

Research on the Cooperative Control of a Group of Autonomous Underactuated Vessels

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Abstract

This paper considers the cooperative control of a group of autonomous underactuated vessels (AUV). The control objective for each vessel is to maintain its position in the formation while Formation Reference Point (FRP) tracks a predefined spatial path. In order to achieve this goal, we use vectorial backstepping to solve two subproblems: one is geometric task, another is dynamic task. The former guarantees that the FRP tracks the spatial path, while the latter ensures accurate speed control along the path. A dynamic guidance system with feedback from the state of all AUV ensures that they have the same priority (no leader) when moving along the path. The controller is proposed basically based on Lyapunov direct method and backstepping technique. Simulation results are presented and discussed.

Keywords: *AUV, FRP, cooperative control, backstepping technique*

1. Introduction

Nowadays, the researchers pay more attention to Control of underactuated systems than before, due to its intrinsic non-linear nature and practical applications, such as rescue mission, large object moving, troop hunting, formation control and satellite clustering. Since the controlling of autonomous underactuated vessels (AUV) is one typical example of underactuated systems, we mainly discuss the cooperative control of a group of autonomous underactuated vessels in this paper.

We can find extensively studied about surface vessel stabilization problem in reference [1–6], and tracking control problem in [7–10]. We also can find some results about designing cooperative control laws for a group of surface vessels in a lot of references. Such as, In [11], Dynamic positioning (DP) control of 3-DOF surface vessels was considered. Under the assumption that the kinematic equations, by applying the backstepping design methodology and gain scheduling techniques, the globally exponentially stable (GES) nonlinear control law was proposed. In [12], decentralized formation control schemes for a fleet of vessels with a small amount of inter vessel communication are proposed and investigated. An individual maneuvering problem is solved for each vessel, with an extension of a synchronization feedback function in the dynamic control laws to ensure that the vessels stay assembled in the desired formation. In [13], the problems of steering a group of vehicles along given spatial paths while holding a desired time-varying geometrical formation pattern were discussed. The solution to this problem, henceforth referred to as the Coordinated Path-Following problem, unfolds in two basic steps. First, a path-following control law is designed to drive each vehicle to its assigned path, with a nominal speed profile that may be path dependent. In the second step, the speeds of the virtual targets (also called coordination states) are adjusted about their nominal values so as to synchronize their positions and achieve, indirectly, vehicle coordination.

The control laws for coordination of a group of AUV based on the virtual structure approach applied by [14] to a formation control of a mobile robot, which were particularly proposed in this paper. The technique in [15] is modified to design a global state feedback controller for each AUV to ensure that all AUV follow perfectly their prescribed spatial paths. The AUV model is based on new results in nonlinear AUV modeling [16]. Particularly we solving the formation control problem this paper in two aspects: the kinematic task and the dynamic task. The former ensures that the individual AUV converge to its positions in the formation and stay at its spatial paths which generally called in literature the spatial path following. The latter will ensure that the AUVs will move along the spatial path with the appointed speed. Therefore, the controller is designed in such a way that the derivative of the spatial path parameter is left as an additional control input to synchronize the formation motion.

The rest of the paper is organized as follows. In Section 2, the problem under study is stated. In Section 3, cooperative control laws are proposed for different communication scenarios and several related problems are studied. In Section 4, simulation results are presented. The last section concludes this paper.

2. Problem Statement

Given a desired geometric pattern P defined by constant vectors $l_i = [l_{ix}, l_{iy}, l_{iz}]^T$ ($1 \leq i \leq n$), we consider the following the format control problem.

Design a cooperative controller for vessel i based on the desired trajectory, its own state and its neighbor's states such that:

$$\lim_{t \rightarrow \infty} (\eta_{li} - \eta_{lid}) = 0 \quad (1)$$

$$\lim_{t \rightarrow \infty} (\eta_{li} - \eta_{lj} - \Delta_{ij}) = 0 \quad (2)$$

$$\lim_{t \rightarrow \infty} (v_{li} - v_{lid}) = 0 \quad (3)$$

Where $i, j = 1, 2, \dots, n$, $\Delta_{ij} = l_i - l_j$, $\eta_{lid} = \eta_{ld} + l_i$, $v_{lid} = R^T(\eta_{ld})\dot{\eta}_{ld} + \delta_{li}$.

The control goals (1) and (2) are individual trajectory tracking control goals for every vessel i to asymptotically converge to the trajectory parallel to the formation reference point. The control goal (3) is synchronization control goals stating that all vessels should synchronize along the paths by converging to the desired inter-vessel distances specified by parameters Δ_{ij} , $i, j = 1, 2, \dots, n$, and move along the paths with desired formation speed v_{lid} .

In this paper, we consider a group of n underactuated autonomous underwater vehicle (AUV). The equations of motion for a i 'th vessel described by 6-DOF models as follows:

$$\dot{\eta}_{li} = R(\eta_{2i})v_{li} \quad (4)$$

$$\dot{\eta}_{2i} = T(\eta_{2i})v_{2i} \quad (5)$$

$$M_{1i}\dot{v}_{li} = B_{1i}\tau_{ui} - S_i(v_{2i})M_{1i}v_{li} - D_{1i}v_{li} \quad (6)$$

$$M_{2i}\dot{v}_{2i} = -S_i(v_{li})M_{1i}v_{li} - S_i(v_{2i})M_{2i}v_{2i} - D_{2i}(v_{2i})v_{2i} + g_{2i}(\eta_{2i}) + B_{2i}\tau_{2i} \quad (7)$$

Where, the subscript i denotes the i 'th vessel. $\eta_{li} = [x_i, y_i, z_i]^T$ is the position of the vehicle in the inertial coordinate frame, $\eta_{2i} = [\phi_i, \theta_i, \psi_i]^T$ is the Euler-angle representation of the orientation of the body-fixed coordinated frame with respect to inertial frame,

where ϕ_i is the roll angle, θ_i is the pitch angle and ψ_i is the yaw angle. $v_{li} = [u_i, v_i, w_i]^T$ is the linear velocity of the vehicle in the body-fixed frame, where u_i , v_i and w_i are the surge, sway and heave velocities, respectively. $v_{2i} = [p_i, q_i, r_i]^T$ is the angular velocity of the vehicle in the body frame, where p_i , q_i and r_i are the roll, pitch and yaw angular velocities, respectively. τ_{ui} is surge control input, $\tau_{2i} = [\tau_{qi}, \tau_{ri}]^T$ are the control input vector, containing the pitch and heave controls, respectively.

3. Cooperative Control Law Design

In the procedure that follows, a recursive backstepping design is proposed to solve the formation maneuvering problem for n vessels with the dynamics given in (4):

Step 1: Define the error variables for the i 'th AUV:

$$z_{li}(\eta_i, \eta_{2i}, \vartheta) = [x_{ei}, y_{ei}, z_{ei}]^T = R_i^T [\eta_{li}(t) - \eta_{lid}(\vartheta)] \quad (8)$$

$$\dot{z}_{li} = -S(v_{2i})z_{li} + v_{li} - R_i^T \dot{\eta}_{lid}(\vartheta) \quad (9)$$

$$\omega_s(\vartheta) = v_s(\dot{\eta}_{ld}, \vartheta) - \dot{\vartheta} \quad (10)$$

Where α_{li} are virtual controls to be specified later, z_{li} For the i 'th the position error of the AUV in the coordinates of the projection. Differentiating (8) with respect to time result in

$$\dot{z}_{li} = -S(v_{2i})z_{li} + v_{li} - R_i^T \dot{\eta}_{lid}(\vartheta) \quad (11)$$

Where $z_1 = [z_{11}^T, z_{12}^T, \dots, z_{1n}^T]^T$,

$$\text{we used } R^T(\eta_2)R(\eta_2) = I_{3n \times 3n} \quad \text{and} \quad \dot{R}(\eta_2) = R(\eta_2)S(v_2) \quad (12)$$

The first virtual control α_{li} are chosen as

$$\alpha_{li} = -K_{li}z_{li} + R_i^T \dot{\eta}_{lid}(\vartheta) v_s \quad (13)$$

Where $K_{li} = K_{li}^T > 0$, (4) becomes

$$\dot{z}_{li} = -S(v_{2i})z_{li} - K_{li}z_{li} + z_{2i} + R_i^T \dot{\eta}_{lid}(\vartheta) \omega_s \quad (14)$$

Define the first control Lyapunov function (CLF) as:

$$V_1 = \frac{1}{2} z_1^T z_1 \quad (15)$$

And computing its time derivative to obtain:

$$V_1 = \sum_{i=1}^n V_{li} = \sum_{i=1}^n z_{li}^T \dot{z}_{li} = \sum_{i=1}^n [z_{li}^T (-K_{li}z_{li} - S(v_{2i})z_{li} + z_{2i}) + z_{li}^T R_i^T \dot{\eta}_{lid}(\vartheta) \omega_s] \quad (16)$$

Due to skew-symmetry of $S(v_2) = -S(v_2)^T$, giving $z_1^T S(v_2) z_1 = 0$, define the first tuning functions ρ_{1i} :

$$\rho_{1i} = \rho_{1i}(\eta_{1i}, \eta_{2i}, \mathcal{G}) = z_{1i}^T R_i^T \eta_{1id}^g(\mathcal{G}), \quad (17)$$

the result of Step 1 becomes:

$$V_1 = \sum_{i=1}^n \left(-z_{1i}^T K_{1i} z_{1i} + z_{1i}^T z_{2i} + \rho_{1i} \omega_s \right) \quad (18)$$

Leaving the terms containing z_{1i} and ω_s for the next step. To aid the next step, let:

$$\sigma_{1i} = K_{1i} S(v_{2i}) z_{1i} - K_{1i} v_{1i} - S(v_{2i}) R_i^T \eta_{1id}^g(\mathcal{G}) v_s \quad (19)$$

$$\alpha_{1i}^g = K_{1i} R_i^T \eta_{1id}^g(\mathcal{G}) + R_i^T \left[\eta_{1id}^{gg}(\mathcal{G}) v_s + \eta_{1id}^g(\mathcal{G}) v_s^g \right] \quad (20)$$

Step 2: Differentiating (6) with respect to time yields:

$$M_{1i} \dot{z}_{2i} = M_{1i} \dot{v}_{1i} - M_{1i} \dot{\alpha}_{1i} = B_{\delta i} v_{2i} + B_{1i} \tau_{1i} - N_{1i} - M_{1i} \alpha_{1i}^g \dot{\mathcal{G}} \quad (21)$$

Where $B_{\delta i} = (M_{1i}^{-1} B_{1i} - \dot{M}_{1i}^{-1} M_{1i})$, $B_{\delta i} \in \mathbb{R}^{3 \times 3}$, $N_{1i} = M_{1i} \dot{\alpha}_{1i} - \dot{M}_{1i} \alpha_{1i}$

It turns out that it will not always be possible to drive z_{2i} to zero. Instead, we will drive z_{2i} to the constant $\delta_i = [\delta_{1i}, 0, 0]^T$ in Assumption 2. To achieve this we define $z_{2\delta i} = z_{2i} - \delta_i$ as a new error variable that we drive to zeros and the second augmented control-Lyapunov function:

$$V_2 = \sum_{i=1}^n V_{2i} = \sum_{i=1}^n \left(V_{1i} + \frac{1}{2} z_{2\delta i}^T M_{1i} z_{2\delta i} \right) = \sum_{i=1}^n \left(\frac{1}{2} z_{1i}^T z_{1i} + \frac{1}{2} z_{2\delta i}^T M_{1i} z_{2\delta i} \right) \quad (22)$$

Whose time derivative is :

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^n \dot{V}_{2i} = \sum_{i=1}^n \left(\dot{V}_{1i} + z_{2\delta i}^T M_{1i} \dot{z}_{2\delta i} \right) \\ &= \sum_{i=1}^n \left(-z_{1i}^T K_{1i} z_{1i} + z_{1i}^T \delta_i + \rho_{2i} \omega_s + z_{2\delta i}^T \left(B_{\delta i} v_{2i} + B_{1i} \tau_{1i} + z_{1i} - N_{1i} - M_{1i} \alpha_{1i}^g v_s \right) \right) \end{aligned} \quad (23)$$

Where B_i is define in $B_i = [B_{1i} \quad B_{\delta i}]$. The second tuning functions ρ_{2i} as:

$$\rho_{2i} = \rho_{2i}(v_{1i}, \eta_{1i}, \eta_{2i}, \mathcal{G}) = \rho_{1i} + z_{2\delta i}^T M_{1i} \alpha_{1i}^g = \eta_{1di}^g(\mathcal{G})^T R_i z_{1i} + (\alpha_{1i}^g)^T M_{1i} z_{2\delta i} \quad (24)$$

The second virtual control $\alpha_{2i} \in \mathbb{R}^{4 \times 1}$ are chosen as:

$$\alpha_{2i} = \left(\begin{matrix} -z_{1i} \\ z_{2\delta i} \\ \tau_{1i} \\ \tau_{2i} \end{matrix} \right) \quad (25)$$

Where, $K_{2i} = K_{2i}^T > C$. The control law τ_{1i} are chosen as:

$$\tau_{1i} = h_1 \alpha_{2i} \quad (26)$$

Define the third error variable z_{3i} as

$$z_{3i} = z_{3i} = (v_{1i}, v_{2i}, \eta_{1i}, \eta_{2i}, \mathcal{G}) = v_{2i} - h_2 \alpha_{2i}, \quad (27)$$

Where $h_1 = [1 \quad 0_{3 \times 3}]$ and $h_2 = [0_{3 \times 1} \quad I_{3 \times 3}]$ are the projection matrix.

We can now rewrite (21) and (23) as:

$$\begin{aligned} \dot{z}_{3i} &= \left(\begin{matrix} -z_{1i} \\ z_{2\delta i} \\ \tau_{1i} \\ \tau_{2i} \end{matrix} \right) \\ &= \left(\begin{matrix} -z_{1i} \\ z_{2\delta i} \\ h_1 \alpha_{2i} \\ h_2 \alpha_{2i} \end{matrix} \right) \end{aligned} \quad (28)$$

$$\begin{aligned} \dot{V}_3 &= \sum_{i=1}^n \dot{V}_{3i} = \sum_{i=1}^n \left(\dot{V}_{2i} + z_{3i}^T M_{2i} \dot{z}_{3i} \right) \\ &= \sum_{i=1}^n \left(-z_{1i}^T K_{1i} z_{1i} - z_{2\delta i}^T K_{2i} z_{2\delta i} + z_{1i}^T \delta_i + \rho_{3i} \omega_s \right. \\ &\quad \left. + z_{3i}^T \left[B_{\delta i}^T z_{2\delta i} - N_{2i}(\eta, \nu) + B_{2i} \tau_{2i} - M_{2i} h_2 \sigma_{2i} - M_{2i} h_2 \alpha_{2i}^g v_{si} \right] \right) \end{aligned} \quad (29)$$

Step3: Differentiating (29) with respect to time yields:

$$\begin{aligned} \dot{V}_3 &= \sum_{i=1}^n \dot{V}_{3i} \\ &= \sum_{i=1}^n \left(\dot{V}_{2i} + z_{3i}^T M_{2i} \dot{z}_{3i} \right) \\ &= \sum_{i=1}^n \left(-z_{1i}^T K_{1i} z_{1i} - z_{2\delta i}^T K_{2i} z_{2\delta i} + z_{1i}^T \delta_i + \rho_{3i} \omega_s \right. \\ &\quad \left. + z_{3i}^T \left[B_{\delta i}^T z_{2\delta i} - N_{2i}(\eta, \nu) + B_{2i} \tau_{2i} - M_{2i} h_2 \sigma_{2i} - M_{2i} h_2 \alpha_{2i}^g v_{si} \right] \right) \end{aligned} \quad (30)$$

Where τ_{2i} are chosen as: Define the third control Lyapunov function as:

$$V_3 = \sum_{i=1}^n V_{3i} = \sum_{i=1}^n \left(V_{2i} + \frac{1}{2} z_{3i}^T M_{2i} z_{3i} \right) = \sum_{i=1}^n \left(\frac{1}{2} z_{1i}^T z_{1i} + \frac{1}{2} z_{2\delta i}^T M_{1i} z_{2\delta i} + \frac{1}{2} z_{3i}^T M_{2i} z_{3i} \right) \quad (31)$$

Whose time derivative is:

$$\begin{aligned} \dot{V}_3 &= \sum_{i=1}^n \dot{V}_{3i} = \sum_{i=1}^n \left(\dot{V}_{2i} + z_{3i}^T M_{2i} \dot{z}_{3i} \right) \\ &= -z_{1i}^T K_{1i} z_{1i} - z_{2\delta i}^T K_{2i} z_{2\delta i} + z_{1i}^T \delta_i + \rho_{3i} \omega_s \\ &\quad + z_{3i}^T \left[B_{\delta i}^T z_{2\delta i} - N_{2i}(\eta, \nu) + B_{2i} \tau_{2i} - M_{2i} h_2 \sigma_{2i} - M_{2i} h_2 \alpha_{2i}^g v_{si} \right] \end{aligned} \quad (32)$$

Where, the third tuning functions ρ_{3i} as:

$$\begin{aligned} \rho_{3i} &= \left(z_{1i}^T K_{1i} z_{1i} + z_{2\delta i}^T K_{2i} z_{2\delta i} - z_{1i}^T \delta_i - \rho_{3i} \omega_s \right) \\ &= \left(z_{1i}^T K_{1i} z_{1i} + z_{2\delta i}^T K_{2i} z_{2\delta i} - z_{1i}^T \delta_i - \rho_{3i} \omega_s \right) \end{aligned} \quad (33)$$

The control law τ_{2i} are chosen as:

$$\tau_{2i} = \left(B_{2i}^{-1} \left[z_{3i}^T M_{2i} \dot{z}_{3i} + z_{1i}^T \delta_i + \rho_{3i} \omega_s - z_{3i}^T \left(B_{\delta i}^T z_{2\delta i} - N_{2i}(\eta, \nu) - M_{2i} h_2 \sigma_{2i} - M_{2i} h_2 \alpha_{2i}^g v_{si} \right) \right] \right) \quad (34)$$

Where, $K_3 = K_3^T > C$, we get:

$$\dot{V}_3 = \sum_{i=1}^n \dot{V}_{3i} = \sum_{i=1}^n \left(-z_{1i}^T K_{1i} z_{1i} - z_{2\delta i}^T K_{2i} z_{2\delta i} - z_{3i}^T K_{3i} z_{3i} + z_{1i}^T \delta_i + \rho_{3i} \omega_s \right) \quad (35)$$

4. Simulations

This section illustrates the effectiveness of the control law (26,34) by simulating it on an underwater vehicle with a length of 5.56 m, a mass of 1089.8 kg, and other parameters taken as follows:

$$\begin{aligned} M &= \begin{bmatrix} 1089.8 & 0 & 0 \\ 0 & 1089.8 & 0 \\ 0 & 0 & 1089.8 \end{bmatrix}, \quad M_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_{\delta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ N &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad N_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ B_{21} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ h_2 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \alpha_{21}^g = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ v_{s1} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \omega_s = 1 \end{aligned}$$

This vehicle has a maximum surge force of 2 104 N, a maximum yaw moment of 1:5 104 Nm, a maximum pitch moment of 1:5 104 Nm, and the maximum roll moment of 120 Nm. The reference trajectory is chosen as:

$$x_d = 0.5t, \quad y_d = 0.5t, \quad z_d = 0.5t, \quad u_d = 1.$$

The initial conditions are picked as follows:

$$\begin{aligned} \eta(0) &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \nu(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \\ \tau_{21}(0) &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \tau_{22}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \\ \tau_{23}(0) &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \tau_{24}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

Based on Theorem.1, the design constants are chosen as:

$$K_1 = 37 \frac{1}{s}, \quad K_2 = 84.5 \frac{1}{s}, \quad K_3 = 3.7 \frac{1}{s}, \quad \delta = 1.924, \quad \kappa = 0.$$

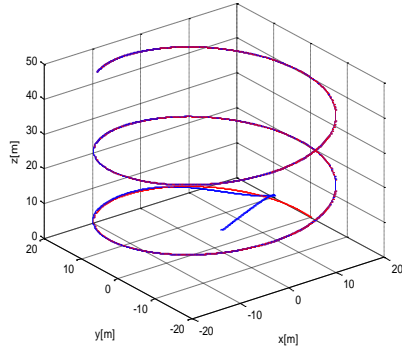


Figure 1. AUV Space Path Tracking

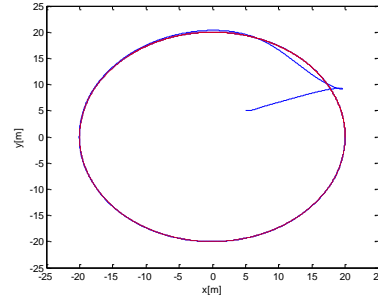


Figure 2. AUV Space Path Tracking Projection in the X-Y Plane

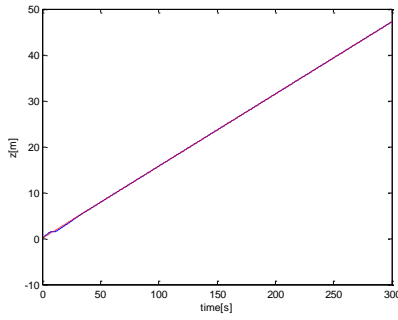


Figure 3. z

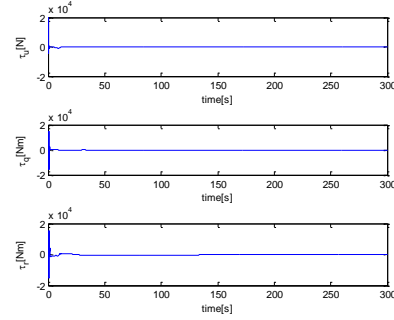


Figure 4. Control Input

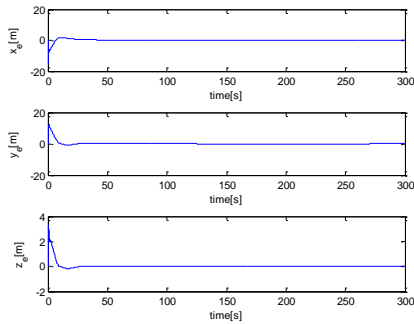


Figure 5. Tracking Error

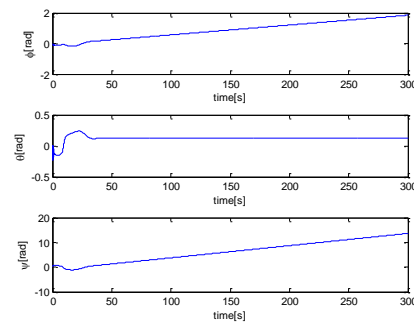


Figure 6. Attitude Angle

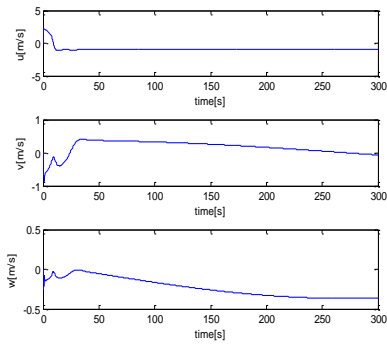


Figure 7. Linear Velocity

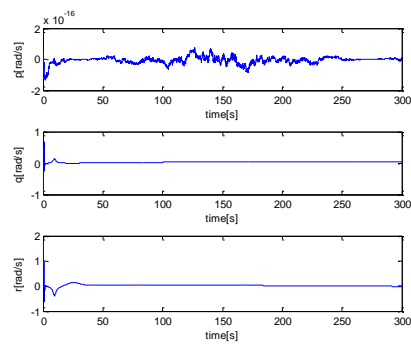


Figure 8. Angular Velocity

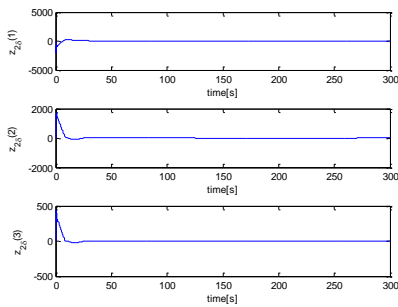


Figure 9. Error Change $z_{2\delta}$

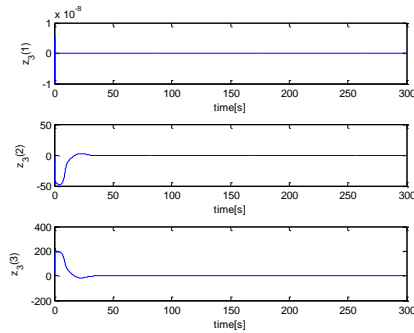


Figure 10. Error Change z_3

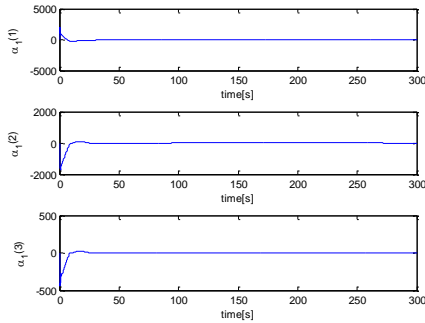


Figure 11. Virtual Control α_1

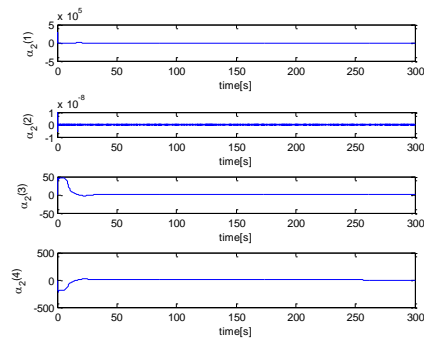


Figure 12. Virtual Control α_2

5. Conclusion

We proposed a solution to the cooperative control of a group of autonomous underactuated vessels (AUV). We illustrated our results in the context of vessel control applications: underwater vessels moving in three-dimensional space. Simulations show that the control objectives were accomplished.

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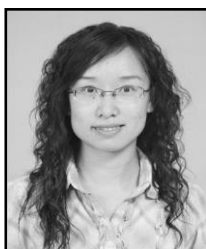
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