International Journal of Control and Automation
Vol.8, No.5 (2015), pp.1-12
http://dx.doi.org/10.14257/ijca.2015.8.5.01

Multisensor Distributed Fusion Wiener Deconvolution Estimator for Linear Stochastic Multichannel ARMA Signal

Yun Li[1,2], Ming Zhao[1], Gang Hao[2]*, Junling Li[1], Hao Jin[1]

1 School of Computer and Information Engineering
Harbin University of Commerce
HeiLongJiang, Harbin, 150001, China
2 Electronic Engineering Institute*
Heilongjiang University
Heilongjiang, Harbin, 150080, China
liyunhd@hrbcu.edu.cn[1], haogang@hlju.edu.cn[2]

Abstract

Multisensor distributed fusion Wiener deconvolution estimator is presented in this paper. It does not need to solve the Diophantine equation, and the steady-state Kalman filter gain of the augmented system. It can handle processing of nonstationary signal. White noise estimator and Aström predictor are used in the algorithm. Gevers-Wouters (G-W) algorithm is also used in this paper. In order to improve the estimation precision, multisensor information fusion Wiener deconvolution estimator for multichannel system is proposed in this paper by using the modern time series analysis method. The information fusion algorithms included matrix weighted, diagonal matrices weighted, scalar weighted and covariance intersection fusion in this paper. Under the linear minimum variance optimal information fusion criterion, the calculation formula of optimal weighting coefficients have be given. The algorithm analyzes the relationship between the accuracy and the computation complexities of four fusion algorithms. Compared with the single sensor case, the accuracy of the fused filter is greatly improved. It can be applied in signal processing, communication, control field and other fields. A simulation example for multichannel ARMA signal shows its correctness and effectiveness.

***Keywords**: Distributed information fusion, Multichannel ARMA signal, wiener deconvolution estimator*

1. Introduction

Signal estimation problems often arise in many application problems. For example, it includes signal filtering, smoothing and prediction problems, estimation of signal deconvolution, white noise deconvolution problem etc. Signal estimation can be regarded as a special form of state estimation [1-3]. For example, estimation of ARMA signals can be transformed into a state estimation problem, while the signal is the component of the state. How to find the optimal information fusion signal estimators has important theoretical and practical significance when there are multiple sensors measurement signals.

At present, the research and development of multisensor information fusion technology have been attached great importance. In last decades, the application field of information fusion technology has increased widely. Its fields include guidance, GPS, defense, medical, integrated navigation, target tracking, robot technology, communications and signal processing [4-6].

Multichannel optimal estimator problems occur widely in the field of signal processing, communication and control. Multichannel Wiener estimator that use the

*Corresponding author: Gang Hao

ISSN: 2005-4297 IJCA

polynomial method is proposed by Robers, Newmann, Ahlen, Sternad and Grimble [7-9]. The polynomial method will eventually come down problem to spectral factorization and solve Diophantine equation. The previous results need assume that the signal and noise is stationary time series. While the other previous results need solve two coupling Diophantine equation. The optimal multichannel optimal estimator and deconvolution estimator require the solution of Diophantine equation or require the calculation of gain of steady-state Kalman estimators for the augmented system, so require the estimated signal being the stationary time series. In this paper, multisensor information fusion Wiener deconvolution estimator for multichannel system is proposed by using the modern time series analysis method.

It takes ARMA model and Lyapunov equation as the basic tool, and avoids solving the Riccati equation. The theory is only suitable for dealing with the constant system optimal information fusion state or signal estimation problems, and it can't handle the time-varying system. The advantage of this theory is that it can handle the optimal fusion estimation problem by the classical Kalman filtering method, and can get the new results of different forms completely. It also can handle the problems that the classical Kalman filtering method is not easily solved and unsolved, and can get many new results. The characteristic of this theory is to calculate the gain matrix of steady Kalman estimators based on the ARMA innovation model, and it avoids the Riccati equation. It can calculate the error variance matrix of local sensor state or signal estimation by the Lyapunov equation, or can calculate the state or signal estimation error covariance and cross covariance matrix by calculating the cross covariance matrix of the process noise, measurement noise and innovation directly. Another advantage of this theory is that it can handle the self-tuning information fusion state or signal estimators with unknown model parameters and noise statistics system.

In this paper, distributed fusion Wiener deconvolution estimator is weighted by matrix, diagonal matrices, scalars, covariance intersection fusion for linear stochastic multichannel ARMA signal [10-13]. The algorithm is under the linear minimum variance sense, and the optimal information fusion criterion includes weighted by matrix, diagonal matrices, scalars, covariance intersection fusion. For matrix, diagonal matrices, scalars, covariance intersection fusion, the accuracies of above four kinds of weighted fusion estimator are from high to low. But the computational burden is on the contrary. Fusion estimator weighted by matrix has a large computational burden, and weighted by covariance intersection fusion with minimal computational burden, and it is suitable for real-time applications. A simulation example for linear stochastic multichannel ARMA signal with 2-sensor shows the correctness and validity of training.

The main structure of this paper is as follows: Problem formulation is given in Section 2. Local optimal Wiener deconvolution estimator is obtained in Section 3. In Section 4 distributed information fusion optimal Wiener deconvolution estimator for linear stochastic multichannel ARMA signal is presented. A simulation example with 2-sensor is given is Section 5. In Section 6 the conclusions of this paper are given.

2. Problem Formulation

Consider the multisensor discrete-time linear time-invariant multichannel system

$$A(q^{-1})s(t) = B(q^{-1})w(t) \tag{1}$$

$$C^{(i)}(q^{-1})y_i(t) = D^{(i)}(q^{-1})s(t) + E^{(i)}(q^{-1})v_i(t),\ i = 1,\cdots,l \tag{2}$$

Where l is the number of the sensor, $l \geq 2$. And $s(t) \in R^n$ is the ARMA signal to be estimated, $y_i(t) \in R^{mi}$ is the measurement (output) of the *i*th sensor subsystem, $v_i(t) \in R^{mi}$ is the measurement noise of the *i*th sensor subsystem, $w(t) \in R^r$ is the input noise, $A(q^{-1})$, $B(q^{-1})$, $C^{(i)}(q^{-1})$, $D^{(i)}(q^{-1})$, $E^{(i)}(q^{-1})$ is the known constant matrix polynomial.

Assumption 1 $w(t) \in R^r$ and $v_i(t) \in R^{mi}, i = 1, \cdots, L$ are independence white noises with zero mean and covariance are Q_w and Q_{vi} individually.

$$\mathrm{E}\left\{\begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix}\begin{bmatrix} w(k) & v_i(k) \end{bmatrix}\right\} = \begin{bmatrix} Q_w & 0 \\ 0 & Q_{vi} \end{bmatrix}\delta_{tk} \tag{3}$$

where E is the mathematical expectation, $\delta_{tt} = 1, \delta_{tk} = 0 (t \neq k)$.

Assumption 2 The initial observation time $t_0 = -\infty$.

The aim is to obtain the local optimal Wiener deconvolution estimator $\hat{s}_i(t \mid t+N), i = 1, 2 \cdots l$ and the optimal distributed fusion Wiener deconvolution estimator $\hat{s}_0(t \mid t+N)$ based on the measurement $(y_i(t+N)), y_i(t+N-1), \cdots), N \geq 0$. For $N = 0$, $N > 0$ or $N < 0$, we referred to as Wiener deconvolution filter, smooth or predictor.

3. Local optimal Wiener Deconvolution Estimator

From (1) and (2) having

$$y_i(t) = C^{(i)-1}(q^{-1})D^{(i)}(q^{-1})A^{-1}(q^{-1})B(q^{-1})w(t) + C^{(i)-1}(q^{-1})E^{(i)}(q^{-1})v_i(t) \tag{4}$$

where q^{-1} is the unit delay operator. And letting $X^{(i)-1}(q^{-1}) = (X^{(i)}(q^{-1}))^{-1}$. This leads to left coprime factorization

$$[C^{(i)-1}(q^{-1})D^{(i)}(q^{-1})A^{-1}(q^{-1})B(q^{-1}), C^{(i)-1}(q^{-1})E^{(i)}(q^{-1})] = \beta^{(i)-1}(q^{-1})[\bar{D}^{(i)}(q^{-1}), \bar{E}^{(i)}(q^{-1})] \tag{5}$$

Substituting (5) into (4), having

$$y_i(t) = \beta^{(i)-1}(q^{-1})[\bar{D}^{(i)}(q^{-1}), \bar{E}^{(i)}(q^{-1})]\begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix} \tag{6}$$

This leads to

$$\beta^{(i)}(q^{-1})y_i(t) = \bar{D}^{(i)}(q^{-1})w(t) + \bar{E}^{(i)}(q^{-1})v_i(t) \tag{7}$$

Assuming that $[\bar{D}^{(i)}(q^{-1}), \bar{E}^{(i)}(q^{-1})]$ is left prime or no determinant zero in the unit circle on the left factor, while having the ARMA innovation model

$$\beta^{(i)}(q^{-1})y_i(t) = \alpha^{(i)}(q^{-1})\varepsilon_i(t) \tag{8}$$

where $\alpha^{(i)}(q^{-1})$ is stable, $\alpha_0^{(i)} = 1$, and innovation $\varepsilon_i(t)$ is white noises with zero mean and covariance are $Q_{\varepsilon i}$, and having

$$\alpha^{(i)}(q^{-1})\varepsilon_i(t) = \bar{D}^{(i)}(q^{-1})w(t) + \bar{E}^{(i)}(q^{-1})v_i(t) \tag{9}$$

$\alpha^{(i)}(q^{-1})$ and $Q_{\varepsilon i}$ can be computed by Gevers-Wouters [14].

Lemma 1 For the multichannel system (1) and (2) under the Assumption 1-2, the asymptotically stability local optimal white wiener estimator $\hat{w}_i(t \mid t+N)$ of the *i*th sensor is as follows [14]

$$\hat{w}(t \mid t+N) = F_N^{(i)w}(q^{-1})\tilde{\beta}^{(i)}(q^{-1})\tilde{\alpha}^{(i)-1}(q^{-1})y_i(t+N) \tag{10}$$

$$\hat{v}_i(t \mid t+N) = F_N^{(i)v}(q^{-1})\tilde{\beta}^{(i)}(q^{-1})\tilde{\alpha}^{(i)-1}(q^{-1})y_i(t+N) \tag{11}$$

having the pseudo exchange

$$\alpha^{(i)-1}(q^{-1})\beta^{(i)}(q^{-1}) = \tilde{\beta}^{(i)}(q^{-1})\tilde{\alpha}^{(i)-1}(q^{-1}) \tag{12}$$

where $\tilde{\beta}_0^{(i)} = I_m$, $\tilde{\alpha}_0^{(i)} = I_m$, and $\det\tilde{\alpha}^{(i)}(q^{-1}) = \det\alpha^{(i)}(q^{-1})$, $n_\alpha = n_{\tilde{\alpha} i}$, $n_\beta = n_{\tilde{\beta} i}$.

$$F_N^{(i)w}(q^{-1}) = \sum_{j=0}^{N} Q_w G_j^{(i)\mathrm{T}} Q_{\varepsilon i}^{-1} q^{j-N}, \quad N \ge 0 \tag{13}$$

$$F_N^{(i)w}(q^{-1}) = 0, \quad N < 0 \tag{14}$$

$$F_N^{(i)v}(q^{-1}) = \sum_{j=0}^{N} Q_{vi} H_j^{(i)\mathrm{T}} Q_{\varepsilon i}^{-1} q^{j-N}, \quad N \ge 0 \tag{15}$$

$$F_N^{(i)v}(q^{-1}) = 0, \quad N < 0 \tag{16}$$

The coefficient $G_j^{(i)}$ and $H_j^{(i)}$ can be recursive calculated as

$$G_j^{(i)} = -\alpha_1^{(i)}G_{j-1}^{(i)} - \cdots - \alpha_{n_{\alpha i}}^{(i)}G_{j-n_{\alpha i}}^{(i)} + \bar{D}_j^{(i)} \tag{17}$$

$$H_j^{(i)} = -D_1^{(i)}H_{j-1}^{(i)} - \cdots - -D_{n_{\alpha i}}^{(i)}H_{j-n_{\alpha i}}^{(i)} + \bar{E}_j^{(i)} \tag{18}$$

where letting $G_j^{(i)} = 0(j<0)$, $H_j^{(i)} = 0(j<0)$, $\bar{D}_j^{(i)} = 0(j > n_{\bar{d}i})$, $\bar{E}_j^{(i)} = 0(j > n_{\bar{e}i})$.

Lemma 2 The ARMA innovation model (6) has the unified $Astr\ddot{o}m$ predictor [14]

$$\hat{y}_i(t \mid t+N) = \psi_{-N}^{(i)}(q^{-1})\tilde{\alpha}^{(i)-1}(q^{-1})y_i(t+N) \tag{19}$$

where $\psi_N^{(i)}(q^{-1})$ is defined as

$$\tilde{\alpha}^{(i)}(q^{-1}) = \chi_N^{(i)}(q^{-1})\tilde{\beta}^{(i)}(q^{-1}) + q^{-N}\psi_N^{(i)}(q^{-1}), \quad N > 0 \tag{20}$$

$$\chi_N^{(i)}(q^{-1}) = I_m + \chi_1^{(i)}q^{-1} + \cdots + \chi_{N-1}^{(i)}q^{-(N-1)} \tag{21}$$

$$\psi_N^{(i)}(q^{-1}) = \tilde{\alpha}^{(i)}(q^{-1})q^N, \quad N \le 0 \tag{22}$$

$$\psi_j^{(i)}(q^{-1}) = \psi_0^{(i)} + \psi_1^{(i)}q^{-1} + \cdots + \psi_{n_{ji}}^{(i)}q^{-n_{ji}}, \quad N > 0$$

$$n_{\psi i} = \max(\tilde{n}_{\beta i} - 1, \tilde{n}_{\alpha i} - N) \tag{23}$$

$F_N^{(i)w}(q^{-1})$、$F_N^{(i)v}(q^{-1})$、$\tilde{\beta}^{(i)}(q^{-1})$ and $\tilde{\alpha}^{(i)}(q^{-1})$ can be computed by **Lemma 1**.

Predictor error covariance $\tilde{y}_i(t \mid t+N)$ can be Calculated as

$$\tilde{y}_i(t \mid t+N) = y_i(t) - \hat{y}_i(t \mid t+N) = E_{-N}^{(i)}(q^{-1})\varepsilon_i(t) \tag{24}$$

Lemma 3 For the multichannel system (1) and (2) under the Assumption 1-2, signal $s(t)$ has non recursive expression [14]

$$s(t)=\sum_{j=0}^{n_{ci}-1}\varpi_j^{(ia)}B(q^{-1})w(t-j)+\sum_{j=0}^{n_a-1}\varpi_j^{(ib)}[C^{(i)}(q^{-1})y_i(t-j)-E^{(i)}(q^{-1})v_i(t-j)],\quad i=1,\cdots,l \tag{25}$$

where $\Omega_j^{(ia)}$ is a $n\times n$ matrix, $\Omega_j^{(ib)}$ is a $n\times m$ matrix, and it is composed of the following block representation:

$$[\varpi_0^{(ia)}\cdots\varpi_{n_{ci}-1}^{(ia)}\varpi_0^{(ib)}\cdots\varpi_{n_a-1}^{(ib)}]=[I_n\ 0\cdots 0]\varpi_i^{+} \tag{26}$$

where ϖ_i^{+} is the pseudo inverse of ϖ_i, and ϖ_i^{+} is defined as

$$\varpi_i^{+}=(\varpi^{\mathrm{T}}\varpi)^{-1}\varpi_i^{\mathrm{T}} \tag{27}$$

And a non-recursive optimal deconvolution filter is obtained as follows:

$$\hat{s}_i(t|t+N)=\sum_{j=0}^{n_{ci}-1}\varpi_j^{(ia)}\sum_{k=0}^{n_b}B_k\hat{w}(t-j-k|t+N)+\sum_{j=0}^{n_a-1}\varpi_j^{(ib)}[\sum_{k=0}^{n_{ci}}C_k^{(i)}\hat{y}_i(t-j-k|t+N)$$
$$-\sum_{k=0}^{n_{ei}}E_k^{(i)}\hat{v}_i(t-j-k|t+N)] \tag{28}$$

Lemma 4 The following formula is established [14]

$$\mathrm{E}[v_i(t)\varepsilon_i^{\mathrm{T}}(j)]=Q_{vi}H_{j-t}^{\mathrm{T}} \tag{29}$$
$$\mathrm{E}[w(t)\varepsilon_i^{\mathrm{T}}(j)]=Q_wG_{j-t}^{\mathrm{T}} \tag{30}$$

Theorem 1 For the multichannel system (1) and (2) under the Assumption 1-2, the asymptotically stability local optimal wiener deconvolution estimator $\hat{s}_i(t|t+N)$ of the *i*th sensor is as follows

$$\hat{s}_i(t|t+N)=\lambda_N^{(i)}(q^{-1})\tilde{\alpha}^{(i)-1}(q^{-1})y_i(t+N) \tag{31}$$

or having ARMA recursive form

$$\det\tilde{\alpha}^{(i)}(q^{-1})\hat{s}_i(t|t+N)=\lambda_N^{(i)}(q^{-1})\,\mathrm{adj}\,\tilde{\alpha}^{(i)}(q^{-1})y_i(t+N) \tag{32}$$

where the matrix polynomial $\lambda_N^{(i)}(q^{-1})$ is defined as

$$\lambda_N^{(i)}(q^{-1})=\sum_{j=0}^{n_{ci}-1}\varpi_j^{(ia)}\sum_{k=0}^{n_b}B_kF_{N+j+k}^{(i)w}(q^{-1})\tilde{\beta}^{(i)}(q^{-1})+\sum_{j=0}^{n_a-1}\varpi_j^{(ib)}[\sum_{k=0}^{n_{ci}}C_k^{(i)}\psi_{-j-k-N}^{(i)}(q^{-1})$$
$$-\sum_{k=0}^{n_{ei}}E_k^{(i)}F_{N+j+k}^{(i)v}(q^{-1})\tilde{\beta}^{(i)}(q^{-1})] \tag{33}$$

Proof: Substitutting (10), (11) and (19) into (28), (31) ~ (33) are obtained. Because of the stability of $\alpha_i(q^{-1})$ leads to (31) and (32) are asymptotically stable.

The proof is completed.

Theorem 2 For the multichannel system (1) and (2) under the Assumption 1-2, the local steady-state optimal estimation error covariance $P_i(N) = \mathrm{E}[\tilde{s}_i(t|t+N)\tilde{s}_i^{\mathrm{T}}(t|t+N)]$ is given as

$$P_i(N) = \sum_{r=0}^{n_0+N}\sum_{s=0}^{n_0+N}\begin{bmatrix}\varpi_r^{(i1)} & \varpi_r^{(i2)} & \varpi_r^{(i3)}\end{bmatrix}\begin{bmatrix} Q_w\delta_{rs} & 0 & Q_w G_{r+s-n_0}^{(i)\mathrm{T}} \\ 0 & Q_{vi}\delta_{rs} & Q_{vi}H_{r+s-n_0}^{(i)\mathrm{T}} \\ G_{r+s-n_0}^{(i)}Q_w & H_{r+s-n_0}^{(i)}Q_{vi} & Q_{\varepsilon i}\delta_{rs}\end{bmatrix}\begin{bmatrix}\varpi_s^{(i1)\mathrm{T}} \\ \varpi_s^{(i2)\mathrm{T}} \\ \varpi_s^{(i3)\mathrm{T}}\end{bmatrix},\quad N \ge 0 \quad (34)$$

$$P_i(N) = \sum_{r=0}^{n_3}\sum_{s=0}^{n_3}\begin{bmatrix}\varpi_r^{(i3)} & \varpi_r^{(i4)} & \varpi_r^{(i6)}\end{bmatrix}\begin{bmatrix} Q_w\delta_{rs} & 0 & Q_w G_{r-s}^{(i)\mathrm{T}} \\ 0 & Q_{vi}\delta_{rs} & Q_{vi}H_{r-s}^{(i)\mathrm{T}} \\ G_{s-r}^{(i)}Q_w & H_{s-r}^{(i)}Q_{vi} & Q_{\varepsilon i}\delta_{rs}\end{bmatrix}\begin{bmatrix}\varpi_s^{(i3)\mathrm{T}} \\ \varpi_s^{(i4)\mathrm{T}} \\ \varpi_s^{(i6)\mathrm{T}}\end{bmatrix},\quad N < 0 \quad (35)$$

Proof: By (25) and (28) having

$$\hat{s}_i(t|t+N) = \sum_{j=0}^{n_0+N}[\varpi_j^{(i1)}w(t-j) + \varpi_j^{(i2)}v_i(t-j) + \varpi_j^{(i3)}\varepsilon_i(t-n_0+j)],\quad N \ge 0 \quad (36)$$

$$\tilde{s}_i(t|t+N) = \sum_{j=0}^{n_3}[\varpi_j^{(i1)}w(t-j) + \varpi_j^{(i2)}v_i(t-j) + \varpi_j^{(i3)}\varepsilon_i(t-j)],\quad N < 0 \quad (37)$$

where setting $\varpi_j^{(i1)} = 0(j > n_1)$, $\varpi_j^{(i2)} = 0(j > n_2)$.

By (36) and (37) can be written as

$$\tilde{s}_i(t|t+N) = \sum_{j=0}^{n_0+N}\begin{bmatrix}\varpi_j^{(i1)} & \varpi_j^{(i2)} & \varpi_j^{(i3)}\end{bmatrix}\begin{bmatrix} w(t-j) \\ v_i(t-j) \\ \varepsilon_i(t-n_0+j)\end{bmatrix},\quad N \ge 0 \quad (38)$$

$$\tilde{s}_i(t|t+N) = \sum_{j=0}^{n_3}\begin{bmatrix}\varpi_j^{(i1)} & \varpi_j^{(i2)} & \varpi_j^{(i3)}\end{bmatrix}\begin{bmatrix} w(t-j) \\ v_i(t-j) \\ \varepsilon_i(t-j)\end{bmatrix},\quad N < 0 \quad (39)$$

From (38), (39) and **Lemma 4**, (34) and (35) are obtained.

The proof is completed.

Theorem 3 For the multichannel system (1) and (2) under the Assumption 1-2, the local steady-state optimal estimation error cross covariance $P_{ij}(N) = \mathrm{E}[\tilde{s}_i(t|t+N)$ $\tilde{s}_j(t|t+N)]$ $(i \ne j)$ between any two local sensor is computed by

$$P_{ij}(N) = \sum_{r=0}^{n_0+N}\sum_{s=0}^{n_0+N}\begin{bmatrix}\varpi_r^{(i1)} & \varpi_r^{(i2)} & \varpi_r^{(i3)}\end{bmatrix}\begin{bmatrix} Q_w\delta_{rs} & 0 & Q_w G_{r+s-n_0}^{(j)\mathrm{T}} \\ 0 & 0 & 0 \\ G_{r+s-n_0}^{(i)}Q_w & 0 & \Re_{ij}(r-n_0, s-n_0)\end{bmatrix}\begin{bmatrix}\varpi_s^{(j1)\mathrm{T}} \\ \varpi_s^{(j2)\mathrm{T}} \\ \varpi_s^{(j3)\mathrm{T}}\end{bmatrix},\quad N \ge 0, \quad (40)$$

$$\Re_{ij}(r-n_0, s-n_0) = \sum_{k=0}^{\infty} G_k^{(i)}Q_w G_{s+k-r}^{(j)\mathrm{T}} \quad (41)$$

$$P_{ij}(N) = \sum_{r=0}^{n_3}\sum_{s=0}^{n_3}\begin{bmatrix}\varpi_r^{(i1)} & \varpi_r^{(i2)} & \varpi_r^{(i3)}\end{bmatrix}\begin{bmatrix} Q_w\delta_{rs} & 0 & Q_w G_{r-s}^{(j)\mathrm{T}} \\ 0 & 0 & 0 \\ G_{s-r}^{(i)}Q_w & 0 & \Re_{ij}(-r, -s)\end{bmatrix}\begin{bmatrix}\varpi_s^{(j1)\mathrm{T}} \\ \varpi_s^{(j2)\mathrm{T}} \\ \varpi_s^{(j3)\mathrm{T}}\end{bmatrix},\quad N < 0 \quad (42)$$

$$\Re_{ij}(-r, -s) = \sum_{k=0}^{\infty} G_k^{(i)}Q_w G_{k+r-s}^{(j)\mathrm{T}} \quad (43)$$

Proof: By the (9) with expansion [14]:

$$\varepsilon_i(t) = \sum_{k=0}^{\infty} G_k^{(i)} w(t-k) + \sum_{k=0}^{\infty} H_k^{(i)} v_i(t-k) \tag{44}$$

So having

$$\mathrm{E}[\varepsilon_i(t-r)\varepsilon_j^{\mathrm{T}}(t-s)] = \sum_{k=0}^{\infty} G_k^{(i)} Q_w G_{k+r-s}^{(j)\mathrm{T}} \tag{45}$$

Defining

$$\Re_{ij}(-r,-s) = \sum_{k=0}^{\infty} G_k^{(i)} Q_w G_{k+r-s}^{(j)\mathrm{T}} \tag{46}$$

Therefore the following equation is established

$$\Re_{ij}(r-n_0, s-n_0) = \sum_{k=0}^{\infty} G_k^{(i)} Q_w G_{k+s-r}^{(j)\mathrm{T}} \tag{47}$$

By (38), (39), (46) and (47), so (40) and (43) are obtained.
The proof is completed.

4. Distributed Information fusion optimal Wiener deconvolution estimator

Theorem 4 The multisensor discrete-time linear time-invariant stochastic multichannel system (1) and (2), under the Assumption 1-2, and the optimal fused Wiener deconvolution estimator $\hat{s}_0(t \mid t+N)$ in the linear minimum variance sense is given as [14-15]

$$\hat{s}_0(t \mid t+N) = \sum_{j=1}^{K} \Im_j(N) \hat{s}_j(t \mid t+N) \tag{48}$$

The optimal fusion weight according to the different weighted fusion algorithm has different calculation formula is as follows.

When weighted criteria are weighted by matrices, the weighted coefficients are

$$[\Im_1(N), \cdots, \Im_K(N)] = (e^{\mathrm{T}} P^{-1}(N) e)^{-1} e^{\mathrm{T}} P^{-1}(N) \tag{49}$$

where $P_{\mathrm{tr}} = (\mathrm{tr}\, P_{ij})_{L \times L}$ is a $L \times L$ matrix, P_{ij}, $i, j = 1, 2, \cdots, L$ can be calculated by (40) ~(43), and $e^{\mathrm{T}} = [I_m \quad I_m \quad \cdots \quad I_m]$ is a $L \times 1$ row vector.

The optimal fused variance matrix is given as

$$P_0(N) = (e^{\mathrm{T}} P^{-1}(N) e)^{-1} \text{ and } \mathrm{tr}\, P_0 \le \mathrm{tr}\, P_j, \quad j = 1, 2, \cdots, K \tag{50}$$

When weighted criteria are weighted by diagonal matrices, the optimal weighting coefficients $\Im_j = \mathrm{diag}(\Im_{jl})$, $l = 1, \cdots, n$,while $\Im_{jl}$ are given by

$$\Im_j = [\Im_{1j}, \Im_{2j}, \cdots, \Im_{Lj}], \quad j = 1, \cdots, n \tag{51}$$

$$\Im_j = (e^{\mathrm{T}} P^{jj}(t \mid t)^{-1} e)^{-1} e^{\mathrm{T}} P^{jj}(t \mid t)^{-1}, \quad j = 1, \cdots, n \tag{52}$$

where $e = [1 \ \cdots \ 1]^{\mathrm{T}}$ is a $L \times 1$ row vector, and $L \times L$ matrices is defined as $P^{jj}(t \mid t) = (P_{lk}^{jj}(t \mid t)), \quad l,k = 1,2,\cdots,L$, $P_{lk}^{jj}(t \mid t)$ is the *j*th row and *j*th column diagonal element of $P_{lk}(t \mid t)$. $P_{lk}(t \mid t)$ is computed by **Theorem 3**.

When weighted criteria are weighted by scalars, the weighted coefficients are

$$[\Im_1, \cdots, \Im_L] = \frac{e^{\mathrm{T}} P_{\mathrm{tr(s)}}^{-1}}{e^{\mathrm{T}} P_{\mathrm{tr(s)}}^{-1} e} \tag{53}$$

where $P_{\mathrm{tr}} = (\mathrm{tr}\, P_{ij})_{K \times K}$ is a $L \times L$ matrix.

The optimal fused variance matrix is given as

$$P_0(N) = \sum_{i,j=1}^{L} \Im_i \Im_j P_{ij}(N) \ \text{ and } \ \mathrm{tr}\, P_0 \le \mathrm{tr}\, P_j, \quad j = 1,2,\cdots,K \tag{54}$$

Proof: The optimal information fusion weighted formula (48) ～ (54) is obtained by literature [14]. The proof is completed.

Lemma 5 For system (1) and (2), under the same conditions, when the variance of P_1 and P_2 are known, but the cross covariance P_{12} is unknown, using the covariance intersection (CI) fusion method, this paper proposes a suboptimal fusion Kalman estimators is as follows [16]:

$$\hat{w}_{CI}(t \mid t) = \sum_{i=1}^{L} \aleph_i(t) \hat{w}_i(t \mid t) \tag{55}$$

Fusion weight is calculated as follows

$$\aleph_i(t) = \hbar_i(t) (\sum_{i=1}^{L} \hbar_i(t) P_i^{-1}(t \mid t))^{-1} P_i^{-1}(t \mid t) \tag{56}$$

where

$$\hbar_i(t) = \frac{\mathrm{tr}(P_i^{-1}(t \mid t))}{\sum_{i=1}^{L} \mathrm{tr}(P_i^{-1}(t \mid t))}, \ 0 \le \hbar_i(t) \le 1, \ \sum_{i=1}^{L} \hbar_i(t) = 1 \tag{57}$$

It is proved in document [14] that the accuracy of above four kinds of weighted fusion estimator from high to low is weighted by matrix, diagonal matrices, scalars and covariance intersection fusion. But the computational burden is on the contrary, fusion estimator weighted by matrix has a large computational burden. And covariance intersection fusion avoids solving cross-covariance matrices and has the minimal computational burden, and it is suitable for real-time applications.

5. Simulation Example

Consider 2-sensor tracking system

$$\begin{gathered} (I_2 + A_1 q^{-1}) s(t) = B_1 q^{-1} w(t) \\ y_i(t) = s(t) + v_i(t), i = 1,2,3 \\ (I_2 + P_1^{(i)} q^{-1}) v_i(t) = (I_2 + R_1^{(i)} q^{-1}) \xi_i(t), i = 1,2,3 \end{gathered} \tag{58}$$

where $T = 0.25$ is the sampled period, $A_1 = -\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}$, $s(t) = [s_1(t) \quad s_2(t)]^{\mathrm{T}}$ is the signal, $y_i(t)$ is the measurement of the ith subsystem, $y_i(t)$ is the measurement noise of the ith subsystem.

And $w(t)$ and $\xi_i(t)$ are assumed to be independent Gaussian white noises with zero mean and variances are $Q_w = 0.3$ and $Q_{\xi i}$ individually, $Q_{\xi i} = \begin{bmatrix} \sigma^2_{\xi i1} & 0 \\ 0 & \sigma^2_{\xi i2} \end{bmatrix}$, $i = 1,2,3$.

The problem is to find the local optimal signal Wiener smooth $\hat{s}_i(t \mid t+1)$ and information fusion optimal signal Wiener smooth $\hat{s}_0(t \mid t+1)$.

The parameters of the model in the simulation is taken for

$$Q_{\xi 1} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.04 \end{bmatrix},\ P_1^{(2)} = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.1 \end{bmatrix},\ R_1^{(1)} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$Q_{\xi 2} = 3 * Q_{\xi 1},\ P_1^{(2)} = \begin{bmatrix} 0.13 & 0 \\ 0 & 0.1 \end{bmatrix},\ R_1^{(2)} = \begin{bmatrix} 0.14 & 0 \\ 0 & 0.12 \end{bmatrix}. \tag{59}$$

The simulation results are shown in Figure 1-Figure10. Figure 1 to 8 gives the fusion smooth weighted by matrix, diagonal matrices, scalars and covariance intersection fusion. Figure 9 and Figure 10 are the curves of sum of the absolute error for the state smooth and fusion states of position and velocity weighted by matrix, diagonal matrices, scalars and covariance intersection fusion. In the figure, the accuracy of the fusion signal estimator is higher than any of the single sensor. Simulation results show no significant difference between the three kinds of distributed fusion algorithm (matrix, diagonal matrices, scalars), while the accuracy of covariance intersection fusion is lower than the other three fusion algorithm. But the covariance intersection fusion weighting fusion estimator can significantly reduce the computational burden, and provides a fast information fusion estimation algorithm.

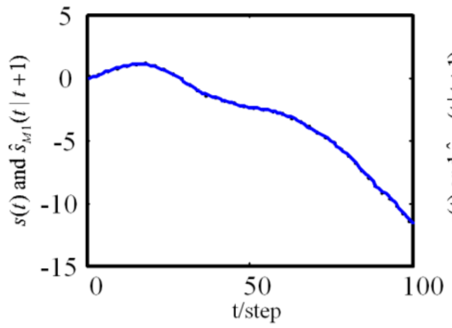


Figure 1. The Position and Fusion Signal Smooth Weighted by Matrix

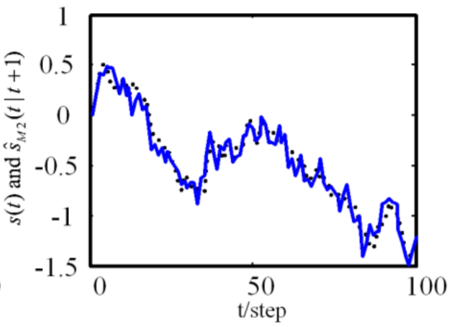


Figure 2. The Velocity and Fusion Signal Smooth Weighted by Matrix

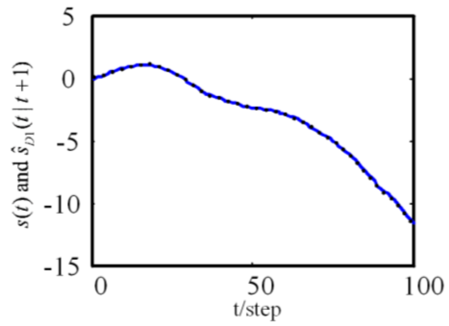


Figure 3. The Position and Fusion Signal Smooth Weighted by Diagonal Matrices

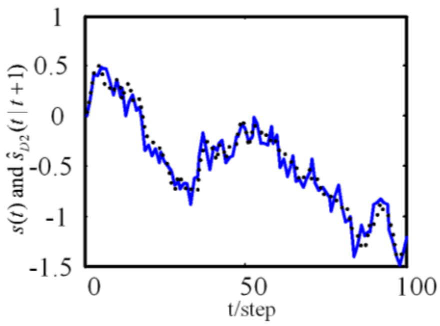


Figure 4. The Velocity and Fusion Signal Smooth Weighted buy Diagonal Matrices

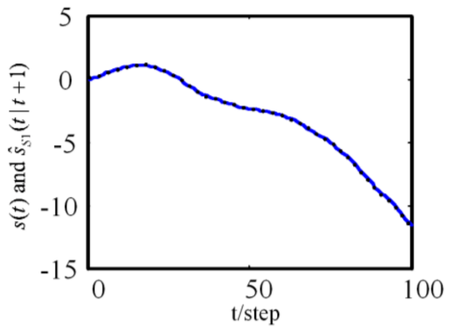


Figure 5. The Position and Fusion Signal Smooth Weigghted by Scalars

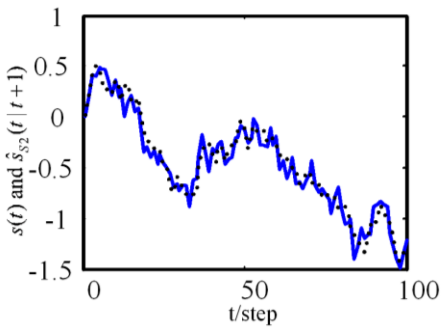


Figure 6. The Velocity and Fusion Signal Smooth Weighted by Scalars

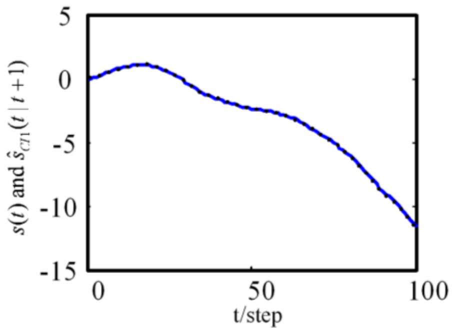


Figure 7. The Position and Fusion Signal Smooth Weighted by ci Fusion

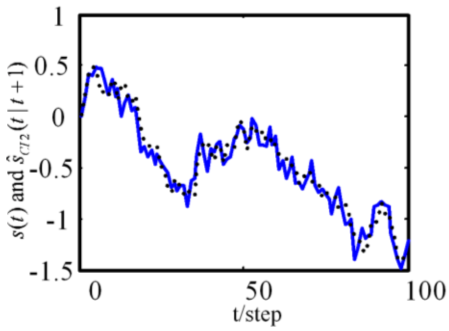


Figure 8. The Velocity and Fusion Signal Smooth Weighted by ci Fusion

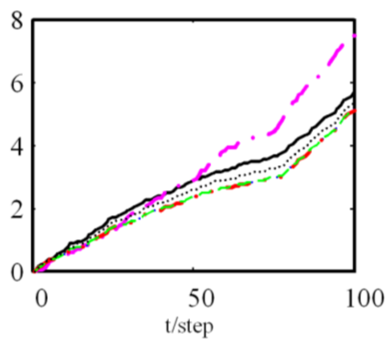


Figure 9. The Curves of the Sum of Absolute Error for Local and Fusion Filters of the Position

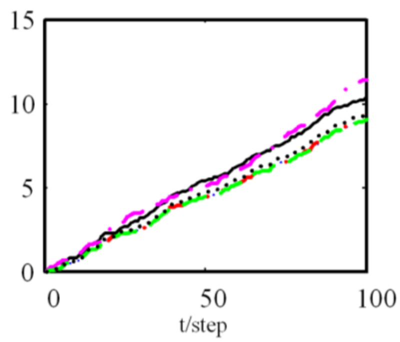


Figure 10. The Curves of the Sum of Absolute Error for Local and Fusion Filters of the Velocity

6. Conclusions

In this paper, the multisensor multichannel ARMA signal distributed information fusion Wiener estimator is obtained by using the modern time series analysis method, and it can handle the fusion filtering, smoothing and prediction problems uniformity. The algorithm presented in this paper has advantages as below:

(1) Distributed information fusion rule, which adopted in this paper, is weighted by matrix, diagonal matrices, scalars, covariance intersection fusion. The estimation accuracy for the system is greatly improved compared with the single local sensor. For matrix, diagonal matrices, scalars, covariance intersection fusion, the accuracy of above four kinds of weighted fusion estimator is from high to low. But the computational burden is on the contrary. Covariance intersection fusion has minimal computational burden because of avoiding computing the cross covariance matrix. Therefore, it is suitable for real time applications from the viewpoint of engineering application.

(2) The advantages of this method can design self-tuning information fusion Wiener deconvolution estimator with unknown model parameters and noise variance based on the online identification of ARMA innovation model.

(3) In this paper, the proposed algorithm can be applied to signal processing in oil seismic exploration, communication and other fields.

Acknowledgments

This work is supported by Key Laboratory of Electronics Engineering, College of Heilongjiang Province, (Heilongjiang University), P. R. China, by science and technology research foundation of Heilongjiang education department under Grant 12531159, by the Natural Science Foundation of Heilongjiang Province of China (Grant Nos. F201426), by Harbin University of Commerce Youth Fund.

*Corresponding author: Gang Hao.

References

[1] S. L. Sun, L. F. Liu. Optimal linear estimation for ARMA signals with stochastic multiple packet dropouts. Control and Decision, vol. 28, no. 2, (**2013**). pp. 223-228.

[2] O. Straka, J. Duník and M. Šimandl, "Truncation nonlinear filters for state estimation with nonlinear inequality constraints. Automatica, vol. 48, no. 27, (**2012**), pp. 273-286.

[3] Y. H. Yang, M. Y. Fu and H. S. Zhang, "State Estimation Subject to Random Communication Delays. Acta Automatica Sinica, vol. 39, no. 3, (**2013**). pp. 237-243.

[4] M. Oussalah, Z. Messaoudi and A. Ouldali, "Track-to-Track Measurement Fusion Architectures and Correlation Analysis", Journal of Universal Computer Science, vol. 16, no. 1, (**2010**), pp. 37-61.

[5] P. J. Dua, S. C. Liu, J. S. Xia and Y. D. Zhao, "Information fusion techniques for change detection from multi-temporal remote sensing images", Information Fusion, vol. 14, no. 1, (**2013**), pp. 19-27.

[6] B. Khaleghi, A. Khamis, F. O. Karray and S. N. Razavi, "Multisensor data fusion: A review of the state-of-the-art", Information Fusion, vol. 14, no. 1, (**2013**), pp. 28-44.

[7] P. Cui, H. S. Zhang, H. X. Wang and J. E. Feng, "Globally optimal real-time distributed fusion of multi-channel observation systems", Control Theory & Applications, vol. 8, no. 6, (**2014**), pp. 384 – 388.

[8] M. B. Trawicki and M. T. Johnson, "Distributed multichannel speech enhancement with minimum mean-square error short-time spectral amplitude, log-spectral amplitude, and spectral phase estimation", Signal Processing, vol. 92, no. 2, (**2012**), pp. 345-356.

[9] C. J. Ran and Z. L. Deng, "Self-tuning distributed measurement fusion Kalman estimator for the multi-channel ARMA signal", Signal Processing, vol. 91, no. 8, (**2011**), pp. 2028-2041.

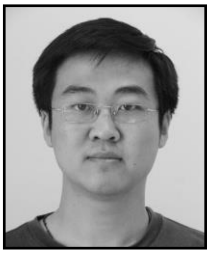
[10] J. X. Feng, Z. D. Wang and M. Zeng, "Distributed weighted robust Kalman filter fusion for uncertain systems with auto correlated and cross-correlated noises", Information Fusion, vol. 14, no. 1, (**2013**). pp. 78-86.

[11] Z. L. Deng, P. Zhang, W. J. Qi, J. F. Liu and Y. Gao, "Sequential covariance intersection fusion Kalman filter", Information Sciences, vol. 189, no. 15, (**2012**), pp. 293-309.
[12] W. J. Qi, P. Zhang and Z. L. Deng, "Two–level robust sequential covariance intersection fusion Kalman predictors over clustering sensor networks with uncertain noise variances", International Journal of Sensor Networks, vol. 14, no. 4, (**2013**). pp. 251-261.
[13] W. J. Qi, P. Zhang and Z. L. Deng, "Covariance Intersection Fusion Kalman Estimators for Multi-Sensor System with Colored Measurement Noises", Research Journal of Applied Sciences, Engineering and Technology, vol. 6, no. 10, (**2013**), pp. 1872-1878.
[14] Z. L. Deng, "Information Fusion Estimation Theory and its Application", Harbin, Science Press, (**2012**).
[15] S. L. Sun and Z. L. Deng, "Multi-sensor optimal information fusion Kalman filter", Automatica, vol. 40, no. 6, (**2004**), pp. 1017-1023.
[16] L. J. Chen, P. O. Arambel and R. K. Mehra, "Estimation under Unknown Correlation: Covariance Intersection Revisited", IEEE Transactions on Automatic Control, vol. 47, no. 11, (**2002**), pp. 1879-1882.

Authors

Yun Li, she is associate professor at Harbin University of Commerce now. She obtained her bachelor's degree and master's degree in Heilongjiang University. Her major researches are state estimation, information fusion, etc.

Ming Zhao, she is associate professor at Harbin University of Commerce now. She obtained her bachelor's degree and master's degree in Harbin Engineering University. Her major researches are pattern recognition, information fusion, etc.



Gang Hao, he is associate professor at Heilongjiang University now. He obtained his bachelor's degree and master's degree in Heilongjiang University, and obtained his Ph.D. in Harbin Engineering University. His major researches are state estimation, information fusion, etc.

