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Pinning Synchronization Control of Two Fuzzy Complex Dynamical Networks based on Output Signals

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Abstract

In recent years, researchers found that the complex dynamical network is an effective tool to describe and understand the complex nonlinear systems. Because of the difficulty in establishing accurate mathematical models for nonlinear systems, the classical linear control methods are difficult to be applied to them. T-S fuzzy model has been proved to be an all-purpose approximation for nonlinear system, and has ability to combine linear control theory with fuzzy theory. In this paper, a scheme for synchronization between two fuzzy networks based on output signals under pinning control is proposed. Unlike other common methods,it only requires output signals to be transmitted from driving network to response network, and part of nodes in response network to be controlled. The synchronization criteria is given by using Lyapunov stability theory and linear matrix inequality technology, and the number of pinning nodes can be calculated by using the proposed synchronization theorem. Finally, a simulation example is given to verify the effectiveness of this scheme.

Keywords: *T-S fuzzy model, complex dynamical networks, pinning synchronization control, output signals*

1. Introduction

With the rapid development of science and technology, more and more complex dynamical networks come out in many fields of daily life, such as World Wide Web, power grid, communication network, social network and so on. It has attracted much attention of researchers from different disciplines[1-24].

It is known that synchronization can be classified into inner synchronization and outer synchronization [14]. Inner synchronization is a collective behavior within a single network, while outer synchronization concerns about a collective behavior between two networks or among more networks. A variety of synchronization control methods have been proposed for synchronization and outer synchronization [1-24].It can be concluded that traditional synchronization control schemes generally use state variables of network based on full-state coupling complex networks to achieve synchronization (see Figure 1). However, it is obviously impractical for real engineering applications, for the state variables of complex networks are usually unmeasurable and it is difficult to transmit so many state variable signals at the same time. Furthermore, when the quantities of nodes in network are too large, it is not economical to give a controller to every node.

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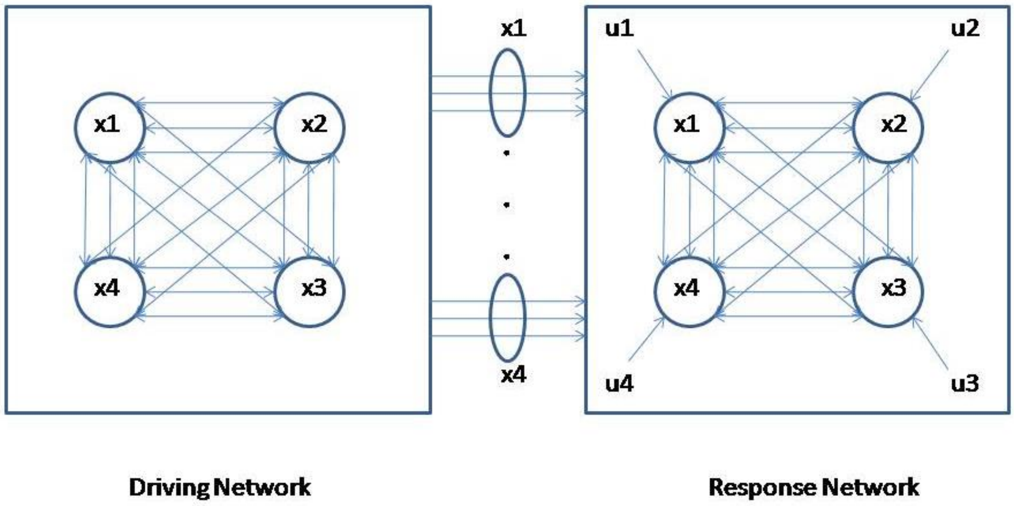

Figure 1. Traditional Outer Synchronization Control Using State Signals of Network

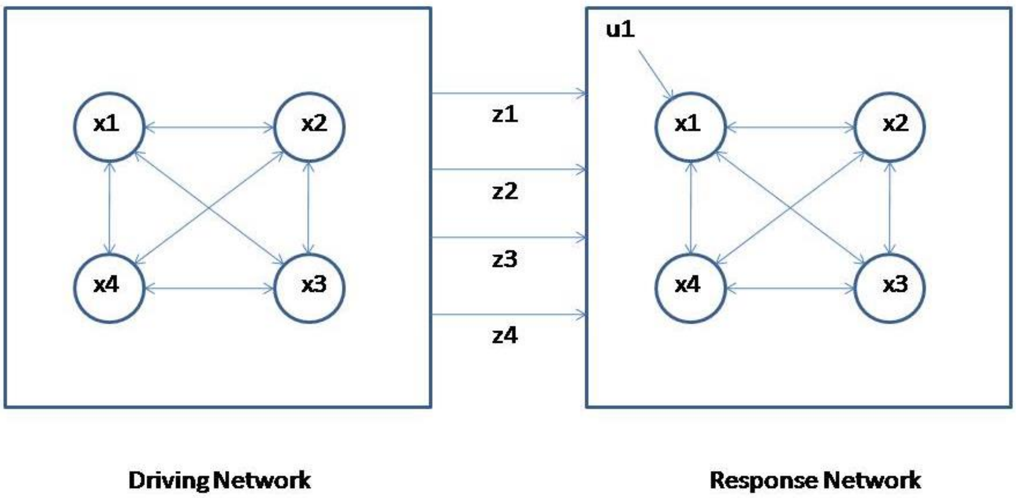

Figure 2. Proposed Outer Synchronization Control Using Output Signals of Network Inner

To reduce the number of transmitted signals and controlled nodes, is there a scheme to synchronize two complex networks coupled by output variables under pinning control (see Figure 2)? The answer is positive, and this paper proposes a pinning synchronization scheme for two fuzzy dynamical complex networks based on state observer. Zhou *et al.* [21] proposed an adaptive pinning method for inner synchronization, and based on this, Fan *et al.* [22] proposed a pinning state observer scheme for outer synchronization. In this paper, the pinning state observer is extended, and applied to synchronization of two fuzzy complex dynamical networks. On the other hand, nowadays, as the dynamical networks become more and more complex, it becomes more and more difficult to establish accurate mathematical models for complex networks. Here, T-S fuzzy model is a perfect choice, for it has been proved to be an all-purpose approximation for nonlinear dynamical systems. For the different dynamics of every zone from the complex network, one can use T-S fuzzy model to firstly construct local linear models, and then connect local linear models with fuzzy membership function in order to attain a global nonlinear model. Based on T-S fuzzy model, one can use classical linear control theory to design and analyze control system for the complex dynamical network. In recent two years, researchers started to study the control problems of complex network based on fuzzy models. Mukhija *et al.* [23] studied a class of fuzzy complex networks with time-varying delay, and proposed new synchronization criteria. Mahdavi *et al.* [24] designed an adaptive pulse controller for synchronization of fuzzy complex network, and proposed a method to choose suitable nodes to be controlled. However, pinning synchronization control has not been applied to two fuzzy complex dynamical networks. Therefore, pinning two fuzzy complex networks under state observer with output signals has important theoretical significance and practical value. In this paper, a new synchronization theorem for calculating the specific number of pinning nodes will be given by using Lyapunov stability theory and linear matrix inequality technology.

The left paper is organized as follows. Several mathematical preliminaries and a fuzzy complex dynamical network model are introduced in Section 2. A new pinning synchronization for two fuzzy complex networks is proposed in Section 3. In Section 4, a numerical simulation example will be given to verify the effectiveness of the proposed outer synchronization scheme. Finally, conclusions are drawn in Section 5.

2. Preliminaries

This section proposes a method to construct a fuzzy complex dynamical network model, and gives several necessary mathematical preliminaries for theoretical derivation.

Consider a complex dynamical network consisting of J identical nodes, which can be described as follows:

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + \sum_{j=1}^{J} g_{ij}(z_j(t)) \\ z_i(t) &= F(x_i(t)) \end{aligned} \tag{1}$$

where $1 \le i \le J$, $x_i \in R^n$ is the state vector of i th node, $z_j = F(x_i) \in R$ is output variable, $\dot{x}_i(t) = f(x_i)$ is the nonlinear dynamic, $g_{ij}(z_j)$ is nonlinear coupling, let $x_{i0} \in R^n$, $z_{j0} \in R$ be the point of Taylor series expansion. By using Taylor series expansion algorithm, constructing precision indicators and applying Lagrange multiplier method to minimize the precision indicators. Here, the detailed locally linearized procedure is omitted. Then, one can attain

$$\begin{aligned} \dot{x}_{i0}(t) &= f(x_{i0}(t)) + \sum_{j=1}^{J} g_{ij}(z_{j0}(t)) = A_i x_{i0}(t) + \sum_{j=1}^{J} c_{ij} L z_{j0}(t) \\ z_{i0}(t) &= D_i x_{i0}(t) \end{aligned} \tag{2}$$

Here, $A_i \in R^{n\times n}$ is system matrix, $L \in R^{n\times 1}$ is the inner coupling matrix, $C = (c_{ij})_{J\times J} \in R^{J\times J}$ is the coupling matrix. If there is a link between i th node and j th node, then $c_{ij} = 1$; otherwise, $c_{ij} = 0$ .Let C be a diffusive matrix satisfying $\sum_{j=1 j\neq i}^{J} c_{ij} = -c_{ii}$.

In order to improve precision of modeling, one should repeatedly linearize the nonlinear models at the feature points, and increase fuzzy rules. Then, T-S fuzzy complex network can be described by

$$\begin{cases} If\ \xi_{i1}\ is\ M_{i1}^{l}\ and \cdots and\ \xi_{ig_i}\ is\ M_{ig_i}^{l}, \\ Then\ \dot{x}_i(t) = A_i^l x_i(t) + \sum_{j=1}^{J} c_{ij} L z_j(t) \\ z_i(t) = D_i^l x_i(t), i = 1, 2, \cdots, J, l = 1, 2, \cdots, r_i. \end{cases} \tag{3}$$

After applying product inference engine, singleton fuzzification and center average defuzzification to (3), it can be rewritten by

$$\begin{aligned} \dot{x}_i(t) &= \sum_{l=1}^{r_i} h_i^l(\xi_{iq}(t))[A_i^l x_i(t) + \sum_{j=1}^{J} c_{ij} L z_j(t)\ ] \\ z_i(t) &= \sum_{l=1}^{r_i} h_i^l(\xi_{iq}(t))(D_i^l x_i(t)) \end{aligned} \tag{4}$$

where

$$\lambda_i^l(\xi_{iq}(t)) = \prod_{q=1}^{g_i} M_{iq}^l(\xi_{iq}(t)), h_i^l(\xi_{iq}(t)) = \frac{\lambda_i^l(\xi_{iq}(t))}{\sum_{l=1}^{r_i} \lambda_i^l(\xi_{iq}(t))}$$

$\xi_{i1}, \cdots, \xi_{ig_i}$ are antecedent variables, $M_{iq}^l(\xi_{iq}(t))$ is membership, $h_i^l(\xi_{iq}(t))$ is membership function, r_i is the number of the i th node's fuzzy rules.

Lemma 1: If matrix $C \in R^{n \times n}$ satisfies $c_{ij} > 0$ and $\sum_{j=1\, j \neq i}^{n} c_{ij} = -c_{ii}$, suppose that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ are the eigenvalues of the matrix C and $\bar{\lambda}_1 \geq \bar{\lambda}_2 \geq \cdots \geq \bar{\lambda}_n$ are the eigenvalues of the matrix $\bar{C}$,where $\bar{C}$ is a modified matrix of C by replacing diagonal elements c_{ii} with $\bar{c}_{ii}$ $(\bar{c}_{ii} > c_{ii})$,then one can obtain $\bar{\lambda}_i \geq \lambda_i (1 \leq i \leq n)$.

Lemma 2[25]: For any vector $x \in R^n$, if $Q \in R^{n \times n}$ is a symmetric matrix and $P \in R^{n \times n}$ is a positive definite matrix, then one can obtain

$$\lambda_{\min}(P^{-1}Q) x^T P x \leq x^T Q x \leq \lambda_{\max}(P^{-1}Q) x^T P x$$

where $\lambda_{\min}(P^{-1}Q)$ and $\lambda_{\max}(P^{-1}Q)$ respectively denote the minimum and maximum eigenvalues of the matrix $P^{-1}Q$.

3. Pinning Synchronization Scheme based State Observer Using Output Signals

This section proposes a new synchronization scheme based on state observer using output signals, and the synchronization criteria will be derived by Lyapunov stability theory and linear matrix inequality technology.

Suppose that network (4) is the driving network, and select N nodes as the pinning nodes in the response network based on state observer. Without the loss of generality, the first N nodes are selected to be controlled in the state observer. Then the response network can be described as follows:

$$\dot{\hat{x}}_i(t)=\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))[A_i^l\hat{x}_i(t)+\sum_{j=1}^{J}c_{ij}L\hat{z}_j(t)]+u_i(t) \qquad (1\le i\le N)$$
$$\dot{\hat{x}}_i(t)=\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))[A_i^l\hat{x}_i(t)+\sum_{j=1}^{J}c_{ij}L\hat{z}_j(t)] \qquad (N+1\le i\le J)$$
$$\hat{z}_i(t)=\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))(D_i^l\hat{x}_i(t)) \qquad (1\le 1\le J) \qquad (5)$$

Then, one can easily attain the error system between driving network (4) and response network (5), and it can be given by

$$\dot{e}_i(t)=\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))A_i^le_i(t)+\sum_{j=1}^{J}\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))c_{ij}LD_j^le_j(t)+u_i(t) \quad (1\le i\le N)$$
$$\dot{e}_i(t)=\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))A_i^le_i(t)+\sum_{j=1}^{J}\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))c_{ij}LD_j^le_j(t) \qquad (N+1\le i\le J)$$
$$e_{zi}(t)=\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))(D_i^le_i(t)) \qquad (1\le i\le J) \qquad (6)$$

where $e_i(t)=\hat{x}_i(t)-x_i(t)$, $e_{zi}(t)=\hat{z}_i(t)-z_i(t)$.

In order to synchronize driving network (4) and response network (5), the pinning controllers should be designed to guide the error system (6) to achieve asymptotic stability, and that is $\lim_{t\to\infty}\|e_i(t)\|=0$. The fuzzy synchronization controller can be designed by using output signals from each node in the network as follows:

$$u_i(t)=-\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))K_i^lB(\hat{z}_i(t)-z_i(t))=-\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))K_i^lBe_{zi}(t) \qquad (7)$$

where $K_i^l=diag(k_{i1}^l,k_{i2}^l,\cdots,k_{in}^l)\in R^{n\times n}, B\in R^{n\times 1}$.

Remark 1: This synchronization scheme only requires part of the nodes to be controlled, and not all. Different from common synchronization control scheme, the proposed method uses the output signals from the nodes, not the state variables, which is useful for real engineering applications, and can save the resources and cost.

Assume that $\|LD_i^l\|_2=\eta$, and $\mu_{\max}$ denotes the maximal eigenvalue of the matrix $(LD_i^l+(LD_i^l)^T)/2$, $(i=1,2,\cdots,J;l=1,2,\cdots,r_i)$. $\alpha_{\max}^l$ denotes the maximal eigenvalue of the matrix $(A_i^l+(A_i^l)^T)/2$.Let $M_i^{lm}=(K_i^mBD_i^l+(K_i^mBD_i^l)^T)/2(i=1,2,\cdots,N)$,and $\|M_i^{lm}\|_2=\zeta^{lm}$.Suppose that $\hat{\lambda}_1\ge\hat{\lambda}_2\ge\cdots\ge\hat{\lambda}_J$ are the eigenvalues of the matrix $(\hat{C}+\hat{C}^T)/2$,where $\hat{C}$ is the modified matrix of C by replacing the diagonal elements c_{ii} by $(\mu_{\max}/\eta)c_{ii}$.Let $h^l=h_i^l(\xi_{iq}(t))(i=1,2,\cdots,J;l=1,2,\cdots,r_i)$.

Theorem1: For any set of rule l and rule $m(l=1,2,\cdots,r_i;m=1,2,\cdots,r_i)$, if there exists a natural number $N(1\le N<J)$ satisfying $\hat{\lambda}_{N+1}\le -(\alpha_{\max}^{l}/\eta)$, a positive constant k and a constant matrix $B\in R^{n\times 1}$ satisfying the following inequality:

$$\alpha_{\max}^{l}+\eta\hat{\lambda}_i-\zeta^{lm}<0 \qquad 1\le i\le N \tag{8}$$

then the driving network (4) and the response network (5) can be asymptotically synchronized under the fuzzy pinning synchronization controller (7).
Proof: Construct a Lyapunov candidate as below:

$$V(t)=\frac{1}{2}\sum_{i=1}^{J}e_i^T(t)e_i(t) \tag{9}$$

Substituting (7) into (6), and then the differential coefficient of $V(t)$ can be described by

$$\begin{aligned}\dot{V}(t)&=\frac{1}{2}\sum_{i=1}^{J}(\dot{e}_i^T(t)e_i(t)+e_i^T(t)\dot{e}_i(t))\\&=\frac{1}{2}\sum_{i=1}^{J}\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))(e_i^T(t)(A_i^l+(A_i^l)^T)e_i(t))\\&+\frac{1}{2}\sum_{i=1}^{J}\sum_{j=1}^{J}\sum_{l=1}^{r_i}h_i^l(\xi_{iq}(t))c_{ij}(e_i^T(t)LD_i^le_j(t)+e_j^T(t)(LD_i^l)^Te_i(t))\\&-\frac{1}{2}\sum_{i=1}^{N}\sum_{l=1}^{r_i}\sum_{m=1}^{r_i}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))e_i^T(t)(K_i^mBD_i^l+(K_i^mBD_i^l)^T)e_i(t)\end{aligned} \tag{10}$$

For any set of rule l and rule $m(l=1,2,\cdots,r_i;m=1,2,\cdots,r_i)$, let

$$\begin{aligned}H^{lm}&=\sum_{i=1}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))e_i^T(t)(\frac{A_i^l+(A_i^l)^T}{2})e_i(t)\\&-\sum_{i=1}^{N}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))e_i^T(t)(\frac{K_i^mBD_i^l+(K_i^mBD_i^l)^T}{2})e_i(t)\\&+\frac{1}{2}\sum_{i=1}^{J}\sum_{j=1}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))c_{ij}(e_i^T(t)LD_i^le_j(t)+e_j^T(t)(LD_i^l)^Te_i(t))\end{aligned} \tag{11}$$

By using Lemma 1 and Lemma 2, one can easily attain

$$\begin{aligned}H^{lm}&\le\sum_{i=1}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))\lambda_{\max}(\frac{A_i^l+(A_i^l)^T}{2})e_i^T(t)e_i(t)\\&-\sum_{i=1}^{N}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))\left\|\frac{K_i^mBD_i^l+(K_i^mBD_i^l)^T}{2}\right\|_2e_i^T(t)e_i(t)\\&+\frac{1}{2}\sum_{i=1}^{J}\sum_{j=1}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))c_{ij}(e_i^T(t)LD_i^le_j(t)+e_j^T(t)(LD_i^l)^Te_i(t))\\&=h^lh^m(\alpha^l-M^{lm})e^Te\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\sum_{i=1}^{J}\sum_{j=1,j\neq i}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))c_{ij}(e_i^T(t)LD_i^le_j(t)+e_j^T(t)(LD_i^l)^Te_i(t))\\
&+\sum_{i=1}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))e_i^T(t)c_{ii}(\frac{LD_i^l+(LD_i^l)^T}{2})e_i(t)\\
\le\ & h^lh^m(\alpha^l-M^{lm})e^Te\\
&+\frac{1}{2}\sum_{i=1}^{J}\sum_{j=1,j\neq i}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))c_{ij}(\|e_i^T(t)\|_2\,\eta\,\|e_j(t)\|_2+\|e_j^T(t)\|_2\,\eta\,\|e_i(t)\|_2)\\
&+\sum_{i=1}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))\lambda_{\max}(\frac{LD_i^l+(LD_i^l)^T}{2})c_{ii}e_i^T(t)e_i(t)\\
=\ & h^lh^m(\alpha^l-M^{lm})e^Te\\
&+\frac{1}{2}\sum_{i=1}^{J}\sum_{j=1,j\neq i}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))(\|e_i^T(t)\|_2\|e_j(t)\|_2+\|e_j^T(t)\|_2\|e_i(t)\|_2)\eta c_{ij}\\
&+\sum_{i=1}^{J}h_i^l(\xi_{iq}(t))h_i^m(\xi_{iq}(t))\mu_{\max}c_{ii}e_i^T(t)e_i(t)\\
=\ & h^lh^me^T(\alpha^l+\eta\frac{\hat{C}+\hat{C}^T}{2}-M^{lm})e
\end{aligned}
\tag{12}
$$

where $\alpha^l=diag(\underbrace{\alpha_{\max}^l,\alpha_{\max}^l,\cdots,\alpha_{\max}^l}_{J})$

$,M^{lm}=diag(\underbrace{\zeta^{lm},\zeta^{lm},\cdots,\zeta^{lm}}_{N},\underbrace{0,0,\cdots,0}_{J-N})\in R^{J\times J}$, $e=(\|e_1(t)\|_2,\|e_2(t)\|_2,\cdots,\|e_J(t)\|_2)^T$.

Since α^l and M^{lm} are both diagonal matrices, and $(\hat{C}+\hat{C}^T)/2$ is a real symmetric matrix, there must exist an orthogonal matrix P satisfying

$$
\begin{aligned}
H^{lm}&\le h^lh^me^T(\alpha^l+\eta\frac{\hat{C}+\hat{C}^T}{2}-M^{lm})e\\
&=(Pe)^T diag\{(\alpha_{\max}^l+\eta\hat{\lambda}_1-\zeta^{lm}),\cdots,(\alpha_{\max}^l+\eta\hat{\lambda}_N-\zeta^{lm}),\\
&\quad(\alpha_{\max}^l+\eta\hat{\lambda}_{N+1}),\cdots,(\alpha_{\max}^l+\eta\hat{\lambda}_J)\}(Pe)\\
&=(Pe)^TQ(Pe)
\end{aligned}
\tag{13}
$$

where

$Q=diag\{(\alpha_{\max}^l+\eta\hat{\lambda}_1-\zeta^{lm}),\cdots,(\alpha_{\max}^l+\eta\hat{\lambda}_N-\zeta^{lm}),(\alpha_{\max}^l+\eta\hat{\lambda}_{N+1}),\cdots,(\alpha_{\max}^l+\eta\hat{\lambda}_J)\}$.

According to the condition of Theorem1, all the diagonal elements of matrix Q are negative, and so Q is a negative define matrix. Therefore, $Pe\to 0$ as $t\to\infty$. Since P is a orthogonal matrix, $e\to 0$ as $t\to\infty$. Then one can obtain

$$\dot{V}(t)=\sum_{l=1}^{r_i}\sum_{m=1}^{r_i}H^{lm}<0 \tag{14}$$

From (13) and (14), one can conclude that error system is asymptotically stable under the fuzzy controller (7).That is, the driving network and response network can be asymptotically synchronized by the pinning synchronization controller (7).

Thus the proof is completed.

4. Numerical Simulation

In this section, a numerical simulation example will be taken to verify the effectiveness of the proposed synchronization scheme. Consider a complex dynamical network consisting of three sub-systems described by T-S fuzzy model as follows:

$$\begin{cases} If\ x_{11}(t)\ is\ M_{i1}^l\ , \\ Then\ \dot{x}_i(t) = A_i^l x_i(t) + \sum_{j=1}^{3} c_{ij} L z_j(t) \\ z_i(t) = D_i^l x_i(t), i = 1,2,3, l = 1,2. \end{cases} \tag{15}$$

The complex dynamical network parameters are showed as below:
The membership functions are $h_i^1(t) = [1+\cos(x_{11}(t))]/2$, $h_i^2(t) = [1-\cos(x_{11}(t))]/2$, $i = 1,2,3$.

The system parameters are chosen as below:

$$A_i^1 = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}, D_i^1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, A_i^2 = \begin{bmatrix} -3.4 & 2.1 \\ 2.1 & -1.2 \end{bmatrix}, D_i^2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, L = \begin{bmatrix} 3 \\ 6 \end{bmatrix},$$

$$C = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}, i = 1,2,3.$$

One can easily attain

$$\alpha_{\max}^1 = \lambda_{\max}\left(\frac{A_i^1 + (A_i^1)^T}{2}\right) = 0.2361, \alpha_{\max}^2 = \lambda_{\max}\left(\frac{A_i^2 + (A_i^2)^T}{2}\right) = 0.0707,$$

$$\eta = \left\| LD_i^l \right\|_2 = 6.7082, \mu_{\max} = \lambda_{\max}((LD_i^l + (LD_i^l)^T)/2) = 4.8541, i = 1,2,3; l = 1,2,$$

$$\hat{C} = \begin{bmatrix} -2*\mu_{\max}/\eta & 1 & 1 \\ 1 & -2*\mu_{\max}/\eta & 1 \\ 1 & 1 & -2*\mu_{\max}/\eta \end{bmatrix} = \begin{bmatrix} -1.4472 & 1 & 1 \\ 1 & -1.4472 & 1 \\ 1 & 1 & -1.4472 \end{bmatrix},$$

then, $\hat{\lambda}_1 = 0.5528, \hat{\lambda}_2 = -2.4472, \hat{\lambda}_3 = -2.4472$.

According to the Theorem 1, there exists a natural number $N = 1$, satisfying $\hat{\lambda}_{N+1} = -2.4472 \le -(\alpha_{\max}^1/\eta) = -0.0352$, $\hat{\lambda}_{N+1} = -2.4472 \le -(\alpha_{\max}^2/\eta) = -0.0105$. That means only one controller is required to pin two complex networks to synchronization. The controller parameters are chosen as below:

$$K_1^1 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, K_1^2 = \begin{bmatrix} 5.5 & 0 \\ 0 & 5.5 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

By calculating the parameters above, the designed synchronization pinning controller satisfies (8) according to the Theorem 1. Let the initial synchronization errors be $e_1(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T, e_2(0) = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}^T, e_3(0) = \begin{bmatrix} 0.7 & 0.7 \end{bmatrix}^T$.

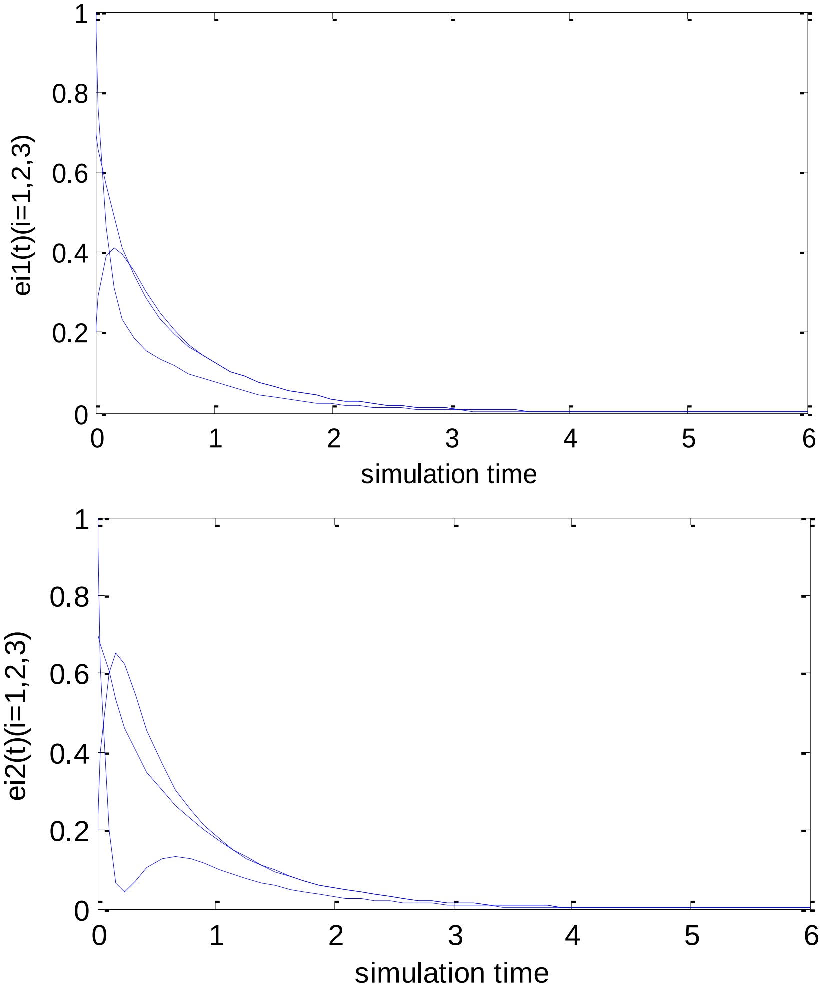

Figure 3. Synchronization Errors between Two Fuzzy Complex Dynamical Networks

The simulation results are shown in Figure 3. It is obvious that synchronization errors rapidly approach zero, and finally two complex dynamical networks achieve synchronization.

5. Conclusion

A new pinning synchronization control scheme based on state observer using output signals has been proposed in this paper. It only requires part of the nodes in the network to be controlled and output signals to be transmitted from driving network to response network. That is practical and economical for real engineering application. By Lyapunov stability theory and linear matrix inequality technology, the synchronization criteria has been derived and given. A numerical simulation example has been taken to verify the effectiveness of the proposed pinning synchronization control scheme. In the future, we will extend our proposed scheme, and further study the outer synchronization control problem with different network structure and outer synchronization robust control problem.

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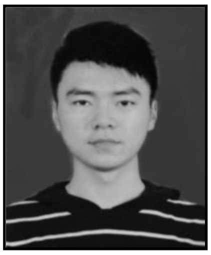

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