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UAV Flyable Trajectory Generation and Its Tracking Control

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Abstract

For the complex combat environment, considering the flyable performance constrains and the safety constrains, an online trajectory planning algorithm of UAVs(Unmanned Aerial Vehicles) based on Pythagorean Hodograph (PH) curve is put forward, which is easy to traced. When the unexpected obstacles are detected, based on the velocity obstacles avoidance method, the collision avoidance trajectory is planned online. The position of the interrupt point and the direction of the UAV turn are calculated. To fully use the PH trajectories performance that the curvature of the curve is continuous and precisely known, the location and crossing heading error equations were established within the Serret-Frenet coordinates. The corresponding asymptotically stable convergence backstepping trajectory tracking control law was designed based on the error formulations. The autonomous collision avoidance trajectory simulation result and its tracking control result show that the proposed trajectory generation method and the corresponding adaptive tracking control method are effective and can improve the tracking accuracy.

Keywords: *Unmanned Aerial Vehicle; PH curve, Trajectory Tracking; Backstepping*

1. Introduction

At present, the independent combat ability to adapt to the unstructured environment of Unmanned Aerial vehicle (UAV) is gradually being strengthened. When UAV encounters unforeseen threats or task changes, UAV must be able to automatically plan new trajectory online, and can track online.

From the perspective of the theory of differential geometry, in order to get a smooth flight trajectory, the first two derivative of the path is at least existed, that is, the curvature of the curve is continuous, the change of path’s curvature is small, and the path is easy to track, which is the really flight path [1]. Literature [2-4] gives the path planning method online of aerial vehicle using the Pythagorean Hodograph (PH) curve, which is based on the position and direction of the UAV. Besides, the curvature of the curve can be calculated in closed form and can be calculated precisely.

Unmanned Aerial vehicles (UAVs) hold good promise for autonomously carrying out various operations in unstructured environment, which should have the ability of autonomous trajectory planning. In the process of obstacle avoidance, if the relative movement trend between the UAVs and the objects can be considered, then better effects of collision avoidance can be got. There have been many attempts to apply kinematic methods to solve the problem of collision detection. In paper [5, 6], the notion of velocity collision cones(CC) is introduced for collision detection and the navigation laws are established based on differential geometry for obstacle avoidance. In this paper, according to the characteristics of PH curve, the principle of velocity collision is used, and the collision avoidance trajectory is generated.

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To fully use the PH trajectories performance that the curvature of the curve is continuous and precisely known, the location and crossing heading error equations were established within the Serret-Frenet coordinates.

To some extent, the traditional path tracking control methods, such as the dynamic inverse method [7,8], input or output feedback linearization method [9] and so on , are depends on the precise system model, and get some limits when these methods are used in UAVs that exist uncertainty. In this paper, backstepping is used to path tracking control. In order to overcome the external disturbance torque and parameter uncertainty of UAV's track control.

2. PH Trajectory Generation

2.1. Problem Presentation

Let the starting pose of the *i*th UAV is $pose_{si}(x_{si}, y_{si}, \theta_{si})$ and the final pose of the same UAV is $pose_{fi}(x_{fi}, y_{fi}, \theta_{fi})$, then the path of the UAV is defined as the parametric curve $r_i(t) = [x(t), y(t)]$ with curvature κ_i connecting the initial and final pose[7]: $Pose_{si}(x_{si}, y_{si}, \theta_{si}) \xrightarrow{r_i(t)} Pose_{fi}(x_{fi}, y_{fi}, \theta_{fi})$.

In this case Pythagorean Hodograph curve in parametric form subjected to the following constraints:

1) $|\kappa_i(t)| \leq \kappa_{\max}$

Where $\kappa_{\max}$ is the maximum curvature attainable by the UAVs.

2) The vehicles must keep safe distance to avoid inter collision while tracing their corresponding trajectories. This can be written in mathematical form as follow:

$R_{si} \cap R_{sj} = \varphi$

That is, the intersection of the safety circles of radius R_{si} , R_{sj} corresponding to *i*th and *j*th UAVs must be empty.

3) The vehicles must keep safe distance to avoid collisions with known obstacles while tracing their corresponding trajectories. This can be written in mathematical form as follow:

$d(obstacle, UAV) \geq R_{obstacle} + R_s$

Where $d(obstacle, UAV)$ is the distance between the centre of the circle enclosing the obstacle and the centre of the safety circle of the UAV. $R_{obstacle}$ is the radius of the circle enclosing the square obstacle.

2.2. Collision Avoidance Trajectories Generation Using PH Curve

Supposing the path is

$$s(t) = \int_{t_1}^{t_2} |r'(t)| dt = \int_{t_1}^{t_2} \sqrt{x'(t)^2 + y'(t)^2}\, dt \tag{1}$$

Selecting appropriate $\sigma(t)$ to satisfy

$$\sigma^2(t) = x'^2(t) + y'^2(t)$$

Then the length of the path is

$$s(t) = \int_{t_0}^{t_1} |\sigma(t)| dt$$

We can produce PH curve by selecting appropriate $u(t), v(t), w(t)$ to construct $\dot{x}(t), \dot{y}(t)$ and satisfy equation(1). Therefore

$$x'(t) = w(t)\left[u^2(t) - v^2(t)\right]$$

$$y'(t) = 2w(t)u(t)v(t)$$

$$\sqrt{x'^2(t) + y'^2(t)} = \sqrt{\left[w(t)\left\{u^2(t) + v^2(t)\right\}\right]^2}$$

$$\sigma(t) = w(t)\left[u^2(t) + v^2(t)\right]$$

Where $w(t) = 1$. The Bernstein form of $u(t), v(t)$ is

$$u(t) = \sum_{k=0}^{2} u_k \binom{2}{k} t^k (1-t)^{2-k}, t \in [0\ 1]$$

$$u(t) = u_0(1-t)^2 + 2u_1(1-t)t + u_2 t^2$$

$$v(t) = \sum_{k=0}^{2} v_k \binom{2}{k} t^k (1-t)^{2-k}, t \in [0\ 1]$$

$$v(t) = v_0(1-t)^2 + 2v_1(1-t)t + v_2 t^2$$

Considering the numeric stability, PH curve can be give by Bézier curve. The curve is

$$r(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \sum_{k=0}^{5} P_k \binom{5}{k} (1-t)^{5-k} t^k \quad (2)$$

Where $P_k = (x_k, y_k)$, k=0, 1, 2, 3, 4, 5 are control points, which have the following relation:

$$P_1 = P_0 + \frac{1}{5}(u_0^2 - v_0^2, 2u_0v_0)$$

$$P_2 = P_1 + \frac{1}{5}(u_0u_1 - v_0v_1, u_0v_1 + u_1v_0)$$

$$P_3 = P_2 + \frac{1}{5}(u_1^2 - v_1^2, 2u_1v_1) + \frac{1}{15}(u_0u_2 - v_0v_2, u_0v_2 + u_2v_0)$$

$$P_4 = P_3 + \frac{1}{5}(u_1u_2 - v_1v_2, u_1v_2 + u_2v_1)$$

$$P_5 = P_4 + \frac{1}{5}(u_2^2 - v_2^2, 2u_2v_2)$$

Giving the initial and final tangent vector length $(\Delta x_{si}, \Delta y_{si})$, $(\Delta x_{fi}, \Delta y_{fi})$, P_1, P_4 can be determinate.

$$r(0) = P_0 = (x_{si}, y_{si})$$

$$\boldsymbol{r}'(0) = 5(\boldsymbol{P}_1 - \boldsymbol{P}_0) = [\Delta \boldsymbol{x}_{si}, \Delta \boldsymbol{y}_{si}]$$

$$r(1) = P_5 = (x_{fi}, y_{fi})$$

$$\boldsymbol{r}'(1) = 5(\boldsymbol{P}_5 - \boldsymbol{P}_4) = [\Delta \boldsymbol{x}_{fi}, \Delta \boldsymbol{y}_{fi}]$$

$$\boldsymbol{P}_1 = \boldsymbol{P}_0 + \frac{1}{5}[\Delta \boldsymbol{x}_{si}, \Delta \boldsymbol{y}_{si}]$$

$$\boldsymbol{P}_4 = \boldsymbol{P}_5 + \frac{1}{5}[\Delta \boldsymbol{x}_{fi}, \Delta \boldsymbol{y}_{fi}]$$

According to paper[10]:

$$\begin{bmatrix} \boldsymbol{u}_0 \\ \boldsymbol{v}_0 \end{bmatrix} = \pm \begin{bmatrix} \sqrt{\dfrac{\sqrt{\Delta \boldsymbol{x}_{si}^2 + \Delta \boldsymbol{y}_{si}^2} + \Delta \boldsymbol{x}_{si}}{2}} \\ \boldsymbol{sign}(5\Delta \boldsymbol{y}_{si})\sqrt{\dfrac{(\sqrt{\Delta \boldsymbol{x}_{si}^2 + \Delta \boldsymbol{y}_{si}^2} - \Delta \boldsymbol{x}_{si})}{2}} \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} \boldsymbol{u}_2 \\ \boldsymbol{v}_2 \end{bmatrix} = \pm \sqrt{\frac{5}{2}} \begin{bmatrix} \sqrt{\sqrt{\Delta \boldsymbol{x}_{fi}^2 + \Delta \boldsymbol{y}_{fi}^2} + \Delta \boldsymbol{x}_{fi}} \\ \boldsymbol{sign}(\Delta \boldsymbol{y}_{fi})\sqrt{\sqrt{\Delta \boldsymbol{x}_{fi}^2 + \Delta \boldsymbol{y}_{fi}^2} - \Delta \boldsymbol{x}_{fi}} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = -\frac{3}{4}\begin{bmatrix} u_0 + u_2 \\ v_0 + v_2 \end{bmatrix} \pm \sqrt{\frac{1}{2}} \begin{bmatrix} c + a \\ sign\ (b)\sqrt{c - a} \end{bmatrix} \quad (5)$$

Where $a = \frac{9}{16}(u_0^2 - v_0^2 + u_2^2 - v_2^2) + \frac{5}{8}(u_0 u_2 - v_0 v_2) + \frac{15}{2}(x_4 - x_1)$, $b = \frac{9}{8}(u_0 v_0 - u_2 v_2) + \frac{5}{8}(u_0 v_2 - u_2 v_0) + \frac{15}{2}(y_4 - y_1)$, $c = \sqrt{a^2 + b^2}$

Therefore P_2, P_3 can be determinate.

From the generation of the PH trajectory, it can be seen that if the start and the final pose are known, the tangent length ε_s, ε_f of the start and the end points can determine the UAV trajectory. The trajectory has continuous curvature. When the obstacle need be avoided, the interrupt point should be given. Through adjusting ε_s, ε_f, the flyable trajectory that satisfy the performance constrains can be given.

2.3. Obstacle Avoidance Trajectory Generation Based on Velocity Obstacles Avoidance Method

When UAV detects the unknown moving threat or obstacle, it will replan the trajectory according to the velocity and the position of the obstacles. On the following, based on the principl**e** of velocity obstacles, combing the characteristic of PH curve, through setting the interrupt path point, the online trajectory planning method is presented.

In the velocity space, find the m obstacles $\{O_1, O_2, \cdots, O_m\}$ that will collide with the UAV. Calculate the collision time $t_{c_k}(k=1,2,\cdots,m)$ that UAV will collide with each obstacle. Set the guidance time for collision avoidance $t=\min(t_{c_1}, t_{c_2}, \cdots, t_{c_m})$.

If collision avoidance is needed, the position of the interrupt point Q of the trajectory should be given. At the same time, according to the movement direction of the obstacle determine the turn direction of the UAV velocity, which should be opposite to the obstacle velocity to avoid the second collision.

Suppose r is the detect distance of UAV sensors, $O_i(x_{oi}, y_{oi})$ is the position coordinate of obstacle O_i, θ_u is the intersection angle contained by the UAV velocity V_u and the axis OX, which is shown in figure 1. θ_{oi} is the intersection angle contained by the obstacle velocity V_{oi} and the axis OX, d_i is the minimum distance between the obstacle O_i and the UAV along the relative velocity direction.

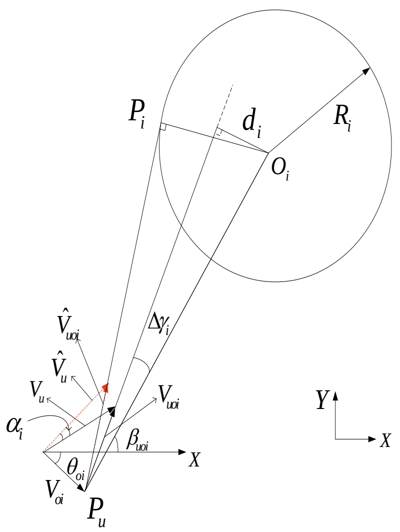

Figure 1. Calculation Sketch Map for UAV Collision Avoidance

(1) Calculation of UAV velocity turn angle α for obstacle avoidance

According to figure 1, determine the turn direction of the UAV velocity V_u. If both sides can avoid the collision, turn V_u to the side with less angle. Calculate the anticlockwise turn angle ψ_i ($i=1,2,\cdots,m$) and clockwise turn angle φ_i for obstacle avoidance.

Suppose $\psi=\max\{\psi_1,\psi_2,\cdots,\psi_m\}$, $\varphi=\max\{\varphi_1,\varphi_2,\cdots,\varphi_m\}$, the turn angle for obstacle avoidance in UAV adjustment time t is

$$\alpha=\min\{\psi,\varphi\}$$

(2) Calculation of interrupt point coordinate $Q(\hat{x}_u, \hat{y}_u)$

After the adjustment time t the interrupt point position $Q(\hat{x}_u, \hat{y}_u)$ should on the direction of the turn angle α. Therefore, UAV can avoid the obstacles and reduce the blindness of the interrupt point selection, which can reduce the calculation complexity. On the following, the detail calculation of the interrupt point is given.

According to the UAV position $P_u(x_u, y_u)$ and the sight angle β_{uoi} between UAV and the obstacle O_i, calculate the position coordinate $\boldsymbol{O}_i(\hat{\boldsymbol{x}}_{oi}, \hat{\boldsymbol{y}}_{oi})$ of every obstacle after the adjustment time t.

So in the opposite direction of the obstacle velocity, the interrupt point position coordinate $Q(\hat{x}_u, \hat{y}_u)$ can be calculated:

$$\begin{bmatrix} \hat{\boldsymbol{x}}_u \\ \hat{\boldsymbol{y}}_u \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{x}}_{oi} \\ \hat{\boldsymbol{y}}_{oi} \end{bmatrix} + \begin{bmatrix} \boldsymbol{R} \times \cos(\pi + \theta_{oi}) \\ \boldsymbol{R} \times \sin(\pi + \theta_{oi}) \end{bmatrix}$$

After the velocity turn angle and the interrupt point coordinate are determined, the new trajectory can be generated between the obstacle detected point and the interrupt point. Then take the interrupt point as the initial point and the object or the next interrupt point as the final point, the next trajectory can also be generated. To keep the continuous of the trajectory, on the detect point and interrupt point, the direction of the tangent line keep unchanged.

3. Establishment of Path Tracking Control Model

Given the UAV target path $\Gamma(s)$, the method of tracking the target path in the horizontal plane is studied. Firstly, three coordinate systems needed to establish, that is, the inertial coordinate system $E-\xi\eta\zeta$, the body coordinate system $B-xyz$ and $Serret-Frenet$ coordinate system, which are shown in Figure 2.

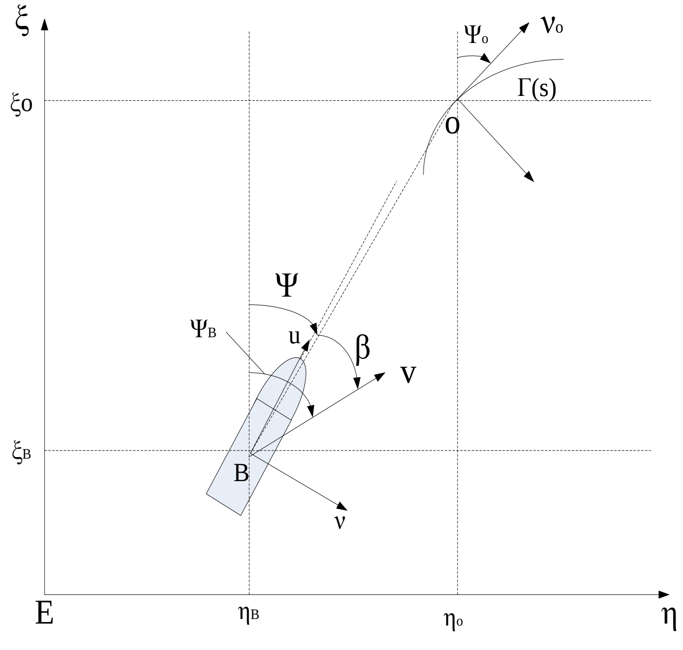

Figure 2. UAV Coordinate Systems

The origin of the inertial coordinate system is a point of space, and the origin of the body coordinate system is the center of gravity of UAV, and the origin of the $Serret-Frenet$ coordinate system is any point on the target path, which axis is composed of the tangential axis, the normal axis, and the secondary normal axis. In the inertial coordinate system, the center of gravity of UAV can be defined as

(ξ_B, η_B) , u and v in figure 2 are defined as UAV's vertical and horizontal velocity components in the body coordinate system. v is the resultant velocity of the two, which size can be defined as $\mathrm{v} = \sqrt{u^2 + v^2}$. ψ is the heading angle and β is the sideslip angle. From the above conditions, the kinematics equation of UAV can be obtained

$$\begin{cases} \dot{\xi}_B = \mathrm{v} \sin \psi_B \\ \dot{\eta}_B = \mathrm{v} \cos \psi_B \\ \dot{\psi} = \omega \end{cases}$$

Where ω is the yaw angular velocity. ψ_B is defined as the angle of the resultant velocity v of UAV and ξ axis of the inertial coordinate system. As a result, the UAV's kinematics equation can be expressed as the following

$$\begin{cases} \dot{\xi}_B = \mathrm{v} \sin \psi_B \\ \dot{\eta}_B = \mathrm{v} \cos \psi_B \\ \dot{\psi}_B = \omega + \dot{\beta} \end{cases} \quad (6)$$

Target path $\Gamma(s)$ is a spatial curve, supposing the arbitrary point o of the target path is the origin of the $Serret-Frenet$ coordinate, which can describe the state of the virtual target's motion state, where o can be regarded as the center of gravity of the virtual target. ψ_o is the angle of v_o and ξ axis of the inertial coordinate system, which can be viewed as virtual target's attitude angle. Therefore, the desired virtual target kinematics equation can be established

$$\begin{cases} \dot{\xi}_0 = \mathrm{v}_0 \sin \psi_0 \\ \dot{\eta}_0 = \mathrm{v}_0 \cos \psi_0 \\ \dot{\psi}_0 = \omega_0 \end{cases} \quad (7)$$

Where ω_o is the derivative of ψ_o. In the inertial coordinate system, the coordinate of the point o is (ξ_o, η_o) , s is the parameter of the target path, the speed of the virtual target is $\mathrm{v}_o = \dot{s}$. When the distance vector of UAV's center of gravity to the virtual target's center of gravity is projected to the $Serret-Frenet$ coordinate system, kinematics equation of the tracking error can be get.

$$x_e = (\xi_o - \xi_B)\cos\psi_o + (\eta_o - \eta_B)\sin\psi_o$$

$$y_e = (\xi_o - \xi_B)\sin\psi_o - (\eta_o - \eta_B)\cos\psi_o$$

$$\psi_e = \psi_o - \psi_B$$

Where ψ_e is called the tracking error of UAV's attitude angle. Derivating both sides of the tracking error kinematics equation and combining the UAV and virtual target kinematics equations, the new error kinematics equations can be get as follows

$$\dot{x}_e = \mathrm{v}_o - y_e\omega_o - \mathrm{v}\cos\psi_e$$

$$\dot{y}_e = x_e\omega_o - \mathrm{v}\sin\psi_e$$

$$\dot{\psi}_e = \omega_o - \omega - \dot{\beta}$$

Suppose the curvature of the path is $c(s)$, which can be defined as

$$c(s) = \frac{\partial \psi_o}{\partial s}$$

We obtain

$$\omega_o = c(s)\dot{s}$$

The error kinematic equations can be deduced as follows

$$\dot{x}_e = \dot{s} - y_e c(s)\dot{s} - \mathrm{v}\cos\psi_e$$

$$\dot{y}_e = x_e c(s)\dot{s} - \mathrm{v}\sin\psi_e$$

$$\dot{\psi}_e = c(s)\dot{s} - \omega - \dot{\beta}$$

According to the error kinematic equations of UAV, supposing the UAV's longitudinal velocity is constant, that is, $u = u_d$, design the control law ω to make the tracking error x_e, y_e and the attitude angle error ψ_e tend to 0 in a limited time, in order to track the target path.

4. Tracking Control Based on Backstepping

Firstly, define the expected value of attitude angle error is ψ_d, which is the expectation value of ψ_e when UAV tracks the target path. Under the appropriate control law, ψ_d should meet the condition $\lim_{t\to\infty}\psi_d = 0$. Therefore, define $\tilde{\psi} = \psi_e - \psi_d$ and supposing

$$\begin{cases} x_1 = x_e \\ x_2 = y_e \\ x_3 = \psi_e \\ u = \omega \end{cases}$$

At the same time

$$\begin{aligned} \sin\psi_e &= \sin(\psi_d + \tilde{\psi}) \\ &= \sin\psi_d + \tilde{\psi}\int_0^1 \cos(\psi_d + \tau\tilde{\psi})d\tau \\ &= \sin\psi_d + \tilde{\psi}\lambda = \sin\psi_d + \lambda\psi_e - \lambda\psi_d \end{aligned}$$

$$\lambda = \int_0^1 \cos(\psi_d + \tau\tilde{\psi})d\tau \tag{8}$$

From the above equation, we know that $\lambda > 0$, so the error kinematics equation can be expressed as

$$\dot{x}_1 = \dot{s} - x_2 c\dot{s} - \mathrm{v}\cos x_3 \tag{9}$$

$$\dot{x}_2 = x_1 c\dot{s} - \mathrm{v}\sin\psi_d - \mathrm{v}(\sin\psi_d + \lambda x_3 - \lambda\psi_d) \tag{10}$$

$$\dot{x}_3 = c\dot{s} - u - \dot{\beta} \tag{11}$$

Based on backstepping design method, define $z_1 = x_1^2 + x_2^2$. Firstly, choosing the first $Lyapunov$ function

$$V_1 = \frac{1}{4}z_1^2 \tag{12}$$

We get

$$\begin{aligned} \dot{V}_1 = z_1\dot{z}_1 &= z_1[x_1(\dot{s} - x_2 c\dot{s} - \mathrm{v}\cos x_3) + x_2(x_1 c\dot{s} - \mathrm{v}(\sin\psi_d + \lambda x_3 - \lambda\psi_d))] \\ &= z_1(x_1\dot{s} - \mathrm{v}x_1\cos x_3 - \mathrm{v}x_2\sin\psi_d - \mathrm{v}\lambda x_2 x_3 + \mathrm{v}\lambda x_2\psi_d) \end{aligned} \tag{13}$$

Taking ψ_d as a virtual control input, and setting virtual error $z_2 = x_3 - \psi_d$, then

$$\psi_d = \frac{1}{\mathrm{v}\lambda x_2}(-x_1\dot{s} + \mathrm{v}x_1\cos x_3 + \mathrm{v}x_2\sin\psi_d + \mathrm{v}\lambda x_2 x_3 - \lambda_1 z_1 + z_2) \tag{14}$$

Substituting the equation (14) into the equation (13), we get

$$\dot{V}_1 = -\lambda_1 z_1^2 + z_1 z_2 \tag{15}$$

Where $\lambda_1 > 0$ is a constant, when $z_2 = 0$,$\dot{V}_1 \le 0$ can be get.

Select the second $Lyapunov$ function:

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{16}$$

So

$$\dot{V}_2 = \dot{V}_1 + z_2 \dot{z}_2 \tag{17}$$

From $z_2 = x_3 - \psi_d$, we get

$$\dot{z}_2 = \dot{x}_3 - \dot{\psi}_d = c\dot{s} - u - \dot{\beta} - \dot{\psi}_d \tag{18}$$

Substituting the equation (18) into the equation (17),we get

$$\dot{V}_2 = -\lambda_1 z_1^2 + z_1 z_2 + z_2 (c\dot{s} - u - \dot{\beta} - \dot{\psi}_d) \tag{19}$$

Design the actual control law

$$u = \omega = c\dot{s} - \dot{\beta} - \dot{\psi}_d + z_1 + \lambda_2 z_2 \tag{20}$$

Substituting the equation (20) into the equation (19), we get

$$\dot{V}_2 = -\lambda_1 z_1^2 - \lambda_2 z_2^2 \le 0 \tag{21}$$

According to the lemma of $Barbala$, $\lim_{t \to \infty} z_2 = 0$, that is to say, when $t \to \infty$, $\psi_e \to \psi_d$. At the same time, ψ_d meets the condition $\lim_{t \to \infty} \psi_d = 0$, so $\psi_e \to 0$. Under the control law (20), $\lim_{t \to \infty} z_1 = 0$ can be get, that is to say, when $t \to \infty$, $x_e^2 + y_e^2 \to 0$, that is $x_e \to 0$, $y_e \to 0$. Therefore, under the control law (20), UAV can effectively track the target path and target attitude angle, and the whole system is global asymptotic stable.

5. Simulation Results

Suppose the start pose P_s is $\left(0, 0, \frac{\pi}{9}\right)$ and the final pose P_f is $\left(200, 500, \frac{17}{18}\pi\right)$, the farthest distance is 100, the average velocity of the UAV is 50. The detect distance is 100. Suppose the radius of the obstacle is 20, its velocity is $v_o = 40$, and the angle to the X axis is $\theta_o = 6.6^\circ$.

According to the velocity obstacle avoidance principle, the UAV will collide with the obstacle. Calculate the interrupt point is $(215, 200)$ and the turn angle anticlockwise is $\alpha = 76.4^\circ$. UAV collision avoidance trajectory is generated, which is shown in Figure 3. When tracking the trajectory using backstepping, taking control parameters $\lambda_1 = 0.01, \lambda_2 = 17$, From Figure 4 to Figure 8 the detailed simulation results are shown. The tracking error is small and the trajectory is tracked perfectly, which show the validity of the tracking control based on backstepping.

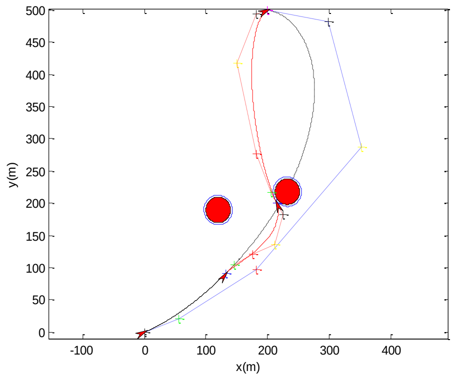

Figure 3. UAV Planning Trajectory Based on PH Curve

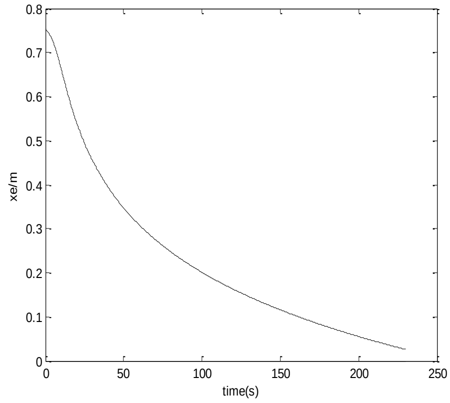

Figure 4. x_e Error Curve

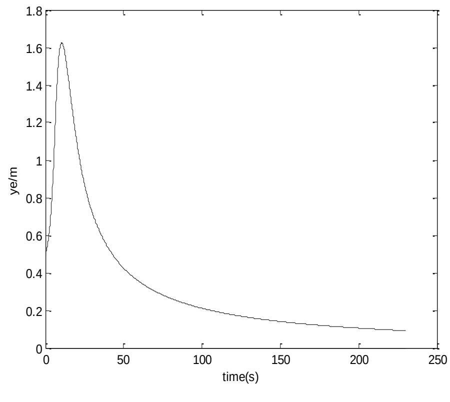

Figure 5. y_e Error Curve

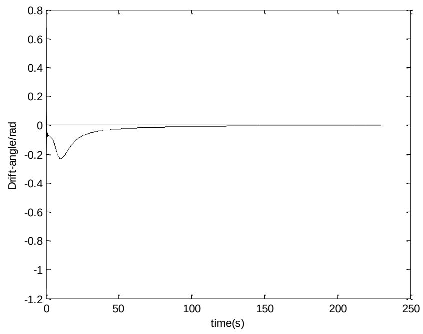

Figure 6. Drifft-angle Error Curve

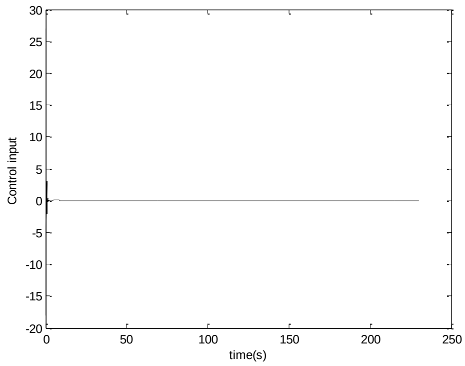

Figure 7. Control Input Curve

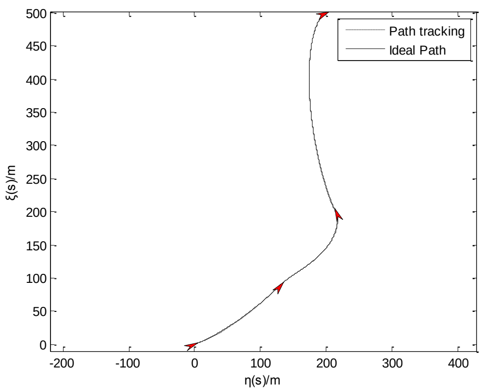

Figure 8. Chart of Path Tracking

6. Conclusion

Pythagorean Hodograph (PH) curve is used to plan the UAV trajectory, which is flyable and satisfy that satisfy the kinematics and dynamics constrains. Based on the principle of velocity collision avoidance, the safe trajectory is replaned. To fully use the performance of PH curve with precise and continuous curvature, the location and crossing heading error equations were established within the Serret-Frenet coordinates. Backstepping control is used to complete the trajectory tracking to overcome the external disturbance torque and parameter uncertainty. Simulation results show that the proposed trajectory generation method and the corresponding adaptive tracking control method are effective and can improve the tracking accuracy.

Acknowledgement

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