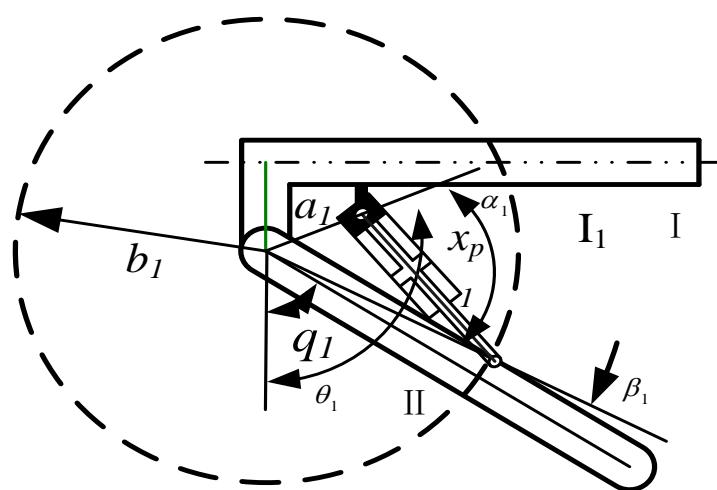
Single leg structure of hydraulic drive quadruped robot was shown in figure 1, there were four active degree of freedom and one passive degree of freedom, and each active degree of freedom was derived by the same type actuator. The stable running of hydraulic quadruped robot realized by the coordinated motion of hydraulic actuator.



1. pitch actuator of hip joint 2. roll actuator of hip joint 3. knee joint actuator 4. ankle joint actuator 5. fuselage 6.link 0 7.link 1 8.link 2 9.link 3 10.damping spring 11.foot end

**Figure 1. Single Structure of Quadruped Robot**

Take account of link 0, link 1 and roll actuator, the geometric relation between electro-hydraulic servo actuator and link was shown in figure 2, where  $a_1$  is the distance between the bottom of actuator and joint axis,  $b_1$  is the distance between joint axis and the connection point of piston rod and link II,  $x_{p1}$  is the distance between the bottom of actuator and the end of piston rod.



**Figure 2. Geometrical relationship between electro-hydraulic**

With reference figure 2, the relationship between joint angle  $q_1$  and piston displacement  $x_{p1}$  is described by

$$x_{p1} = \sqrt{a_1^2 + b_1^2 - 2a_1b_1 \cos(\theta_1 - \beta_1 - q_1)} \quad (1)$$

The relationship between joint torque  $\tau_1$  and piston rod force  $F_1$  is described by

$$F_1 = \frac{\tau_1}{a_i \sin(\arccos(\frac{a_1 - b_1 \cos(\theta_1 - \beta_1 - q_1)}{\sqrt{a_1^2 + b_1^2 - 2a_1b_1 \cos(\theta_1 - \beta_1 - q_1)}}))} \quad (2)$$

As known from equation (1) to equation (2), both the relationship between joint ankle  $q_1$  and piston displacement  $x_{p1}$  and the relationship between joint torque  $\tau_1$  and output force of piston rod  $F_1$  are one to one correspondence, and the relationship can be extended to other joint, namely ,

$$\mathbf{x}_{p1} = \mathbf{f}_q(\mathbf{q}) \quad (3)$$

Where  $\mathbf{x}_{p1} = [x_{p0} \ x_{p1} \ x_{p2} \ x_{p3}]^T$ ,  $x_{p0}, x_{p1}, x_{p2}$  and  $x_{p3}$  is the piston displacement of pitch joint actuator, roll joint actuator, knee joint actuator and ankle joint actuator, respectively,  $\mathbf{f}_q(\mathbf{q})$  is the joint angle function,  $\mathbf{q}$  is joint angle,  $\mathbf{q} = [q_0 \ q_1 \ q_2 \ q_3]^T$ ,  $q_0, q_1, q_2$  and  $q_3$  is the joint angle of corresponding joint.

$$\boldsymbol{\tau} = \mathbf{F}\mathbf{f}(\mathbf{q}) \quad (4)$$

where  $\boldsymbol{\tau}$  is joint torque ,  $\boldsymbol{\tau} = [\tau_0 \ \tau_1 \ \tau_2 \ \tau_3]^T$  ,  $\mathbf{F}$  is the output force of actuator,  $\mathbf{F} = [F_0 \ F_1 \ F_2 \ F_3]^T$  ,  $\tau_0, \tau_1, \tau_2, \tau_3$  and  $F_0, F_1, F_2, F_3$  is the joint torque and output force of pitch joint actuator, roll joint actuator, knee joint actuator and ankle joint actuator, respectively,  $\mathbf{f}(\mathbf{q})$  is function of joint ankle.

Single leg dynamic equation of hydraulic quadruped robot is written by

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) \quad (5)$$

Where  $\mathbf{M}(\mathbf{q})$  is inertial coefficient,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is coefficient matrix of coriolis force and centripetal,  $\mathbf{G}(\mathbf{q})$  is potential energy.

The linearized flow-pressure equation of servo valve, the flow continuity equation of hydraulic cylinder, the force balanced equation of hydraulic cylinder with load is described respectively as follows

$$q_L = K_q x_v - K_c p_L \quad (6)$$

Where  $q_L$  is load flow,  $K_q$  is servo valve flow gain,  $K_c$  is flow pressure coefficient,  $x_v$  is spool displacement,  $p_L$  is pressure drop.

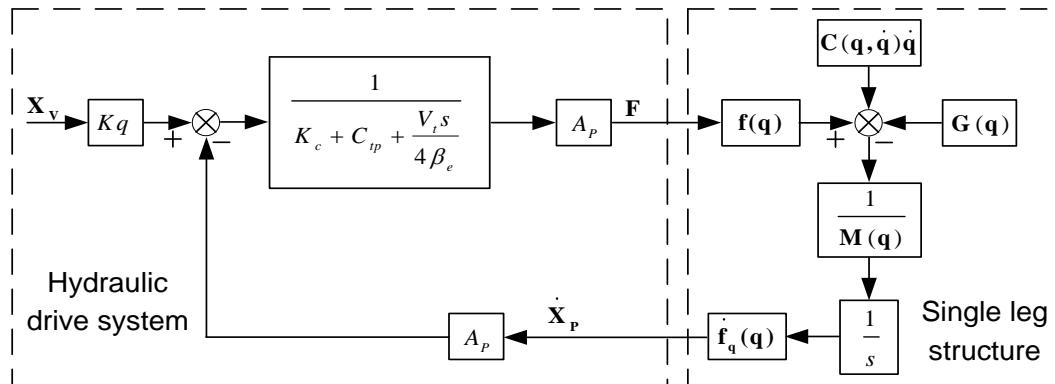
$$q_L = A_p \dot{x}_p + C_{tp} p_L + \frac{V_t}{4\beta e} \dot{p}_L \quad (7)$$

Where  $A_p$  is effective area of cylinder,  $x_p$  is displacement of piston,  $C_{tp}$  is total flow-pressure coefficient defined as  $C_{tp} = C_{ip} + C_{ep} / 2$ ,  $C_{ip}$  is internal leakage coefficient,  $C_{ep}$  is external leakage coefficient,  $V_t$  is total effective volume of hydraulic cylinder ( $\text{m}^3$ ),  $\beta_e$  is bulk modulus of oil (Pa).

$$F = A_p p_L = m \ddot{x}_p + B_c \dot{x}_p + K_L x_p + F_L \quad (8)$$

Where  $F$  is output force of piston rod,  $m$  is equivalent load mass,  $B_c$  is equivalent viscous damping coefficient,  $K_L$  is equivalent load stiffness,  $F_L$  is external disturbance.

From equation (3) to equation (8) [6], the single leg dynamic diagram of hydraulic quadruped was shown in Figure 3.



**Figure 3. Dynamic Diagram of Single Leg**

The relationship between servo valve input current and spool displacement of servo valve can be described as

$$\frac{X_V}{I} = \frac{K_{xv}}{\frac{s^2}{\omega_{sv}^2} + \frac{2\xi_{sv}}{\omega_{sv}} s + 1} \quad (9)$$

Where  $I$  is input current of servo valve,  $K_{xv}$  is servo valve gain,  $\omega_{sv}$  is servo valve natural frequency,  $\xi_{sv}$  is servo valve damping ratio.

The dynamics characteristics of servo valve can be neglected for the natural frequency of servo valve is higher, the equation of (9) can be rewritten as

$$\frac{X_V}{I} = K_{xv} \quad (10)$$

Define  $K_a$  as the servo amplifier gain, the relationship between the input voltage of amplifier and output current of amplifier can be described as

$$I = K_a u \quad (11)$$

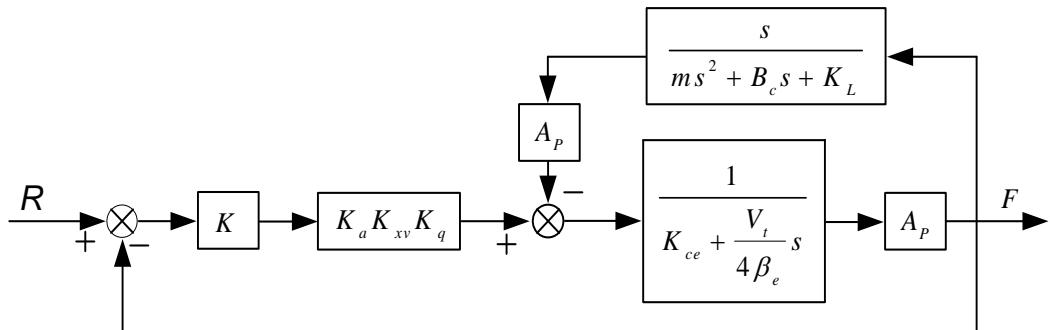
The spool displacement of servo valve is

$$X_V = K_{xv} K_a u \quad (12)$$

The output flow of servo valve is

$$Q = K_q X_V = K_q K_{xv} K_a u \quad (13)$$

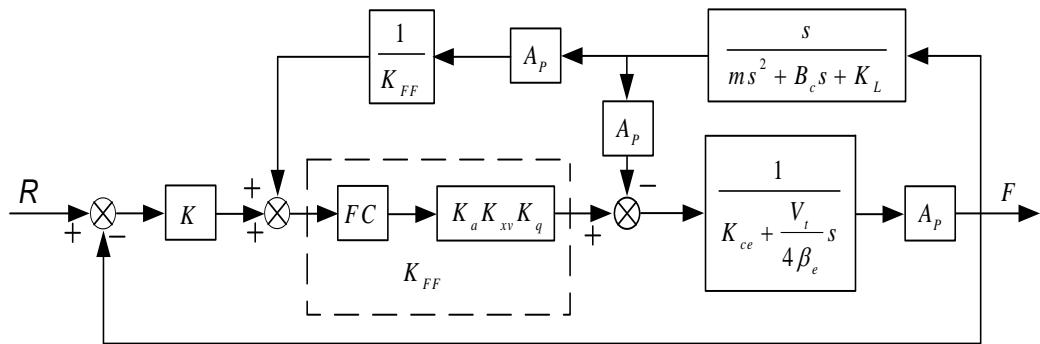
Furthermore, the force control diagram of actuator was shown in figure 4, where  $R$  is command signal,  $K$  is controller. In order to get a good force control performance, the controller should inhibit the disturbance of system parameters, such as equivalent load mass  $m$ , equivalent viscous damping coefficient  $B_c$ , equivalent load stiffness  $K_L$  and so on.



**Figure 4. Force Control Diagram of Actuator**

### 3. Controller Design

The compound control strategy including flow compensator and velocity feed forward was shown in Figure 5, the output flow of servo valve with compensator has no relation with the load force, and then the flow characteristics of servo valve is kept constant, the velocity feed forward controller can inhibit the influence which caused by the parameters time varying on system performance, finally, the higher force servo accuracy can be got.



**Figure 5. Principle Diagram of Compound Control**

The servo valve flow-pressure equation can be described as

$$q_L = C_d w x_v \sqrt{\frac{1}{\rho} \left[ p_s - \frac{x_v}{|x_v|} p_L \right]} \quad (14)$$

Where  $C_d$  is servo valve flow coefficient,  $w$  is servo valve area gradient,  $\rho$  is oil density,  $p_s$  is hydraulic supply pressure.

Define  $K_B = C_d w K_{xv} \sqrt{\frac{1}{\rho}}$ , the equation (14) can be rewritten as

$$q_L = \begin{cases} K_B i \sqrt{p_s - p_L}, & i > 0 \\ -K_B i \sqrt{p_s + p_L}, & i < 0 \end{cases} \quad (15)$$

Define compensation function as

$$f(F_c) = \begin{cases} \sqrt{\frac{\lambda p_s}{p_s - F / A_p}}, & i > 0 \\ \sqrt{\frac{\lambda p_s}{p_s + F / A_p}}, & i < 0 \end{cases} \quad (16)$$

where  $\lambda$  is compensation coefficient.

With the input voltage  $uf(F_c)$ , the output current of servo amplifier is  $i = uf(F_c)K_a$ , and substitute it into formula (15), the load flow is

$$q_L = \begin{cases} K_B u K_a \sqrt{\lambda p_s}, & u > 0 \\ -K_B u K_a \sqrt{\lambda p_s}, & u < 0 \end{cases} \quad (17)$$

As known from equation (17), there is a linear relationship between the servo valve load flow and input current.

The value of  $K_B$  is unknown because the servo valve flow coefficient and area gradient cannot be find from the datasheet of servo valve, the formula (14) can be rewritten as

$$K_B = C_d w K_{xv} \sqrt{\frac{1}{\rho}} = \frac{q_L}{i \sqrt{\Delta P}} \quad (18)$$

$$\text{where } \Delta P = \begin{cases} p_s - p_L, & i > 0 \\ p_s + p_L, & i < 0 \end{cases}$$

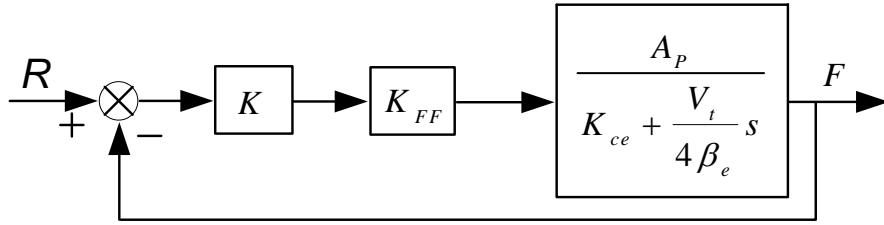
As known from formula (18), the value of  $K_B$  can be calculated by the value of servo valve output flow and servo valve input current. In the formula (18), the value of  $q_L$  is equal to the rated flow of servo valve  $Q_n$ , the value of  $i$  is equal to the rated current of

servo valve  $I_n$ ,  $\Delta P_n$  is valve pressure drop with rated flow, define  $K_{FF} = \frac{Q_n K_a \sqrt{\lambda p_s}}{I_n \sqrt{\Delta P_n}}$ ,

the formula (17) can be rewritten as formula(19), the formula (19) is the flow equation of servo valve with compensator.

$$q_L = \begin{cases} u K_{FF}, & u > 0 \\ -u K_{FF}, & u < 0 \end{cases} \quad (19)$$

In this paper, the value of  $\lambda$  is 2, then, the figure 4 can be changed into the form of Figure 6.



**Figure 6. Simplified Diagram of Force Control**

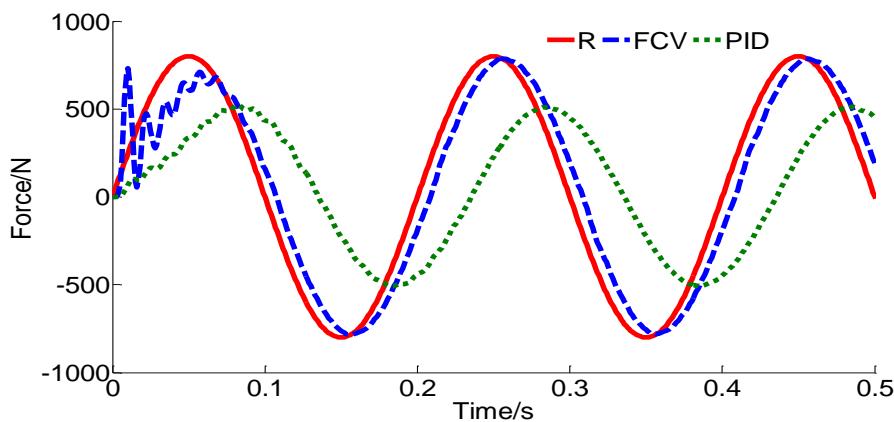
The equivalent transfer function of force control can be written as

$$\frac{F}{R} = \frac{KK_{FF}A_p}{K_{ce} + \frac{V_t}{4\beta_e}s} \quad (20)$$

As known from formula (20), the compound control strategy including flow compensator and velocity feed forward completely eliminate the influence on system performance which caused by the parameter time varying of equivalent load mass, equivalent viscous damping coefficient  $B_c$  and equivalent load stiffness  $K_L$ .

#### 4. Simulation Analysis

The planar mechanism and hydraulic system was designed in AMESim, the controller was designed in Matlab, respectively, the composite strategy was verified on Co-simulation of MATLAB & AMESim platform. The main parameters of the force control system are as follows: rated pressure of servo valve is 21 MPa, rated flow of servo valve is 11.5 L/min, rated current is 15 mA, amplitude-frequency of -3db is 145Hz, phase- frequency of 90° is 205Hz, dimension of actuator piston and piston rod is 25mm and 14mm, respectively, the range of displacement sensor is between 0 and 50 mm, output signal of displacement sensor is between 0 and 10 V, the range of force sensor is between -13.32 KN and 13.32 KN ,output signal of displacement sensor is between -10V and 10 V. The force tracking curve with PID controller and composite strategy was shown in Figure 7, and the pressure of supply oil is 16 MPa, the amplitude and frequency of reference signal is 800N and 5Hz, respectively.



**Figure 7. Force Tracking Curve**

It has been shown from the Figure 6 that the force tracking curves with different controllers had different performances, the force tracking curve with PID controller had bad performance for the stability margin was little and cannot improve the force tracking performance by adjusting the controller parameters, the force tracking curve with composite strategy has good performance, the Amplitude attenuation of the tracking signal is 1.5625% ,the phase lag is 15.48° , respectively, and realized high precision force control.

## 5. Conclusions

- (1) Single leg dynamic equation of hydraulic quadruped robot was derived.
- (2) In order to realize high precision control with model parameters time varying, the compound control strategy including flow compensator and velocity feed forward was proposed.
- (3) The composite strategy was verified by Co-simulation of MATLAB & AMESim, the simulation results showed that the amplitude attenuation of the tracking signal was 1.5625%,the phase lag was 15.48° ,and realized high precision force control.

## Acknowledgements

This work was supported by International cooperative projects (2012DFR70840).

## References

- [1] G. Wu, N. Sepehri and K. Ziae, “Design of a hydraulic force control system using a generalized predictive control algorithm”, IEEE Proceedings: Control theory and applications, United kingdom, (1998) September, vol. 145, pp. 428-436.
- [2] K. K. Ahn and Q. T. Dinh, “Self-tuning of quantitative feedback theory for force control of an electro-hydraulic test machine, control engineering practice”, United kingdom, vol.17, (2009) November, pp. 1291-1306.
- [3] H. Zhao, H. Zhang, S. Y. Zhang and J. W. Han, “Application of  $\mu$ theory in compliant force control”, Chinese journal of mechanical engineering. Beijing, vol. 43, (2007) December, pp.97-101.
- [4] M. H. Chiang, Y. P. Yeh, F. L. Yang and Y. N. Chen, “Integrated control of clamping force and energy-saving in hydraulic injection moulding machines using decoupling fuzzy sliding-mode control”, International journal of advanced manufacturing technology. United kingdom, vol. 27, (2005) January, pp. 53-62.
- [5] W. M. Bessa, M. S. Dutra and E. Kreuzer, “Sliding mode control with adaptive fuzzy dead-zone compensation of an electro-hydraulic servo system”, Journal of intelligent and robotic system: theory and application. Netherlands, vol. 58, (2010) April, pp. 4-5.