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Design and Implementation of Fractional Order Controller for Service Robots

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Abstract

In this paper, the fractional order motion control is designed basing on the Leader I-DX service robotic platform, where the implementation of robust fractional order controller is introduced in details. The fractional order PDμ control is analyzed in frequency domain by using an FIR/IIR method. The system model of the motor of Leader I-DX is established in MATLAB so that the influence of μ to the whole control system can be verified, and the comparisons of PD and PDμ controllers are shown as well. Motivated by the better performance of fractional order controllers, a digital implementation of fractional order PDμ controllers using PRONY technique is applied so that the incremental fractional order controllers can be realized. The real experiments on Leader I-DX platform are illustrated to validate the above concepts.

Keywords: *Service robot; Motion control; Fractional calculus; Fractional order PDμ controller*

1. Introduction

With the development of society and the progress of science and technology, the research and applications of service robots have been paid more attention than ever before, and it is gradually emerged into people's lives. It is well known that the motion performance is an important index to measure the robots. The high precision control of robots are strongly relied on the reliability of control systems, and the advanced motion control strategy. At present, in the process of designing the robot controller, the PID algorithm is the most widely used one that has simple structure, strong adaptability, good robustness and anti-perturbation ability, etc. However, the traditional PID algorithm may not be ideal in the control of nonlinear, time-varying, coupling and other complex systems. Thus the PID algorithm cannot meet the requirements of high performance robots in many respects.

The history of fractional calculus is the same long as the traditional calculus, which adopts the derivatives and integrals with arbitrary real or even complex numbers [1-2]. It has been pointed in [3] that “the fractional order system is ubiquitous in reality”. Besides, it can be seen that the fractional order theory can better reveal the facts of the real systems, therefore it is a better way to system modeling [4-8]. Nowadays, the fractional order control has become an important part of modern control theory and its applications [9-11]. Particularly, the optimal control strategy of fractional order system is always pointing to the fractional order one. It is a meaningful work about the fractional order systems and controls. The CRONE control is proposed by Oustaloup [12]; the fractional order PD control is discussed by Dorcak in [13]; the fractional order $PI^{\lambda}D^{\mu}$ control is developed by Podlubny in [14]. Meanwhile, the applications of fractional calculus to various areas are shown in [15-18].

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In this paper, the fractional order PD$^{\mu}$ control is applied successfully to the Leader I-DX robot [19]. It will be shown that the fractional order dynamics and controls compensates the properties of the system that are ignored in integer order discussions. Some mathematical preliminaries are introduced in Section 1. The model of Leader I-DX service robot's motor is established in Section 2. In Section 3 a robust fractional order PD$^{\mu}$ controllers is designed for the motion control of the Leader I-DX service robotic platform. With the established MATLAB simulation model, Section 4 presents the influence of the fractional derivative operator μ to the control system and the comparisons between PD and PD$^{\mu}$ controllers. Section 5 realizes the discretization of the fractional operator and the implementation of fractional order controller. The experimental application results are shown in Section 6. Finally, the conclusions are shown in Section 7.

2. Fractional Calculus and Fractional Order PI$^{\lambda}$D$^{\mu}$ Control

The Grünwald-Letnicov fractional order derivative is defined as

$$ {}_{t_0}D_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-t_0}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{1} $$

where h is the sample instant, and $[X]$ denotes the integer part of X. The continuous Riemann-Liouville (RL) fractional order operator is ${}_{t_0}D_t^{\alpha}$, which is shown as

$$ {}_{\alpha}D_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}}, & R(\alpha) > 0 \\ 1, & R(\alpha) = 0 \\ \int_{\alpha}^{t} (d\tau)^{(-\alpha)}, & R(\alpha) < 0 \end{cases} \tag{2} $$

Moreover, the integer order PID control strategy is

$$ u(t) = K_p[e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}] \tag{3} $$

that can be extended to the fractional order PI$^{\lambda}$D$^{\mu}$ control:

$$ u(t) = K_p[e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^{\mu} e(t)] \tag{4} $$

The Laplace transform of RL derivative with zero initial condition is

$$ L[{}_0D_t^{\alpha} f(t)] = s^{\alpha} F(s) \tag{5} $$

so that the transfer function of fractional order PI$^{\lambda}$D$^{\mu}$ control is shown as

$$ G\ (s) = \frac{U(s)}{E(s)} = K_p(1 + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}) \tag{6} $$

3. The Modeling of Leader I-DX's DC Motor

The scheme of DC motor is shown in Figure 1, and its transfer function can be denoted as:

$$G(s)=\frac{\theta(s)}{u(s)}=\frac{1}{s(T_m s+1)k_e}=\frac{K}{s(T_m s+1)} \tag{7}$$

where K is the speed constant, $T_m=\frac{R_a J}{k_e k_m}$ is the time constant of the DC motor.

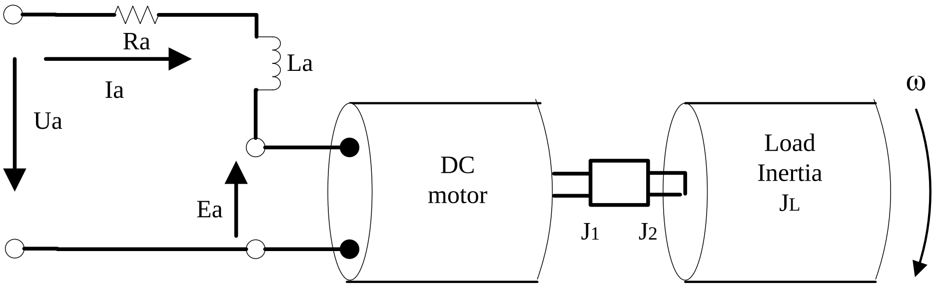

Figure 1. The Scheme of DC Motor

The driven motor of Leader I-DX is the MAXON RE30 DC motor, where R_a=0.198Ω ; L_a=3.45×10^{-5}H ; K=685rpm/v=71.73rad/v.s ; k_m=1.39×10^{-2}Nm/A ; J_1=3.35×10^{-6}Kg.m^2 ; J_2=7×10^{-6}Kg.m^2 ; η=86% ; i=66:1.

The total mass of Leader I-DX is m≈50kg, the driving wheel diameter is D=200mm, and the load moment of inertia is $J_L=\frac{m}{2}.\frac{D^2}{4}=0.25\text{Kg.m}^2$. Besides, the motor shaft inertia in total: $J=J_1+J_2+J_L/i^2\eta=7\times10^{-5}Kg.m^2$. Based on the above parameters, the time constant of the DC motor is T_m=0.07.

Therefore, the transfer function of the DC motor is

$$G(s)=\frac{\theta(s)}{u(s)}=\frac{71.73}{s(0.07s+1)} \tag{8}$$

4. Fractional Order PD$^\mu$ Control

4.1 Design Principles of Robust Fractional Order PD$^\mu$ Controller

In the design of robust controller, analyze the control system based on the desired amplitude margin and phase margin from the aspect of relative stability in frequency domain can meet the requirements of the system. In this paper, the fractional order controller is designed based on the amplitude margin and phase margin.

It follows from (6) that the fractional order PD$^\mu$ controller can be expressed as

$$C(s)=K_P(1+K_d s^{\mu}) \tag{9}$$

where $\mu\in(0,1]$. Obviously, (9) is a special case of fractional order PI$^\lambda$D$^\mu$ control where λ =0.

The magnitude and phase characteristics of DC motor are

$$\arg\left|G(j\omega)\right| = -\arctan(\omega T_m) - \frac{\pi}{2} \tag{10}$$

$$\left|G(j\omega)\right| = \frac{K}{\omega\sqrt{1+(\omega T_m)^2}} \tag{11}$$

Moreover, the PD$^{\mu}$ controller can be expressed in frequency domain that

$$C(j\omega) = K_p[1+K_d(j\omega)^{\mu}] = K_p(1+K_d\omega^{\mu}\cos\frac{\mu\pi}{2} + jK_d\omega^{\mu}\sin\frac{\mu\pi}{2}) \tag{12}$$

where

$$\arg\left|C(j\omega)\right| = \arctan\frac{\sin\frac{(1-\mu)\pi}{2} + K_d\omega^{\mu}}{\cos\frac{(1-\mu)\pi}{2}} - \frac{(1-\mu)\pi}{2} \tag{13}$$

$$\left|C(j\omega)\right| = K_p\sqrt{(1+K_d\omega^{\mu}\cos\frac{\mu\pi}{2})^2 + (K_d\omega^{\mu}\sin\frac{\mu\pi}{2})^2} \tag{14}$$

Thus the open loop transfer function $L(s)$ can be shown as

$$L(s) = C(s)G(s) \tag{15}$$

It follows from (10) and (13) that

$$\arg\left|L(j\omega)\right| = \arctan\frac{\sin\frac{(1-\mu)\pi}{2} + K_d\omega^{\mu}}{\cos\frac{(1-\mu)\pi}{2}} - \frac{(1-\mu)\pi}{2} - \arctan(\omega T_m) - \frac{\pi}{2} \tag{16}$$

In frequency domain, there are three roles to design the fractional order PD$^{\mu}$ control:

(1) Phase margin constraint

$$\arg\left|L(j\omega_{cg})\right| = agr\left|C(j\omega_{cg})G(j\omega_{cg})\right| = -\pi + \phi_m \tag{17}$$

where φ_m is the phase margin and ω_{cg} denotes the cut-off frequency.

(2) Robustness constraint

$$\frac{d(\arg[L(j\omega)])}{d\omega}\Big|_{\omega=\omega_{cg}} = 0 \tag{18}$$

It should be noted that the zero derivative of L(jω) to ω denotes the robustness of the closed control system so that the overshoots can be efficiently restricted for varying system gains. This is the core to design the fractional order robust control of Leader I-DX robot.

(3) Amplitude margin constraint

$$\left|L(j\omega_{cg})\right| = \left|C(j\omega_{cg})\right|\left|G(j\omega_{cg})\right|_{db} = 1 \tag{19}$$

4.2 The Parameters of Fractional Order PD$^{\mu}$ Controller

(1) Considering equation (17) we have

$$\arg\left|L(j\omega)\right| = \arctan\frac{\sin\frac{(1-\mu)\pi}{2}+K_d\omega^{\mu}}{\cos\frac{(1-\mu)\pi}{2}} - \frac{(1-\mu)\pi}{2} - \arctan(\omega T_m) - \frac{\pi}{2} = -\pi+\varphi_m \quad (20)$$

It follows from (20) that K_d and μ are related as

$$K_d = \frac{1}{\omega_{cg}{}^{\mu}}\tan[\varphi_m + \arctan\omega_{cg}T_m - \frac{\mu\pi}{2}]\cos\frac{(1-\mu)\pi}{2} - \frac{1}{\omega_{cg}{}^{\mu}}\sin\frac{(1-\mu)\pi}{2} \quad (21)$$

(2) Using equation (18) we have

$$\frac{d(\arg(L(j\omega)))}{d\omega}\Big|_{\omega=\omega_{cg}} = \frac{\mu K_d\omega_{cg}{}^{\mu-1}\cos\frac{(1-\mu)\pi}{2}}{\cos^2\frac{(1-\mu)\pi}{2}+(\sin\frac{(1-\mu)\pi}{2}+K_d\omega_{cg}{}^{\mu})^2} - \frac{T_m}{1+(T_m\omega_{cg})^2} = 0$$

(22)

It follows from (22) that K_d can be computed by

$$A\omega_{cg}^{2\mu}K_d{}^2 + BK_d + A = 0 \quad (23)$$

So that

$$K_d = \frac{-B\pm\sqrt{B^2-4A^2\omega_{cg}^{2\mu}}}{2A\omega_{cg}^{2\mu}} \quad (24)$$

where $A = \frac{T_m}{1+(T_m\omega_{cg})^2}$, $B = 2A\omega_{cg}{}^{\mu}\sin\frac{(1-\mu)\pi}{2} - \mu\omega_{cg}{}^{\mu-1}\cos\frac{(1-\mu)\pi}{2}$.

(3) The constraint in equation (19) gives

$$\left|L(j\omega_{cg})\right| = \left|C(j\omega)\right|\left|G(j\omega)\right| = \frac{K.K_p\sqrt{(1+K_d\omega_{cg}{}^{\mu}\cos\frac{\mu\pi}{2})^2+(K_d\omega_{cg}{}^{\mu}\sin\frac{\mu\pi}{2})^2}}{\omega_{cg}\sqrt{1+(\omega_{cg}T_m)^2}} = 1$$

(25)

By doing so the K_d and μ can be computed by combing (21) and (24). It should be noted that it is always a very heavy work to compute such parameters. In this paper, a practical strategy is proposed by using MATLAB. The detailed procedure is:

1) Given the desired cut-off frequency ω_{cg}.
2) Given the desired phase margin φ_m.
3) Plot the curve in equation (21), where μ is a independent variable and K_d is a dependent variable.
4) Plot the curve in equation (24), where μ is a independent variable and K_d is a dependent variable.

5) As shown in Figure 2, the intersection of the above two curves gives the values of μ and K_d.
6) Using equation (25) gives the value of K_p.

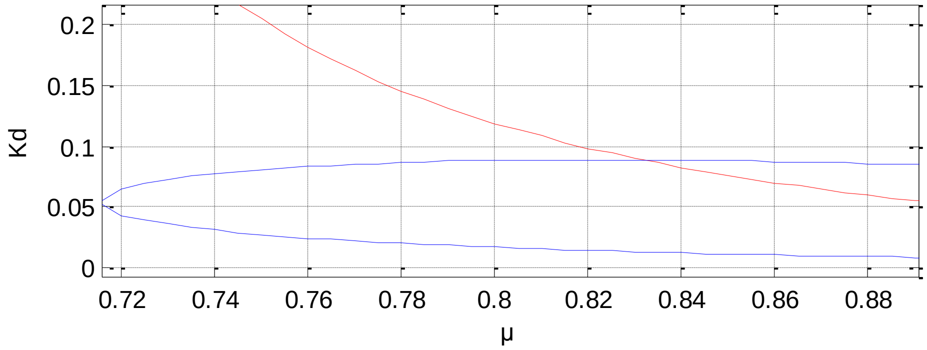

Figure 2. The Curves of μ and K_d

To our experiences, it is assumed that ω_{cg}=60rad/sec and φ_m =70°. It follows from the intersection in Figure 2 that μ=0.83 and K_d=0.09. It then follows from (25) that K_p=1.18. Therefore, the fractional order PD^{μ} control is

$$c(s) = 1.18(1 + 0.09s^{0.83}) \tag{26}$$

5. The Control System and Simulation Results

5.1 The Analysis of Control Systems

The system model of the DC motor of Leader I-DX is built in MATLAB/SIMULINK, which is shown in Figure 3.

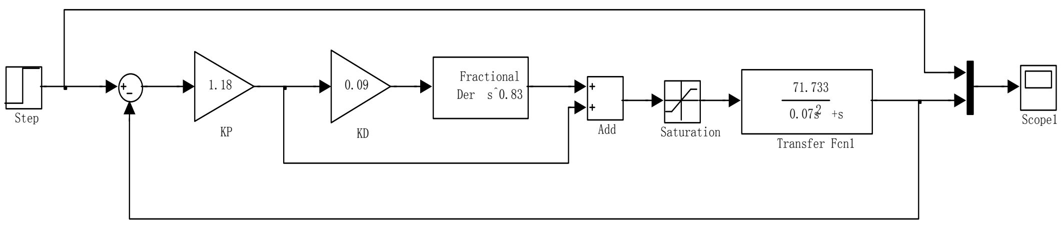

Figure 3. The System Model of Leader I-DX

The Bode plot of the open loop fractional order PD^{μ} contrl system is shown in Figure 4. As shown in this figure, the designed fractional order PD^{μ} controller not only satisfy the phase margin requirement but also remain unchanged in a certain range of the cut-off frequency ω_{cg}.

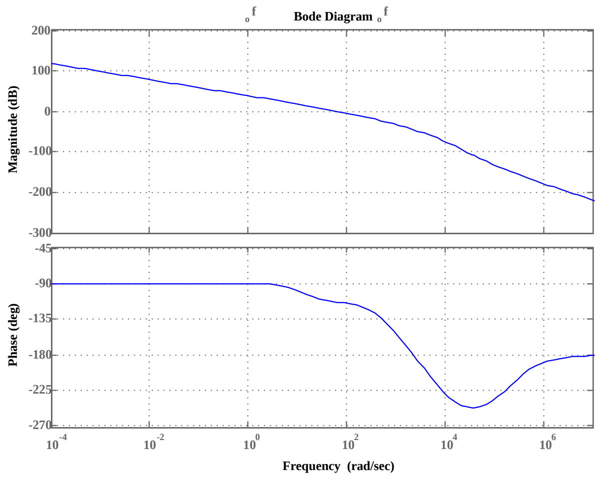

Figure 4. The Bode Plot of the Fractional Order PDμ Control System

5.2 The Influence of μ to the Control System

The step responses of the control system are shown in Figure 5. It can be seen that, for μ less than 0.83, the overshoot, oscillating, tuning time and static accuracy are all improved at the same time if μ is increased. A larger μ may deduce to slow tracking and low accurate static error. When μ is too small, there are too large overshoots, strong oscillating or even non-stability. Nevertheless, a small change of μ in a small field doesn't influence of performance of fractional order control system.

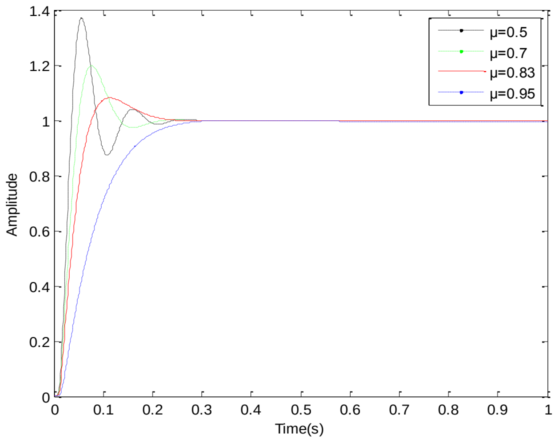

Figure 5. The Step Responses for Different *μ*

5.3 The Comparisons of PD^{μ} and PD Controllers

The comparisons of PD^{μ} and PD controllers are shown in Figure 6. It can be seen that, for various K_d in PD controller, the fractional order PD^{μ} controller (dotted line) performs better than the integer order ones (bold lines), which implies the advantages of fractional order control.

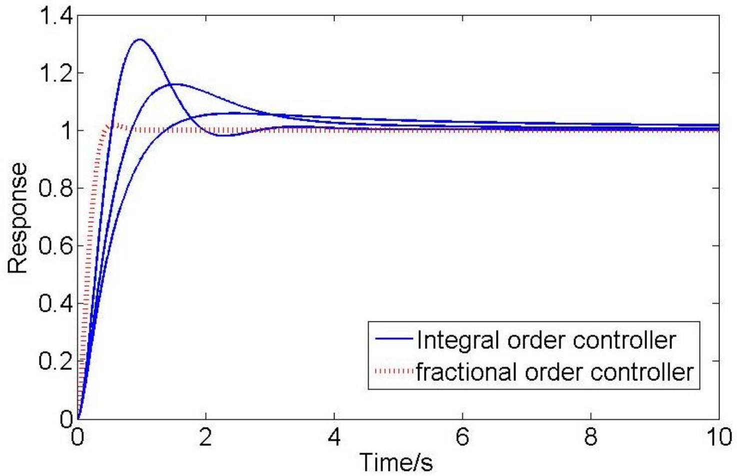

Figure 6. The Step Response Comparison between Fractional Order PD$^{\mu}$ Controller and Integral Order PD Controller

6. The Digital Implementation of Fractional Order Control

To further improve the robustness of fractional order control system, and consider the implementation of fractional order controllers, a low pass filter is cascaded to the fractional order derivative so that the fractional order controller is expressed as:

$$C(s) = K_P(1 + K_d \frac{s^{\mu}}{Ts+1}) = K_P(1 + K_d G_0(s)) \tag{27}$$

where T=1, $G_0(s)$ denotes the differentiator that implemented by adding a low pass filter. Besides, the inverse Laplace transform of $\frac{s^{\alpha-\beta}}{s^{\alpha}+\tau}$ is

$$t^{\beta-1}E_{\alpha,\beta}\left(-\tau t^{\alpha}\right) \tag{28}$$

where $E_{\alpha,\beta}()$ is the Mittag-Leffler function in two parameters. It follows from (28) that the impulse response of $G_0(s)$ is

$$h_0(t) = t^{1-\mu}E_{1,1-\mu}(-t) \tag{29}$$

By using the PRONY function in MATLAB, the approximated discrete transfer function of $G_0(s)$ can be shown as

$$G_0(z) = \frac{-0.008747z - 0.001903}{z - 1.019} \tag{30}$$

$$zU(z) - 1.019U(z) = K_p(z - 1.019 + K_d(-0.008747z - 0.001903))E(z) \tag{31}$$

Given the zero initial condition, the discrete fractional order PD$^{\mu}$ control is

$$\begin{aligned} u(k) &= 1.019u(k-1) + K_P[e(k) - 1.019e(k-1) \\ &+ K_d(-0.008747e(k) - 0.0101903e(k-1))] \end{aligned} \tag{32}$$

Thus the incremental fractional order $PI^{\lambda}D^{\mu}$ controller can be shown as

$$\begin{aligned}\Delta u(k) &= u(k) - u(k-1) \\ &= 0.019u(k-1) + K_P[e(k) - 1.019e(k-1) \\ &+ K_d(-0.008747e(k) - 0.0101903e(k-1))]\end{aligned} \tag{33}$$

7. Experiments

Let the referential speed of Leader I-DX be 600mm/s, and the sample instant be 8ms, the system output is shown in Figures 7 and 8. It can be seen that the fractional order PD^{μ} control strategy has a faster tracking speed. Besides, too many oscillations are observed in the traditional PD control case. Moreover, it should be noted that the weight of Leader I-DX is 50Kg. Thus a slow response is happened for zero initial state, which can be improved by an initial excitation.

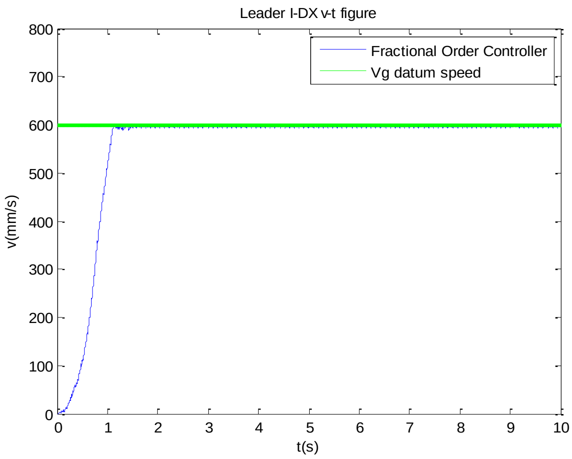

Figure 7. Fractional Order PD^{μ} Control

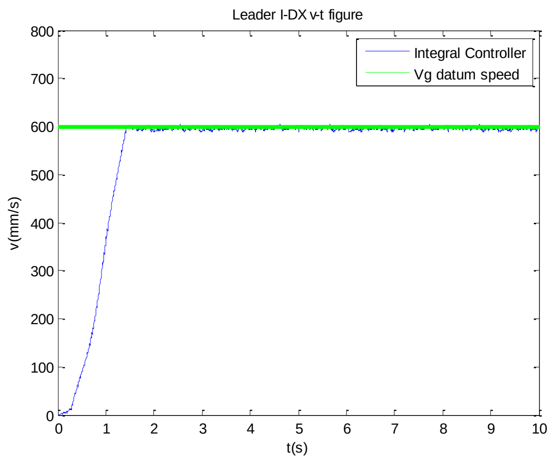

Figure 8. Integer Order PD Control

8. Conclusions

This paper discusses the robust design of fractional order PD^{μ} control, which is applied to the speed tracking control of Leader I-DX service robot. It has been verified in simulations and experiments that the fractional order PD^{μ} control performances better than traditional PD controllers, where the faster tracking speed and the smaller tracking error can be obtained.

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