# Modeling and Simulation of Temperature Field in High Speed Mil ling based on Weighted Residual Method

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# Abstract

In the process of high speed milling of titanium alloy, the milling temperature change had an adverse effect on machining accuracy, machining surface integrity and machining efficiency. To deal with this problem, heat conduction mode, the heat convection between cutting tools and air medium, and heat flux intensity boundary conditions in actual milling process are fully considered. And according to general heat equation, the heat conduction model of transient temperature field is built. The differential form of temperature field model are converted into corresponding form of the integral functional equation by the weighted residual method. A certain accuracy method for calculating the approximate solution is presented. And the transient temperature response of space element in milling system is obtained under equal time intervals. Finally it is proved that the model is correct by experiment result, which laid a solid foundation for accurately analyzing milling temperature.

*Keywords:* weighted residual method, high speed milling, temperature field, heat conduction

### **1. Introduction**

The milling temperature is an important physical quantity of high speed milling. The change of milling temperature had an adverse effect on the tool life, the machining accuracy and machining surface integrity [1]. Obtaining temperature distribution of milling area and transient temperature of single point is an important process to study the effects of milling temperature for high speed milling process. The finite element method is an effective and reliable analysis method. And it can comprehensively consider the material performance, heat conduction and other factors to build temperature field model [2].

Currently, a lot of literature about milling temperature field research are mainly divided into elastic-plastic theory and the finite element method in two ways. The threedimensional finite element analysis of temperature field is little. The hot-elastoplastic constitutive equation was built by finite deformation theory, thus the large deformation heat-elastoplastic coupling control equation was established by Zhitao Tang [3]. The numerical model of temperature field prediction between the straight cutting tool and chip was established by Lazoglu [4]. The temperature field model of basing on the elasticplastic theory could not obtain milling heat which changes over time and the single point of the transient temperature distribution. The two-dimensional finite element model was established by separating the steady state of cutting chip to simulate the temperature field [5]. Titanium alloy shear zone temperature field model is established by using Jeager static heat source theory [6]. 3D orthogonal cutting model is established by using the finite difference method [7]. Two-dimensional temperature field modeling of basing on the finite element method is more, but two-dimensional milling temperature field model could not obtain spatial single-point transient temperature, which makes modeling of 2D milling temperature do not meet the actual requirements of high speed milling process of the temperature field modeling.

In this paper, heat conduction mode, the heat convection between cutting tools and air medium, and heat flux intensity boundary conditions in actual milling process will be fully considered. And according to general heat equation, the heat conduction model of transient temperature field will be built. And a certain accuracy method for calculating the approximate solution will be presented by the weighted residual method. Finally it will be verified by experiment result.

# 2.Establishment of Transient Temperature Field of Heat Transfer Model

#### 2.1 General Equation of System Temperature Field

**2.1.1 Temperature Field:** Under the effect of temperature difference, the heat is always from the high temperature area to pass in low temperature area. In the process of heat conduction, each point in the system temperature T is a function of spatial coordinates and time [8]:

$$T = T\left(x, y, z, t\right) \tag{1}$$

**2.1.2 Temperature Gradient and Heat Flow Strength:** Along an arbitrary isothermal surface temperature is the same, but the temperature of the other direction is different. On the normal direction of isothermal surface, the temperature change is the biggest temperature gradient on this direction:

$$grad(T) = \nabla T = \frac{\partial T}{\partial x}i + \frac{\partial T}{\partial y}j + \frac{\partial T}{\partial z}k = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right)$$
(2)

For the heterogeneous materials, heat q passing per unit area and per unit time is proportional to the temperature gradient:

$$q = \left(-k_x \frac{\partial T}{\partial x}, -k_y \frac{\partial T}{\partial y}, -k_z \frac{\partial T}{\partial z}\right)$$
(3)

Where, q is the heat flux intensity,  $k_x$ ,  $k_y$ ,  $k_z$  is coefficient of thermal conductivity of three directions in the space.

#### 2.2 Establishment of Temperature Field Differential Equation

In the process of high speed milling, the milling speed is much greater than the feed speed, the heat source can be thought as the process for high speed rotary motion on the surface, and high speed milling of titanium alloy generally form a continuous serrated chip. Thus the heat source can be regarded as continuous heat source of high-speed rotation. Regarding the work piece and cutter, chip and the ambient air as a heat conduction system at here, which is shown in Figure 1. The idea of general system temperature field equation is used by scattering the heat conduction system into finite heat conduction elements and establishing differential function with time variable. According to the law of heat conduction, the heat of introducing into element should be equal to its accumulating heat in unit time.



Figure 1. Milling System and Temperature Conduction Mode

**2.2.1 Heat of Introducing into Element:** On the x direction, due to the effect of heat conduction, importing heat  $dq_{S1}$ , exporting heat  $dq_{S2}$  and the heat achieved of x direction  $dQ_x$  have the following relations, which is shown in Figure 2:

$$dq_{s1} = q_x dy dz = -k_x \frac{\partial T}{\partial x} dy dz$$
(4)

$$dq_{S2} = \left(q_x + \frac{\partial q_x}{\partial x}\right) dy dz = -\left(k_x \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) dx\right) dy dz$$
(5)

$$dQ_x = dq_{s_1} - dq_{s_2} = \frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) dx dy dz$$
(6)



#### Figure 2. The Heat Conduction of Element in the x Direction

In the same way, the heat of introducing into element along the y axis and z axis, respectively:

$$dQ_{y} = \frac{\partial}{\partial y} \left( k_{y} \frac{\partial T}{\partial y} \right) dx dy dz$$
(7)

$$dQ_{z} = \frac{\partial}{\partial z} \left( k_{z} \frac{\partial T}{\partial z} \right) dx dy dz$$
(8)

Getting (6) (7) (8) together, the heat of importing the various surfaces of the element in unit time:

$$dQ_{1} = dQ_{x} + dQ_{y} + dQ_{z} = \left[\frac{\partial}{\partial x}\left(k_{x}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_{y}\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_{z}\frac{\partial T}{\partial z}\right)\right]dxdydz$$
(9)

**2.2.2 The Accumulative Heat in Element:** The change of temperature in element from T to  $T + \partial T / \partial t$  in unit time. Therefore, in the element the amount of heat necessary to cause the rise of temperature is:

$$dQ_s = \rho c \frac{\partial T}{\partial t} dx dy dz \tag{10}$$

Where,  $\rho$  is the density of material, *c* is the specific heat materials.

**2.2.3 Build Temperature Field Differential Equation under Heat Convection:** In practical milling process, some work piece due to the factors of using way, such as titanium alloy, high temperature alloy. The milling heat could not be passed to the surrounding air, but the cutting tool material usually has good thermal conductivity. In order to closely response to the temperature field, considering heat convection between tool and air medium in building temperature model is significant. The intensity of heat convection can be represented as:

$$q_c = h \left( T_e - T_s \right) \tag{11}$$

Where,  $q_c$  is the intensity of heat convection, h is the convection coefficient,  $T_e$  is the environment temperature,  $T_s$  is the temperature of the solid surface.

According to the conservation principle of heat conduction, the following equation was established. The formula (9)+(11) equal the formula (10), high speed milling heat conduction model of the transient temperature field is built:

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) + q_c = \rho c \frac{\partial T}{\partial t}$$
(12)

**2.2.4 Confirm the Temperature Field Differential Equation and Boundary Conditions:** Temperature boundary conditions is defined on any part of the milling system boundary  $\Gamma 1$ , the temperature of any heat conduction element at various points is a given function of spatial location and time:

$$T(x, y, z, t) = \overline{T}(x, y, z, t)$$
(13)

The thermal boundary conditions are defined on any part of the milling system boundary  $\Gamma^2$ , heat flow intensity of any heat conduction element at various points is a given function of spatial location and time:

$$-k_n \frac{\partial T}{\partial n} = q\left(x, y, z, t\right) \tag{14}$$

Convection boundary conditions are defined on any part of the milling system border  $\Gamma$ 3, the convection conditions of any heat conduction infinitesimal body at various points are known:

$$k_n \frac{\partial T}{\partial n} = h \left( T_e - T_s \right) \tag{15}$$

# **3. Integral Functional Solution of Milling Temperature based on Weighted Residual Method**

There are some non-homogeneous items in this model. It will have a complex process of integral construction in solving process, but the weighted residual method can be more convenient to solve such problems. The differential form of temperature field model are converted into corresponding form of the integral functional equation by it. And a certain accuracy solution will be obtained:

$$\int_{V} R_{i} W_{i} dv + \int_{S} R_{S} W_{B} ds = 0 \quad (i=1,2,...n)$$
(16)

Where,  $W_i$  is the weight function,  $W_B$  is the weight function on the boundary conditions.

The weighted residual method is used for high speed milling heat conduction model of the transient temperature field in three dimensional space domain, and using the first order variation function, there are:

$$\int_{V} \delta\{T'\}^{T} [k_{t}] \{T'\} dV - \int_{\Gamma^{2}} h(T_{e} - T_{s}) \delta T_{s} dS + \int_{\Gamma^{3}} q \delta T_{s} dS + \int_{V} \overline{q} \delta T_{s} dV = 0$$
(17)

Where,  $\{T'\}^T = \begin{bmatrix} \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{bmatrix}$ ,  $\begin{bmatrix} k_t \end{bmatrix} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$ . V is the milling area of

solving,  $\Gamma 2$  is convection boundary region,  $\Gamma 3$  is heat input boundary,  $[k_t]$  is matrix of coefficient of thermal conductivity.

### 4. High Speed Milling Temperature Field Simulation Analysis

In order to study the change of milling temperature, through the finite element software to establish three-dimensional model of the milling. Radius of cutting tool choice is 4 mm of carbide end milling ball head. Spindle speed is 8000 r/min, and each tooth feed is 0.2 mm/z, milling depth of 0.4 mm, the milling width is 0.6 mm. Simulating titanium alloy milling process, and the temperature field distribution of Step 9 and step 29 are shown in Figure 3, 4.



Figure 3. Step 9 Simulation of Temperature Field



Figure 4. Step 29 Simulation of Temperature Field

Extracting data of work piece temperature, tool and chip temperature, and drawing curves over time, which is shown in Figure 5. It is found that the temperature of chip will increase with progressing of the milling. But after entering a stable state, the temperature maintains at about 1100 °C. The raising temperature of work piece is small in high speed milling due to chip away with a lot of heat, and it is extremely advantageous to obtain good processing quality.



Figure 5. The Temperature Curve in the Process of High Speed Milling

# 5. High Speed Milling Experiments Proof of the Transient Temperature Field Model

(1) Testing facilities: Milling machine is adopted in the experiment XH715 vertical machining center. Data acquisition card and charge amplifier. The infrared thermal image.

(2) Experimental materials and the milling parameters: Experiments in TC4 titanium alloy material, and its main physical properties are shown in Table 1. Titanium alloy TC4 length, width and height respectively is 160 x 100 x 40 mm. Tool for carbide end milling ball head two blade, diameter is 8mm and spiral Angle  $\beta = 40^{\circ}$ . Milling parameters for milling speed 200 m/min, feeding 0.1 mm/z, milling depth of 0.4 mm, the milling width is 0.2 mm. The climb milling and one-way feeding.

Modulus of	Poisson's ratio	Density	Specific heat	Thermal
elasticity (Pa)	(u)	$(Kg/m^3)$	capacity	conductivity
			(J/(Kg•°C))	(W/(m•°C))
$1.0480 \times 10^{11}$	0.34	$4.5 \times 10^{3}$	674	20

Table 1. Titanium Alloy TC4 Parameters

As the heat source will be monitored, using the transient temperature field of the axial heat conduction model to calculate the point contact depth 1 mm work piece temperature changes, as Figure 6 shows. During the process of milling, the maximum error between milling temperature calculation results and the experimental temperature measured is 15.16%, which illustrates that the transient temperature field model of high speed milling is correct.



Figure 6. The Contrast of Milling Temperature Experiment Data and Calculated Data

# **6.**Conclusion

(1) The heat conduction model of transient temperature field is built by considering the milling heat conduction mode, the thermal convection of cutting tools and air medium and heat intensity flux boundary conditions. This model can well describe heat conduction in the process of milling process.

(2) It is a good way to solve the special solution of a certain accuracy in high speed milling transient temperature field model by the weighted residual method. And then the transient temperature response of space element in milling system is obtained under equal time intervals.

(3) The experimental results show that the result error of model is small and can intuitively reflect the temperature field of heat conduction process of high speed milling process.

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